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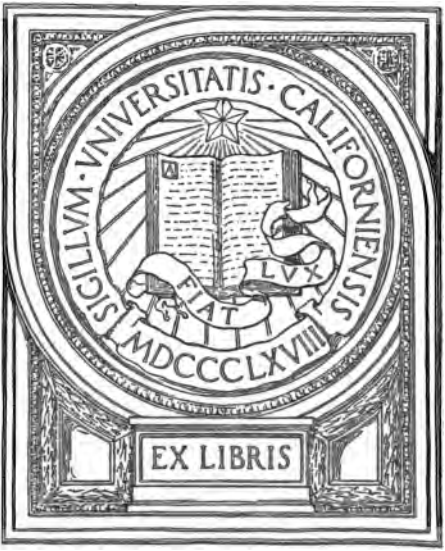
IN MEMORIAM
George Davidson 1825-1911



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Geo Davidson

# A SYNOPSIS

OF

# Practical Philosophy,

ALPHABETICALLY ARRANGED,

CONTAINING

A GREAT VARIETY

OF

THEOREMS, FORMULÆ, AND TABLES,

FROM THE MOST ACCURATE AND RECENT AUTHORITIES,

IN VARIOUS BRANCHES OF

MATHEMATICS AND NATURAL PHILOSOPHY;

TO WHICH ARE SUBJOINED SMALL

TABLES OF LOGARITHMS.

DESIGNED AS A

MANUAL FOR ARCHITECTS, SURVEYORS, ENGINEERS,
STUDENTS, NAVAL OFFICERS, AND OTHER SCIENTIFIC MEN.

BY THE REV. JOHN CARR, M.A.,

LATE FELLOW OF TRINITY COLLEGE, CAMBRIDGE.

SECOND EDITION: 390 PAGES.

LONDON:
JOHN WEALE, 59, HIGH HOLBORN.
1843.

In memoriam

George Davidson

1825-1911

HF

Q121
C3
1843.

TO

JOHN MACNEILL, ESQ.,

CIVIL ENGINEER,

F. R. S., &c.

THE RE-PUBLICATION OF

THIS VERY USEFUL LITTLE WORK

IS INSCRIBED

BY HIS VERY HUMBLE SERVANT,

JOHN WEALE.

DEC. 31, 1842.

M275060

# INTRODUCTION.

This small volume is intended, as its title page imports, partly as a Manual for the scientific man, to aid him in his researches, when, from his distance from home, or other circumstances, he is precluded from having access to more extended and elaborate works; and partly as a convenient appendage to the table of the general reader, for purposes of occasional reference; while to the Student it will supply the place of a syllabus, and furnish him with formulæ for the solution of problems in many useful branches of mixed Mathematics.

With respect to its plan, the reader, on turning to any article, will usually find entered first the Propositions or Formulæ applicable to it, illustrated, if necessary, by examples; to which are appended, such practical results and tables as the subject appeared to require, or the limits of the book to admit of.

The Propositions are very rarely accompanied by proofs; nor is any explanation given of the various terms employed, further than what is necessary to a due understanding of the several symbols introduced. The book professing merely to supply a combination of *facts*, calculated to aid the memory, or exercise the ingenuity, of the reader, any attempt at elementary instruction would have been altogether inconsistent with its scope and principle.

Most of the articles have been compiled and abridged from original sources, as will appear from referring to their several heads, where the names of the respective writers, from whom the extracts have been made, are usually inserted; and particular care has been taken throughout to admit nothing of a practical nature which has not been sanctioned by unexceptionable authorities: at all events, since, in every case which admits not of rigid demonstration, the authority has been most scrupulously quoted, the intelligent reader will at once be able to judge what degree of confidence it is entitled to.

The small Tables of Logarithms will probably be considered a valuable addition: by the help of these, any one, having the proper data, may exhibit arithmetically such formulæ as require a logarithmic computation with sufficient accuracy for all *temporary* purposes.

Some subjects, which, from their practical utility, might seem to claim a place in this Synopsis, have, in cases where long verbal descriptions or an expensive apparatus of plates were necessary for their illustration, been purposely omitted; it having been a leading object in the compilation to confine the volume within such limits as might render it conveniently portable. Other omissions doubtless there are, which may have proceeded from inadvertence, or a want of judgment in the selection; but these last will not, it is hoped, be found very considerable, either in point of number or importance.

To typographical accuracy every possible attention has been paid; without that, a book of this kind would be worse than useless.

# A SYNOPSIS

OF

# *PRACTICAL PHILOSOPHY.*

**ABERRATION** of *Light.—(Woodhouse, Vince.)*

1. If two lines be drawn from the earth, one in the direction of its motion, which will be a tangent to its orbit, and the other through the star, the angle they form is called the ∠ of the *earth's way;* and the aberration will wholly take place in the plane passing through these two lines; which is ∴ called the *plane of aberration.*

2. The greatest effect of aberration = 20″. 232, or in round numbers 20″; and generally the aberration in its own plane = 20″ × sin. of the ∠ of the earth's way.

The velocity of the earth : the velocity of light :: sin. 20″ : rad. :: 1 : 10324.

3. This aberration will affect the apparent position of the stars both in latitude, and longitude; declination, and right ascension. Hence the following Formulæ:—

*Aberration in Latitude.*

Aberrat. in lat. = *o*, when the earth is in syzygy with the star.

In any other position of the earth, aberration in lat. is
20″ × sin. of earth's distance from syzygy × sin. star's lat.

Hence the aberration in lat. is a max. when the earth is in quadratures with the star, and then = 20″ × sin. star's lat.

*Aberration in Longitude.*

Aberrat. in long. = *o*, when the earth is in quadratures with the star.

Aberration of a star [illegible]
Formula p. [illegible]

In any other position of the earth, aberration in longitude is

$$\frac{20'' \times \text{cos. earth's distance from syzygy}}{\text{cos. star's lat.}}$$

Hence aberrat. in long. is a max. when the earth is in syzygy with the star, and $= \dfrac{20''}{\text{cos. star's lat.}}$.

*Aberration in Declination.*

Aberration in declination $= o$, when tang. earth's dist. from syzygy $= \dfrac{\text{tan. position}}{\text{sin. star's lat.}}$.

In any other position of the earth, let $d =$ dist. of the earth, at the time of observation, from the position it had when aberrat. in declin. $= o$, D $=$ earth's dist. from syzygy at the same time, found by the last Article, then aberrat. in declination is

$$\frac{20'' \times \text{sin. } d \times \text{sin. position}}{\text{sin. D}}$$

Hence aberration in declination is a max$^{m}$. when $d = 90^{o}$. and then $= \dfrac{20'' \times \text{sin. position}}{\text{sin. D}}$.

*Aberration in Right Ascension.*

Aberration in right ascension $= o$, when the tang. earth's distance from syzygy $= \dfrac{\text{cotan. position}}{\text{sin. star's lat.}}$.

In any other position of the earth, let $d =$ dist. of the earth, at the time of observation, from the position it had when aberration in right ascension $= o$, D $=$ earth's distance from syzygy at the same time, found by last Art.; then aberrat. in right ascension is

$$20''. \frac{\text{sin. } d \times \text{cos. position}}{\text{cos. decl.} \times \text{sin. D}}.$$

Hence aberrat. in right ascension is a max. when $d = 90^{o}$, and $= 20''. \dfrac{\text{cos. posit.}}{\text{cos. dec.} \times \text{sin. D}}$.

4. The following are the Formulæ given by M. Cagnoli, in his Trigonometry, as being the most convenient for practice, and from which M. de Lambre has computed his Tables on Aberration.—*(See Vince & Playfair.)*

If L be the longitude of the sun at any time, and L′ the longitude of a star, the aberration of the star in lat. is

$$20''.232 \times \text{sin. } (L' - L) \times \text{sin. lat.}$$

And the aberration in longitude is

$$\frac{-20''.232 \times \cos.(L' - L)}{\cos.\ \text{lat.}}.$$

If A be the right ascension, and D the declination of a star, L being the sun's longitude as before, the aberration in declination is

$$\sin. D\left(19''.17 \sin.(A - L) - 0''.83 \sin.(A + L)\right) - 8'' \cos. L \times \cos. D.$$

And the aberration in right ascension is

$$-\frac{19''17 \times \cos.(A - L) - 0''.83 \times \cos.(A + L)}{\cos. D}.$$

From these four last Formulæ all the effects of aberration may be computed.

5. In consequence of the aberration of light, the apparent place of a star will trace out upon a plane parallel to the ecliptic a circle, in which the true place of the star is similar to that of the sun in the circle described on the axis major of the earth's orbit as a diameter.

This circle, projected upon the plane of vision, is an ellipse, the $\frac{1}{2}$ ax. maj. $= 20''232$, and $\frac{1}{2}$ ax. min. $= 20''232 \times$ sin. star's lat. Hence a star in the pole of the ecliptic describes a circle, and a star in the ecliptic a straight line.

6. To make allowance for the aberration of a planet, let T be the instant for which the geocentric place is to be computed, $t =$ time light takes to move from the planet to the earth. Compute its geocentric place by the common rules for the time $T - t$, and it will be its geocentric place at the time T, corrected for aberration.

The aberration of the sun in longitude always $= 20''$, that being the space moved through by the sun or earth in $8'.\ 7\frac{1}{2}''$, which is the time in which light passes from the sun to the earth.

7. *Aberratic Curve.—(Wright's sol. Camb. Prob.)*

Let $y$ and $p$ denote the rad. vect. and perpendicular upon the tangent of the given orbit; $y'$ and $p'$ the corresponding ones to the aberratic curve. Also let $c =$ twice area described dat. tem.; then

$$p' = \frac{c}{y},$$

$$\& \ y' = \frac{c}{p}.$$

These two equations will give the equation to the aberratic curve.

*Ex.* 1. Let the given orbit be a parabola; then $p^2 = \frac{L y}{4}$ (L = lat. rect.) $\therefore y' = \frac{c}{p} = \frac{2c}{\sqrt{L y}} = 2\sqrt{\frac{c p'}{L}}$, $\therefore p' = \frac{L}{4c} y'^2$, but $p =$

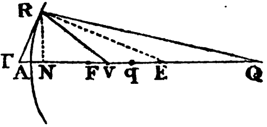
$\frac{r'^2}{2r}$ is an equation to the circle when the centre of the polar coordinates is in the circumference *(see Circle Equation to)*, ∴ the aberratic curve is a circle, whose rad. is $\frac{2c}{L}$.

2. Let the orbit be an ellipse, whose equation is $p^2 = \frac{b^2\,y}{2a - y}$; Then $y' = \frac{c}{p} = \frac{c}{b}\sqrt{\frac{2a-y}{y}} = \frac{c}{b}\sqrt{\frac{2a}{c}p' - 1}$, $\therefore\ p' = \frac{y'^2 + \frac{c^2}{b^2}}{\frac{2ac}{b^2}}$; so that the aberratic curve *(see Circle Equation to)* is a circle, whose radius is $\frac{ac}{b^2}$, and the distance of whose centre from the centre of coordinates is $\frac{c}{b}\sqrt{a^2 - b^2}$.

3. Let the orbit be the log. spiral, whose equation is $p = m\,y$, then $y' = \frac{c}{p} = \frac{c\,m}{y} = \frac{c\,m}{\frac{c}{p'}} = m\,p'$, $\therefore\ p' = \frac{y'}{m}$; ∴ the aberratic curve is also a log. spiral.

ABERRATION in *Optics*.—*(Coddington.)*

I. *Aberration in reflection at spherical surfaces.*



Let $EQ = q$, $Eq = q'$,
$EF = f$, $AN = v$
and the point $v$ in the figure being the actual intersection of the reflected ray and axis, let $Ev = q^{\backprime}$, then $q^{\backprime} = \frac{q\,f}{q+f} + \frac{q^2\,f\,v}{(q+f)^2} + \frac{q^3\,f\,v^2}{(q+f)^3} + \text{\&c.}$

This in geometrical terms is equivalent to

$$E\,q + \frac{QE^2}{QF^2}\cdot\frac{AN}{2} + \frac{QE^3}{QF^3}\cdot\frac{AN^2}{4\,EF} + \text{\&c.}$$

& ∴ $q^{\backprime} - q'$, or aberration in longitude, is $\frac{QE^2}{QF^2}\cdot\frac{AN}{2} + \frac{QE^3}{QF^3}\cdot\frac{AN^2}{4\,EF}$ &c. or, because $v$ is small, is

$$\frac{QE^2}{QF^2}\cdot\frac{AN}{2}\ ;\ \text{or}\ \frac{QE^2}{QF^2}\cdot\frac{AT}{2}\ \text{nearly.}$$

*Cor.* When Q E and Q F are given, aberration varies as A N varies as R N$^2$ nearly.

The following series for aberration is a little different from the preceding; but amounts nearly to the same thing; putting $\theta = \angle$ R E A.

$$\text{Aberrat.} = \frac{q^2}{(q+f)^3} f\,(\sec.\,\theta - 1) - \frac{q^2}{(q+f)^3} f^2\,(\sec.\,\theta - 1)^2 + \&c.$$

*Cor.* When the incident rays are parallel, aberration $= \frac{1}{2}$ A T .

II. *Aberration in refraction at spherical surfaces.*

Let $\Delta$ & $\Delta'$ be the perpendicular distances of Q and $q$ from the refracting surface, $m$ the ratio of the sine of incidence : sine of refraction, $v =$ ver. sin. A N (see preceding figure;) then

$$\text{Aberrat.} = (\Delta' - r)^2 \left(\frac{m}{\Delta} - \frac{1}{\Delta'}\right) v$$

$$\text{or} = (\Delta' - r)^2 \times \frac{m-1}{m} \left(\frac{m+1}{\Delta} - \frac{1}{r}\right) v,$$

& is ∴ positive, if $\Delta$ be less than $(m + 1)\,r$, & negative when $\Delta$ is above that value. When $\Delta = (m + 1)\,r$, there is no aberration.

When the incident rays are parallel, or $\frac{1}{\Delta} = o$, this reduces to

$-\frac{(\Delta' - r)^2}{\Delta'}\,v$; or if F be the principal focal distance, it is

$-\frac{(F - r)^2}{F}\,v$,

III. *Aberration in a lens.*

We may consider this as consisting of two parts :—

(1.) The variation in the second focal distance arising from the aberration in the first ($\alpha$.)

(2.) The additional aberration in the refraction at the second surface ($\beta$).

Let $\Delta''$ be the distance of the focus after the 2d refraction, the rest as before; then

$$\alpha = \frac{m\,\Delta''^2}{\Delta'^2}\,(\Delta' - r)^2 \left(\frac{m}{\Delta} - \frac{1}{\Delta'}\right) v.$$

For the 2d part we must alter our formula, by putting $\frac{1}{m}$ for $m$, $v'$ for $v$, $\Delta'$ for $\Delta$, $\Delta''$ for $\Delta'$, $r'$ for $r$;

$$\beta = (\Delta'' - r')^2 \left(\frac{1}{m\,\Delta'} - \frac{1}{\Delta''}\right) v'$$

The whole aberration is therefore

$$\frac{m\,\Delta''^2}{\Delta'^2}\,(\Delta - r)^2 \left(\frac{m}{\Delta} - \frac{1}{\Delta'}\right) v + (\Delta'' - r')^2 \left(\frac{1}{m\,\Delta'} - \frac{1}{\Delta''}\right) v'.$$

The aberration for a particular value of $\Delta$ varies as $v$, or as the *square of the radius of the aperture* nearly.

Let us examine what kinds of value the aberration in a lens assumes in different cases.

(1.) For the meniscus or concavo-convex lens, ($r$ & $r'$ being both positive.)

The aberration

$$A = \left\{ m \,.\, \frac{\Delta''^2}{\Delta'^2} \,.\, (\Delta' - r)^2 \left(\frac{m}{\Delta} - \frac{1}{\Delta'}\right) + (\Delta'' - r')^2 \left(\frac{1}{m\Delta'} - \frac{1}{\Delta''}\right) \right\} v.$$

(2.) For the double concave lens $r'$ is negative,

$$A = \left\{ m \,.\, \frac{\Delta''^2}{\Delta'^2} \,.\, (\Delta' - r)^2 \left(\frac{m}{\Delta} - \frac{1}{\Delta'}\right) + (\Delta'' + r')^2 \left(\frac{1}{m\Delta'} - \frac{1}{\Delta''}\right) \right\} v.$$

(3.) For the double convex lens $r$ is negative,

$$A = \left\{ m \,.\, \frac{\Delta''^2}{\Delta'^2} \quad (\Delta' + r)^2 \left(\frac{m}{\Delta} - \frac{1}{\Delta'}\right) + (\Delta'' - r')^2 \,.\! \left(\frac{1}{m\Delta'} - \frac{1}{\Delta''}\right) \right\} v$$

To find the least circle of aberration into which all the homogeneal rays of the same pencil, refracted by a lens, are collected.

Let $a = \frac{1}{2}$ aperture of the lens, $b =$ distance of the point where the extreme ray cuts the axis from the focus of refracted rays, $c =$ distance of the same point from the lens, $x =$ rad. of the least circle of aberration, then

$$x = \frac{a \times b}{4\,c}$$

*Cor.* If the focal length of the refractor, and the focus of incidence be given, $c$ is given and $x$ varies as $a\,b$ varies as $a^3$; and on the same supposition the area of the least circle of aberration varies as $a^6$.

**ACCELERATION** of *Falling Bodies.—See Motion.*

**ACCELERATION** of *Stars on Mean Solar Time.—See Time.*

**ACCELERATION** of the *Moon.—See Moon.*

**ADHESION**, a term chiefly used to denote the force, with which the surface of a solid remains attached to the surface of a liquid, after they have been brought into contact.

In the year 1773, Guyton-Morveau ascertained experimentally the force of adhesion of eleven different metals to mercury. The surface of each metal was an inch (French) in diameter and polished. The following Table exhibits the weight in French grains necessary to separate each metal from the mercury.

| | | | |
|---|---|---|---|
| Gold | 446 | Zinc | 204 |
| Silver | 429 | Copper | 142 |
| Tin | 418 | Antimony | 126 |
| Lead | 397 | Iron | 115 |
| Bismuth | 372 | Cobalt | 8 |
| Platinum | 282 | | |

**ÆRAS**, list of the most remarkable:—

| | *Julian Period.* | *B.C.* |
|---|---|---|
| Creation of the world | 706 | 4007 |
| Deluge | 2362 | 2351 |
| Olympiads of the Greeks | 3937 | 776 |
| Rome built, or Roman æra | 3961 | 752 |
| Æra of Nabonassar of Chaldæns and Egyptians | 3967 | 746 |
| Death of Alexander | 4390 | 323 |
| Æra of the Seleucidæ | 4401 | 312 |
| First of Julius Cæsar | 4669 | 44 |
| Vulgar æra of Christ's birth | 4713 | *A.C.* |
| Hegira, Mahometan æra | 5335 | 622 |
| Yesdegird, Persian æra | 5344 | 631 |

**ÆRONAUTICS.**

To calculate the height to which a balloon will ascend, under given circumstances.—(*Wright's solut. Camb. Prob.*)

Let W = weight of the balloon, and all its appendages in ounces, D = density of mercury at the time, $\delta$ the spec. grav. of the atmosphere at the surface of the earth, when the barometer stands at $b$ feet, and $\frac{\delta}{n}$ that of the gas; $c^3$ the capacity of the balloon in cubic feet, $x$ = height to which it will ascend in feet; then

$$x = \frac{b\,D}{\delta} \times \log. \frac{n\,\delta\,c^3}{n\,W + \delta\,c^3}.$$

*Cor.* If the gas be hydrogen or $n = 1\,3$, $b = 30$ inches $= \frac{5}{2}$ feet, D = 14019 (density of water being 1000), and $\delta = \frac{6}{5}$, then

$$x = 42057 \times \log. \frac{78\,c^3}{65\,W + 6\,c^3}.$$

*Ex.* Given W = 20 stone, and the other elements as in the Cor. to determine the magnitude of the balloon necessary just to lift that weight from the ground.

$$\text{Here } x = o, \therefore \frac{78\,c^3}{65 \times 4480 + 6\,c^3} = 1$$

$\therefore c^3 = 4041$ cubic feet.

Short historical notice :—

October 15, 1783. M. Pilatre de Rozier was the first person who ever ascended in a balloon; it was inflated with heated air. He perished in a subsequent ascent, being the first who did so.

December 1st, 1783. M. M. Roberts and Charles first ascended with an hydrogen gas balloon.

September 15th, 1784. The first aerial voyage in England performed by Lunardi.

Jan. 7, 1785. M. Blanchard and Dr Jeffries passed from Dover to Calais.

August, 1785. Blanchard in one of his excursions from Lisle, traversed a distance of more than 300 miles without halting.

Sept. 21, 1802. Garnerin first descended in a parachute from London.

September 15, 1804. Gay Lussac ascended from Paris for scientific purposes, and rose to the enormous height of 22,912 feet; or 23,040, *i. e.* more than 4½ miles above the level of the sea; being 1600 feet above the summit of the Andes; the barometer sunk to 12,95 inches. From this last ascent two results were obtained; (1) that the intensity of the magnetic power continues the same at all accessible distances from the earth's surface: and (2) that the proportions of oxygen and nitrogen, which constitute the atmosphere, do not vary sensibly in the most extended limits.

AIR *Atmospheric.—See Atmosphere.*

AIR *Pump.—See Pump.*

ANGULAR *Velocity.—See Central Forces.*

ANIMAL *Strength.—(Playfair.)*

1. The strength of men, and of all animals, is most powerful when directed against a resistance that is at rest: when the resistance is overcome, and when the animal is in motion, its force is diminished; lastly, with a certain velocity the animal can do no work, and can only keep up the motion of its own body.

2. A formula, having the three properties just mentioned, will afford an approximation to the law of animal force. Let P be the weight which the animal exerting itself to the utmost, or at a *dead pull*, is just able to overcome, W any other weight with which it is actually loaded, and $v$ the velocity with which it moves when so loaded; $c$ the velocity at which the power of drawing or carrying a load entirely ceases; then, till experience has led to a more accurate result, we may suppose the strength of animals to follow the law expressed by the formula

$$W = P\left(1 - \frac{v}{c}\right)^2.$$

This is Euler's Formula.

*Cor.* Hence the effect of animal force, or the quantity of work done in a given time, will be proportional to W $v$, or to P $v\left(1-\frac{v}{c}\right)^2$, and will be a maximum when $v=\frac{c}{3}$, and $W=\frac{4\,P}{9}$, *i. e.* when the animal moves with one-third of the speed with which it is able only to move itself, and is loaded with $\frac{4}{9}$ of the greatest load it is able to put in motion.

3. The quantities P and $c$ can only be determined by experience. Euler supposes that for the work of men, P may upon an average be taken $=$ 60lb, and $c=6$ feet per second, or a little more than four miles an hour.

4. A man, according to this estimate, when working to the greatest advantage, should carry a load of 27lb, and walk at the rate of two feet in a second, or a mile and one-third an hour.

5. A horse, according to Desaguliers, drawing a weight out of a well, over a pulley, can raise 200lb. for eight hours together, at the rate of 2¼ miles an hour. Supposing in this case the horse to work to the greatest advantage, $P=450$, and $c=6\frac{3}{4}$ miles per hour. This estimate, however, seems to give too high a value to P. It will suit better with general experience to make $P=420$ and $c=7$.

6. It appears from Cavallo, that a horse can draw 25 cwt. on a level road in a cart weighing 10 cwt., with wheels six feet high. In a common cart, two horses easily draw 30 cwt. In a common waggon, six horses draw 80 cwt.: in three carts they might draw 90; in six, 150 cwt.: and three carts cost less than a waggon. A horse drew three tons up a railway rising 7 inches in 144: the draught was 327 pounds besides friction.—*(Young's Nat. Phil.)*

7. According to Coulomb's experiments on this subject, if $w$ be the weight of the man's body, $l$ an additional load, which he is made to carry, H the height to which he ascends in a given time, when walking freely, and $h$ the height to which he ascends in the same time with the load $l$; then

$$h=\frac{w\,H\left(1-\frac{l}{2\,w}\right)}{w+l}.$$

*Cor.* 1. When $l=2\,w$, $L=o$.

*Cor.* 2. The greatest effect of a man's strength in raising a weight will be, when the weight of the man is to that of his load as $1:-1+\sqrt{3}$, or nearly as $4:3$.

8. The following experiments by Peron, with Regnier's dynamometer, shew that the strength of men depends considerably on the climate.

| | | | |
|---|---|---|---|
| English ............... | 71,4 | Van Diemen's Land | 51,8 |
| French ............... | 69,2 | New Holland ......... | 50,6 |
| Timor ............... | 58,7 | | |

*(Ency. Brit. Supplt.)*

**ANNUAL** *Equation.—See Moon.*

**ANNUITIES.**

### 1. *Annuities at Simple Interest.*

1. Let A be the annuity, $r$ the interest of £1. for one year, M the amount of the annuity for $n$ years, then

$$M = n\,A + n.\ \frac{n-1}{2}.\ r\,A.$$

2. Let P be the present value of an annuity to continue for $n$ years; the rest as before, then

$$P = \frac{n\,A + n.\ \frac{n-1}{2}.\ r\,A.}{1 + n\,r}$$

In these Equations any one of the quantities may be found, the rest being given.

### *Annuities at Compound Interest.*

3. Let R = £1. and its interest for one year, the rest as before, then

$$M = \frac{R^n - 1}{R - 1} \times A.$$

$$\&\ P = \frac{1 - \frac{1}{R^n}}{R - 1} \times A$$

If the No. of years be infinite,

$$P = \frac{A}{R - 1},$$

which gives the value of a freehold estate, A being the annual rent.

4. The present value of an annuity to commence at the expiration of $p$ years, and to continue $q$ years, is the difference between its present value for $p + q$ years, and its present value for $p$ years,

$$\text{or } P = \left(\frac{1}{R^p} - \frac{1}{R^{p+q}}\right) \times \frac{A}{R - 1}.$$

If the annuity, instead of being payable annually, be made payable half-yearly, quarterly, or at any other given interval, the above formulæ are still applicable, by calling R £1. and its interest for the given interval, and $n$ the number of those intervals.

## TABLE I.

*Showing the amount of an Annuity of £1. for any number of years, not exceeding fifty; and for the different rates of interest from 3 to 6 per Cent.—(Encyc. Metrop.)*

| No. of Years. | 3 perCent. | 3½ perCent. | 4 perCent. | 4½ perCent. | 5 perCent. | 6 perCent. |
|---|---|---|---|---|---|---|
| 1 | 1·00000 | 1.00000 | 1·00000 | 1·00000 | 1·00000 | 1·00000 |
| 2 | 2·03000 | 2·03500 | 2·04000 | 2·04500 | 2·05000 | 2·06000 |
| 3 | 3·09090 | 3·10622 | 3·12160 | 3·13702 | 3·15250 | 3·18360 |
| 4 | 4·18362 | 4·21494 | 4·24646 | 4·27819 | 4·31012 | 4·37461 |
| 5 | 5·30913 | 5·36246 | 5·41632 | 5·47070 | 5·52563 | 5·63709 |
| 6 | 6·46840 | 6·55015 | 6·63297 | 6·71689 | 6·80191 | 6·97531 |
| 7 | 7·66246 | 7·77940 | 7·89829 | 8·01915 | 8·14200 | 8·39383 |
| 8 | 8·89233 | 9·05168 | 9·21422 | 9·38001 | 9·54910 | 9·89746 |
| 9 | 10·15910 | 10·36849 | 10·58279 | 10·80211 | 11·02656 | 11·49131 |
| 10 | 11·46387 | 11·73139 | 12·00610 | 12·28820 | 12·57789 | 13·18079 |
| 11 | 12·80779 | 13·14199 | 13·48635 | 13·84117 | 14·20678 | 14·97164 |
| 12 | 14·19202 | 14·60196 | 15·02580 | 15.46403 | 15·91712 | 16·86994 |
| 13 | 15·61779 | 16·11303 | 16·62683 | 17·15991 | 17·71298 | 18·88213 |
| 14 | 17·08632 | 17.67698 | 18·29191 | 18·93210 | 19·59863 | 21·01506 |
| 15 | 18·59891 | 19·29568 | 20·02358 | 20·78405 | 21·57856 | 23·27596 |
| 16 | 20·15688 | 20·97102 | 21·82453 | 22·71933 | 23·65749 | 25·67252 |
| 17 | 21·76158 | 22·70501 | 23·69751 | 24·74170 | 25·84036 | 28·21287 |
| 18 | 23·41443 | 24·49969 | 25·64541 | 26·85508 | 28·13238 | 30·90565 |
| 19 | 25·11686 | 26·35718 | 27·67122 | 29·06356 | 30·53900 | 33·75999 |
| 20 | 26·87037 | 28·27968 | 29·77807 | 31·37142 | 33·06595 | 36·78559 |
| 21 | 28·67648 | 30·26947 | 31·96920 | 33·78313 | 35·71925 | 39·99272 |
| 22 | 30·53678 | 32·32890 | 34·24796 | 36·30337 | 38·50521 | 43·39229 |
| 23 | 32·45288 | 34·46041 | 36·61788 | 38·93702 | 41·43047 | 46·99582 |
| 24 | 34·42647 | 36·66652 | 39·08260 | 41·68919 | 44·50199 | 50·81557 |
| 25 | 36·45926 | 38·94985 | 41·64590 | 44·56521 | 47·72709 | 54·86451 |
| 26 | 38·55304 | 41·31310 | 44·31174 | 47·57065 | 51·11345 | 59·15638 |
| 27 | 40·70963 | 43·75906 | 47·08421 | 50·71132 | 54·66912 | 63·70576 |
| 28 | 42·93092 | 46·29062 | 49·96758 | 53·99333 | 58·40258 | 68·52811 |
| 29 | 45·21885 | 48·91079 | 52·96628 | 57·42303 | 62·32271 | 73·63979 |
| 30 | 47·57541 | 51·62267 | 56·08493 | 61·00706 | 66·43884 | 79·05818 |
| 31 | 50·00267 | 54·42947 | 59·32833 | 64·75238 | 70·76078 | 84·80167 |
| 32 | 52·50275 | 57·33450 | 62·70146 | 68·66624 | 75·29882 | 90·88977 |
| 33 | 55·07784 | 60·34121 | 66·20952 | 72·75620 | 80.06377 | 97·34316 |
| 34 | 57·73017 | 63·45315 | 69·85790 | 77·03025 | 85·06695 | 104·18375 |
| 35 | 60·46201 | 66·67401 | 73·65222 | 81·49661 | 90·32030 | 111·43477 |
| 36 | 63·27594 | 70·00760 | 77·59831 | 86·16396 | 95·83632 | 119·12086 |
| 37 | 66·17422 | 73·45786 | 81·70224 | 91·04134 | 101·62813 | 127·26811 |
| 38 | 69·15944 | 77.02889 | 85·97033 | 96·13820 | 107·70954 | 135·90420 |
| 39 | 72.23423 | 80·72490 | 90·40914 | 101·46442 | 114·09502 | 145·05845 |
| 40 | 75.40125 | 84·55027 | 95·02551 | 107·03032 | 120·79977 | 154·76196 |
| 41 | 78·66329 | 88·50953 | 99·82653 | 112·84668 | 127·83976 | 165·04768 |
| 42 | 82·02319 | 92·60737 | 104·81959 | 118·92478 | 135·23175 | 175·95054 |
| 43 | 85·48389 | 96.84862 | 110·01238 | 125·27640 | 142·99333 | 187·50757 |
| 44 | 89·04840 | 101·23833 | 115·41287 | 131·91384 | 151·14300 | 199·75803 |
| 45 | 92·71986 | 105·78167 | 121·02939 | 138·84996 | 159·70015 | 212·74351 |
| 46 | 96·50145 | 110·48403 | 126·87056 | 146·09821 | 168·68516 | 226·50812 |
| 47 | 100·39650 | 115·35097 | 132·94539 | 153·67263 | 178·11942 | 241·09861 |
| 48 | 104·40839 | 120·38825 | 139·26320 | 161·58790 | 188·02539 | 256·56452 |
| 49 | 108·54064 | 125·60184 | 145·83373 | 169·85935 | 198·42666 | 272·95840 |
| 50 | 112·79686 | 130·99791 | 152·60708 | 178·50302 | 209·34799 | 290·33590 |

## TABLE II.

*Showing the present value of an Annuity of £1. per annum, for any number of years, not exceeding fifty, and at different rates of interest, from 3 to 6 per cent.—(Encyc. Metrop.)*

| No. of Years. | 3 perCent. | 3½ perCent. | 4 perCent. | 4½ perCent. | 5 perCent. | 6 perCent. |
|---|---|---|---|---|---|---|
| 1 | ·97887 | ·96618 | ·96153 | ·95693 | ·95238 | ·94339 |
| 2 | 1·91346 | 1·89969 | 1·88609 | 1·87266 | 1·85941 | 1·83339 |
| 3 | 2·82861 | 2·80163 | 2·77509 | 2·74896 | 2·72324 | 2·67301 |
| 4 | 3·71709 | 3·67307 | 3·62989 | 3·58752 | 3·54595 | 3·46510 |
| 5 | 4·57970 | 4·51505 | 4·45182 | 4·38997 | 4·32947 | 4·21236 |
| 6 | 5·41719 | 5·32855 | 5·24213 | 5·15787 | 5·07569 | 4·91732 |
| 7 | 6·23028 | 6·11454 | 6·00205 | 5·89270 | 5·78637 | 5·58238 |
| 8 | 7·01969 | 6·87395 | 6·73274 | 6·59588 | 6·46321 | 6·20979 |
| 9 | 7·78610 | 7·60768 | 7·43533 | 7·26879 | 7·10782 | 6·80169 |
| 10 | 8·53020 | 8·31660 | 8·11089 | 7·91271 | 7·72173 | 7·36008 |
| 11 | 9·25262 | 9·00155 | 8·76047 | 8·52891 | 8·30641 | 7·88687 |
| 12 | 9·95400 | 9·66333 | 9·38507 | 9·11858 | 8·86325 | 8·38384 |
| 13 | 10·63495 | 10·30273 | 9·98564 | 9·68285 | 9·39357 | 8·85268 |
| 14 | 11·29607 | 10·92052 | 10·56312 | 10·22282 | 9·89864 | 9·29498 |
| 15 | 11·93793 | 11·51741 | 11·11838 | 10·73954 | 10·37965 | 9·71224 |
| 16 | 12·56110 | 12·09416 | 11·65229 | 11·23401 | 10·83776 | 10·10589 |
| 17 | 13·16611 | 12·65132 | 12·16566 | 11·70719 | 11·27406 | 10·47725 |
| 18 | 13·75351 | 13·18968 | 12·65929 | 12·15999 | 11·68958 | 10·82760 |
| 19 | 14·32379 | 13·70983 | 13·13393 | 12·59329 | 12·08532 | 11·15811 |
| 20 | 14·87747 | 14·21240 | 13·59032 | 13·00793 | 12·46221 | 11·46992 |
| 21 | 15·41502 | 14·69794 | 14·02915 | 13·40472 | 12·82115 | 11·76407 |
| 22 | 15·93691 | 15·16712 | 14·45111 | 13·78442 | 13·16300 | 12·04158 |
| 23 | 16·44360 | 15·62041 | 14·85684 | 14·14777 | 13·48857 | 12·30337 |
| 24 | 16·93554 | 16·05836 | 15·24696 | 14·49547 | 13·79864 | 12·55035 |
| 25 | 17·41314 | 16·48151 | 15·62207 | 14·82820 | 14·09394 | 12·78335 |
| 26 | 17·87684 | 16·89035 | 15·98276 | 15·14661 | 14·37518 | 13·00316 |
| 27 | 18·32703 | 17·28536 | 16·32958 | 15·45130 | 14·64303 | 13·21053 |
| 28 | 18·76410 | 17·66701 | 16·66306 | 15·74287 | 14·89812 | 13·40616 |
| 29 | 19·18845 | 18·03576 | 16·98371 | 16·02188 | 15·14107 | 13·59072 |
| 30 | 19·60044 | 18·39204 | 17·29203 | 16·28888 | 15·37245 | 13·76483 |
| 31 | 20·00042 | 18·73627 | 17·58849 | 16·54439 | 15·59281 | 13·92908 |
| 32 | 20·38876 | 19·06886 | 17·87355 | 16·78889 | 15·80267 | 14·08404 |
| 33 | 20·76579 | 19·39020 | 18·14764 | 17·02286 | 16·00254 | 14·23022 |
| 34 | 21·13183 | 19·70068 | 18·41119 | 17·24675 | 16·19290 | 14·36814 |
| 35 | 21·48722 | 20·00066 | 18·66461 | 17·46101 | 16·37419 | 14·49824 |
| 36 | 21·83225 | 20·29049 | 18·90828 | 17·66604 | 16·54685 | 14·62098 |
| 37 | 22·16723 | 20·57052 | 19·14257 | 17·86223 | 16·71128 | 14·73678 |
| 38 | 22·49246 | 20·84108 | 19·36786 | 18·04999 | 16·86789 | 14·84601 |
| 39 | 22·80821 | 21·10249 | 19·58448 | 18·22965 | 17·01704 | 14·94907 |
| 40 | 23·11477 | 21·35507 | 19·79277 | 18·40158 | 17·15908 | 15·04629 |
| 41 | 23·41239 | 21·59910 | 19·99305 | 18·56610 | 17·29436 | 15·13801 |
| 42 | 23·70135 | 21·83488 | 20·18562 | 18·72354 | 17·42320 | 15·22454 |
| 43 | 23·98190 | 22·06268 | 20·37079 | 18·87421 | 17·54591 | 15·30617 |
| 44 | 24·25437 | 22·28279 | 20·54884 | 19·01838 | 17·66277 | 15·38318 |
| 45 | 24·51871 | 22·49545 | 20·72003 | 19·15634 | 17·77406 | 15·45583 |
| 46 | 24·77544 | 22·70091 | 20·88465 | 19·28837 | 17·88006 | 15·52436 |
| 47 | 25·02470 | 22·89943 | 21·04293 | 19·41470 | 17·98101 | 15·58902 |
| 48 | 25·26670 | 23·09124 | 21·19513 | 19·53560 | 18·07715 | 15·65002 |
| 49 | 25·50165 | 23·27656 | 21·34147 | 19·65129 | 18·16872 | 15·70757 |
| 50 | 25·72976 | 23·45561 | 21·48218 | 19·76200 | 18·25592 | 15·76186 |

## TABLE III.

*Showing the Annuity that £1. will purchase for any number of years, not exceeding fifty; at different rates of interest from 3 to 6 per cent.—(Encyc. Metrop.)*

| No. of Years. | 3 perCent. | 3½ perCent. | 4 perCent. | 4½ perCent. | 5 perCent. | 6 perCent. |
|---|---|---|---|---|---|---|
| 1 | 1·03000 | 1·03500 | 1·04000 | 1·04500 | 1·05000 | 1·06000 |
| 2 | ·52261 | ·52640 | ·53019 | ·53399 | ·53780 | ·54543 |
| 3 | ·35353 | ·35693 | ·36034 | ·36377 | ·36720 | ·37410 |
| 4 | ·26902 | ·27225 | ·27549 | ·27874 | ·28201 | ·28859 |
| 5 | ·21835 | ·22148 | ·22462 | ·22779 | ·23097 | ·23739 |
| 6 | ·18459 | ·18766 | ·19076 | ·19387 | ·19701 | ·20336 |
| 7 | ·16050 | ·16354 | ·16660 | ·16970 | ·17281 | ·17913 |
| 8 | ·14245 | ·14547 | ·14852 | ·15160 | ·15472 | ·16103 |
| 9 | ·12843 | ·13144 | ·13449 | ·13757 | ·14069 | ·14702 |
| 10 | ·11723 | ·12024 | ·12329 | ·12637 | ·12950 | ·13586 |
| 11 | ·10807 | ·11109 | ·11414 | ·11724 | ·12038 | ·12679 |
| 12 | ·10046 | ·10318 | ·10655 | ·10966 | ·11282 | ·11927 |
| 13 | ·09402 | ·09706 | ·10014 | ·10327 | ·10645 | ·11296 |
| 14 | ·08852 | ·09157 | ·09466 | ·09782 | .10102 | ·10758 |
| 15 | ·08376 | ·08682 | ·08994 | ·09311 | ·09634 | ·10296 |
| 16 | ·07961 | ·08268 | ·08582 | .08901 | ·09226 | ·09895 |
| 17 | ·07595 | ·07904 | ·08219 | ·08541 | ·08869 | ·09544 |
| 18 | ·07270 | ·07581 | ·07899 | ·08223 | ·08554 | ·09235 |
| 19 | ·06981 | ·07294 | ·07613 | ·07940 | ·08274 | ·08962 |
| 20 | ·06721 | ·07036 | ·07258 | ·07687 | ·08024 | ·08718 |
| 21 | ·06487 | ·06803 | ·07128 | ·07460 | ·07799 | ·08500 |
| 22 | ·06274 | .06593 | ·06919 | ·07254 | ·07597 | ·08304 |
| 23 | ·06081 | ·06401 | ·06730 | ·07068 | .07413 | ·08127 |
| 24 | ·05904 | ·06227 | ·06558 | ·06898 | ·07247 | ·07967 |
| 25 | ·05742 | ·06067 | ·06401 | ·06743 | ·07095 | ·07822 |
| 26 | ·05593 | ·05920 | ·06256 | ·06602 | ·06956 | ·07690 |
| 27 | ·05456 | ·05785 | ·06123 | ·06471 | ·06829 | ·07569 |
| 28 | ·05329 | ·05660 | ·06001 | ·06352 | ·06712 | ·07459 |
| 29 | ·05211 | ·05544 | ·05887 | ·06242 | ·06604 | ·07357 |
| 30 | ·05101 | ·05437 | ·05783 | ·06139 | ·06505 | ·07264 |
| 31 | ·04999 | ·05337 | ·05685 | ·06044 | ·06413 | ·07179 |
| 32 | ·04904 | ·05244 | ·05594 | ·05956 | ·06328 | ·07100 |
| 33 | ·04815 | ·05157 | ·05510 | ·05874 | ·06249 | ·07027 |
| 34 | ·04732 | ·05075 | ·05431 | ·05798 | ·06175 | ·06959 |
| 35 | ·04653 | ·04099 | ·05357 | ·05727 | ·06107 | ·06897 |
| 36 | ·04580 | ·04928 | ·05288 | ·05660 | ·06043 | ·06839 |
| 37 | ·04511 | ·04861 | ·05223 | ·05598 | ·05983 | ·06785 |
| 38 | ·04445 | ·04798 | ·05163 | ·05540 | ·05928 | ·06735 |
| 39 | ·04384 | ·04738 | ·05106 | ·05485 | ·05876 | ·06689 |
| 40 | ·04326 | ·04682 | ·05052 | ·05434 | ·05827 | ·06646 |
| 41 | ·04271 | ·04629 | ·05001 | ·05386 | ·05782 | ·06605 |
| 42 | ·04219 | ·04579 | ·04954 | ·05340 | ·05739 | ·06568 |
| 43 | ·04169 | ·04532 | ·04908 | ·05298 | ·05699 | ·06533 |
| 44 | ·04122 | ·04487 | ·04866 | ·05258 | ·05661 | ·06500 |
| 45 | ·04078 | ·04445 | ·04826 | ·05220 | ·05626 | ·06470 |
| 46 | ·04036 | ·04405 | ·04788 | ·05184 | ·05592 | ·06441 |
| 47 | ·03996 | ·04366 | ·04752 | ·05150 | ·05561 | ·06414 |
| 48 | ·03957 | ·04330 | ·04718 | ·05118 | ·05531 | ·06389 |
| 49 | ·03921 | ·04296 | ·04685 | ·05088 | ·05503 | ·06366 |
| 50 | ·03886 | ·04263 | ·04655 | ·05060 | ·05477 | ·06344 |

## 2. *Of Life Annuities.—(Wood.)*

1. To find the probability that an individual of a given age will live any number of years.

Let A be the number in the tables of the given age, B, C, D ......... X the number left at 1, 2, 3 ......... $t$ years; then $\frac{B}{A}$ is the probability that the individual will live one year; $\frac{C}{A}$ the probability that he will live two years, $\frac{X}{A}$ that he will live $t$ years. Also $\frac{A-B}{A}$, $\frac{A-C}{A}$, $\frac{A-X}{A}$ are the probabilities that he will die in 1, 2, $t$ years.

2. To find the probability that two individuals P and Q, whose ages are known, will live a year.

Let the probability that P will live a year, determined by the last Art. be $\frac{1}{m}$, and the probability that Q will live a year $\frac{1}{n}$; then the probability that they will both be alive at the end of that time is $\frac{1}{mn}$.

3. To find the probability that one of them at least will be alive at the end of any number of years.

Let $\frac{1}{p}$ be the probability that P will live $t$ years, and $\frac{1}{q}$ the probability that Q will live the same time; then the prob. that one of them at least will be alive at the end of the time is $1-\frac{\overline{p-1}.\ \overline{q-1}}{pq}$, or $\frac{p+q-1}{pq}$.

4. To find the present value of an annuity of £1. to be continued during the life of an individual of a given age, allowing compound interest for the money.

Let $r$ be the amount of £1. for one year; A, B, C, &c. as in Art. 1, then the value required is $\frac{1}{A} \times \left(\frac{B}{r} + \frac{C}{r^2} + \frac{D}{r^3} + \&c.\right)$ to the end of the tables.

De Moivre supposes that out of 86 persons born, one dies every year, till they are extinct. On this supposition, the sum of the above series may be found thus. Let $n$ be the number of years which any individual wants of 86; then will $n$ be the number of persons living of that age, out of which one dies every year; then the sum of the above series or the present value of the annuity is $\frac{\overline{n-1}.\ r - n + \frac{1}{r^{n-1}}}{n.\ (r-1)^2}$ = (if P be the present value of an annuity of £1. to continue certain for $n$ years)

$$\frac{1 - \frac{r}{n}P}{r - 1}$$

5. The present value of the annuity to continue for ever from the death of the proposed individual is $\frac{rP}{n.\overline{r-1}}$.

6. To find the present value of an annuity of £1. to be paid as long as two specified individuals are both living.

Find by Art. 2. the probability that they will both be alive at the end 1, 2, 3, &c. years to the end of the Tables, call these probabilities $a$, $b$, $c$, &c. and $r$ the amount of £1. in one year, then $\frac{a}{r} + \frac{b}{r^2} + \frac{c}{r^3} +$ &c. is the present value of the annuity required.

7. To find the present value of an annuity of £1. to be paid as long as either of two specified individuals is living.

Find by Art. 3. the probability that they will not both be extinct in 1, 2, 3, &c. years to the end of the tables, and call these probabilities A, B, C, &c. then the present value of the annuity is $\frac{A}{r} + \frac{B}{r^2} + \frac{C}{r^3} +$ &c.

## TABLE I.

*Mean Standard Table of the Decrements of Life in Great Britain,* 1824. —(*Dr Young's Phil. Trans.* 1826.)

| Age. | Decrement. | Living. | Age. | Decrement. | Living. | Age. | Decrement. | Living. | Age. | Decrement. | Living. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 20531 | 100003 | 30 | 705 | 46527 | 60 | 938 | 21810 | 90 | 164 | 589 |
| 1 | 9106 | 79472 | 31 | 712 | 45822 | 61 | 942 | 20872 | 91 | 130 | 425 |
| 2 | 4780 | 70366 | 32 | 719 | 45110 | 62 | 943 | 19930 | 92 | 87 | 295 |
| 3 | 2854 | 65586 | 33 | 726 | 44391 | 63 | 944 | 18987 | 93 | 60 | 208 |
| 4 | 1880 | 62732 | 34 | 734 | 43665 | 64 | 943 | 18043 | 94 | 44 | 148 |
| 5 | 1341 | 60852 | 35 | 742 | 42931 | 65 | 942 | 17100 | 95 | 31 | 104 |
| 6 | 979 | 59511 | 36 | 751 | 42189 | 66 | 939 | 16158 | 96 | 19 | 73 |
| 7 | 752 | 58532 | 37 | 759 | 41438 | 67 | 933 | 15219 | 97 | 14 | 54 |
| 8 | 603 | 57780 | 38 | 768 | 40679 | 68 | 926 | 14286 | 98 | 9 | 40 |
| 9 | 494 | 57177 | 39 | 776 | 39911 | 69 | 915 | 13360 | 99 | 6 | 31 |
| 10 | 423 | 56683 | 40 | 785 | 39135 | 70 | 903 | 12445 | 100 | 6 | 25 |
| 11 | 377 | 56260 | 41 | 795 | 38350 | 71 | 888 | 11542 | 101 | 5 | 19 |
| 12 | 349 | 55883 | 42 | 804 | 37555 | 72 | 871 | 10654 | 102 | 5 | 14 |
| 13 | 337 | 55534 | 43 | 813 | 36751 | 73 | 850 | 9783 | 103 | 4 | 9 |
| 14 | 337 | 55197 | 44 | 821 | 35938 | 74 | 826 | 8933 | 104 | 2 | 5 |
| 15 | 347 | 54860 | 45 | 831 | 35117 | 75 | 801 | 8107 | 105 | 1 | 3 |
| 16 | 381 | 54513 | 46 | 839 | 34286 | 76 | 768 | 7306 | 106 | .25 | 2 |
| 17 | 393 | 54132 | 47 | 848 | 33447 | 77 | 733 | 6538 | 107 | .25 | 1.75 |
| 18 | 422 | 53739 | 48 | 857 | 32599 | 78 | 697 | 5805 | 108 | .25 | 1.50 |
| 19 | 458 | 53317 | 49 | 866 | 31742 | 79 | 654 | 5108 | 109 | .25 | 1.25 |
| 20 | 497 | 52859 | 50 | 874 | 30876 | 80 | 610 | 4454 | 110 | .25 | 1.0 |
| 21 | 540 | 52362 | 51 | 882 | 30002 | 81 | 559 | 3844 | 111 | .25 | .75 |
| 22 | 581 | 51822 | 52 | 890 | 29120 | 82 | 513 | 3285 | 112 | .25 | .50 |
| 23 | 621 | 51241 | 53 | 898 | 28230 | 83 | 460 | 2772 | 113 | .25 | .25 |
| 24 | 656 | 50620 | 54 | 906 | 27332 | 84 | 408 | 2312 | 114 | 0 | 0 |
| 25 | 678 | 49964 | 55 | 913 | 26426 | 85 | 357 | 1904 | | | |
| 26 | 682 | 49286 | 56 | 917 | 25513 | 86 | 307 | 1547 | | | |
| 27 | 687 | 48604 | 57 | 923 | 24596 | 87 | 258 | 1240 | | | |
| 28 | 692 | 47917 | 58 | 929 | 23673 | 88 | 215 | 982 | | | |
| 29 | 698 | 47225 | 59 | 934 | 22744 | 89 | 178 | 767 | | | |

Dr Young's formula expressing the decrement of human life is

$$y = 368 + 10x - 11(156 + 20x - x^2)^{\frac{3}{2}} + \frac{1}{285 + 2.05x^2 + 2\left(\frac{x}{10}\right)^6}$$

$$-5.5\left(\frac{x}{50}\right)^{10} + \frac{5.5^2}{4000}\left(\frac{x}{50}\right)^{20} - 5500\left(\frac{x}{100}\right)^{40}.$$

; $y$ being the number of deaths among 100000 persons, in the year that completes the age $x$. ; $y = 368 + 10x$ may be employed as sufficiently correct for the middle portion of life, being certainly much nearer to the truth than Demoivre's hypothesis, who makes $y = \frac{100000}{86} = 1163$ throughout life.

## TABLE II.

*Showing the value of an Annuity on a single life at every age, deduced from the observations made at Northampton.—(Encyc. Metrop.)*

| Age. | 4 per Cent. | 5 per Cent. | 6 per Cent. | Age. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|---|---|---|
| 1 | 13·465 | 11·563 | 10·107 | 49 | 11·475 | 10·443 | 9·563 |
| 2 | 15·633 | 13·420 | 11·724 | 50 | 11·234 | 10·269 | 9·417 |
| 3 | 16·462 | 14·135 | 12·348 | 51 | 11·057 | 10·097 | 9·273 |
| 4 | 17·010 | 14·613 | 12·769 | 52 | 10·849 | 9·925 | 9·129 |
| 5 | 17·248 | 14·826 | 12·962 | 53 | 10·637 | 9·748 | 8·980 |
| 6 | 17·482 | 15·041 | 13·156 | 54 | 10·421 | 9·567 | 8·827 |
| 7 | 17·611 | 15·166 | 13·275 | 55 | 10·201 | 4·382 | 8·670 |
| 8 | 17·662 | 15·226 | 13·337 | 56 | 9·977 | 9·193 | 8·509 |
| 9 | 17·625 | 15·210 | 13·335 | 57 | 9·749 | 8·999 | 8·343 |
| 10 | 17·523 | 15·139 | 13·285 | 58 | 9·516 | 8·801 | 8·173 |
| 11 | 17·393 | 15·043 | 13·212 | 59 | 9·280 | 8·599 | 7·999 |
| 12 | 17·251 | 14·937 | 13·130 | 60 | 9·039 | 8·392 | 7·820 |
| 13 | 17·103 | 14·826 | 13·044 | 61 | 8·795 | 8·181 | 7·637 |
| 14 | 16·950 | 14·710 | 12·958 | 62 | 8·547 | 7·966 | 7·440 |
| 15 | 16·791 | 14·588 | 12·857 | 63 | 8·291 | 7·742 | 7·258 |
| 16 | 16·625 | 14·460 | 12·755 | 64 | 8·030 | 7·514 | 6·052 |
| 17 | 16·462 | 14·334 | 12·655 | 65 | 7·761 | 7.276 | 6·841 |
| 18 | 16·309 | 14·217 | 12·562 | 66 | 7·488 | 7·034 | 6·625 |
| 19 | 16·167 | 14·108 | 12·477 | 67 | 7·211 | 6·787 | 6·406 |
| 20 | 16·033 | 14·007 | 12·398 | 68 | 6·930 | 6·536 | 6·179 |
| 21 | 15·912 | 13·917 | 12·329 | 69 | 6·647 | 6·281 | 5·949 |
| 22 | 15·797 | 13·833 | 12·265 | 70 | 6·361 | 6·023 | 5·716 |
| 23 | 15·680 | 13·746 | 12·200 | 71 | 6·075 | 5·764 | 5·479 |
| 24 | 15·560 | 13·658 | 12·132 | 72 | 5·790 | 5·504 | 5·241 |
| 25 | 15·438 | 13·567 | 12·063 | 73 | 5·507 | 5·245 | 5·004 |
| 26 | 15·312 | 13·473 | 11·992 | 74 | 5·230 | 4·990 | 4·796 |
| 27 | 15·184 | 13·377 | 11·917 | 75 | 4·962 | 4·744 | 4·542 |
| 28 | 15·053 | 13·278 | 11·841 | 76 | 4·710 | 4·511 | 4·326 |
| 29 | 14·918 | 13·177 | 11·763 | 77 | 4·457 | 4·277 | 4·109 |
| 30 | 14·781 | 13·072 | 11·682 | 78 | 4·197 | 4·035 | 3·884 |
| 31 | 14·639 | 12·965 | 11·598 | 79 | 3·921 | 3·776 | 3·641 |
| 32 | 14·495 | 12·854 | 11·512 | 80 | 3·643 | 3·515 | 3·394 |
| 33 | 14·347 | 12·740 | 11·423 | 81 | 3.377 | 3·263 | 3·156 |
| 34 | 14·195 | 12·623 | 11·331 | 82 | 3·122 | 3·026 | 2·926 |
| 35 | 14·039 | 12·502 | 11·236 | 83 | 2·887 | 2·797 | 2·713 |
| 36 | 13·889 | 12·377 | 11·137 | 84 | 2·708 | 2·627 | 2·551 |
| 37 | 13·716 | 12·249 | 11·035 | 85 | 2·543 | 2·471 | 2·402 |
| 38 | 13·548 | 12·116 | 10·929 | 86 | 2·393 | 2·328 | 2·266 |
| 39 | 13·375 | 11·979 | 10·819 | 87 | 2·251 | 2·193 | 2·138 |
| 40 | 13·197 | 11·837 | 10·705 | 88 | 2·131 | 2·080 | 2·031 |
| 41 | 13·018 | 11·695 | 10·589 | 89 | 1.967 | 1·924 | 1·882 |
| 42 | 12·838 | 11·551 | 10·473 | 90 | 1·758 | 1·723 | 1·689 |
| 43 | 12·657 | 11·407 | 10·356 | 91 | 1·474 | 1·447 | 1·422 |
| 44 | 12·472 | 11·258 | 10·235 | 92 | 1·171 | 1·153 | 1·136 |
| 45 | 12·283 | 11·105 | 10.110 | 93 | ·827 | ·816 | ·806 |
| 46 | 12·089 | 10·947 | 9·980 | 94 | ·530 | ·524 | ·518 |
| 47 | 11·890 | 10·784 | 9·846 | 95 | ·240 | ·238 | ·236 |
| 48 | 11·685 | 10·616 | 9·707 | 96 | ·000 | ·000 | ·000 |

## TABLE III.

*Showing the value of an Annuity on two joint lives, deduced from observations made at Northampton, the difference of ages being* 5 *years.—(Encyc. Metrop.)*

| Ages. | 4 per Cent. | 5 per Cent. | Ages. | 4 per Cent. | 5 per Cent. |
|---|---|---|---|---|---|
| 1- 6 | 10·741 | 9·479 | 47-52 | 8·147 | 7·582 |
| 2- 7 | 12·581 | 11·100 | 48-53 | 7·965 | 7·424 |
| 3- 8 | 13·319 | 11·755 | 49-54 | 7·780 | 7·262 |
| 4- 9 | 13·775 | 12·165 | 50-55 | 7·593 | 7·098 |
| 5-10 | 13·933 | 12·315 | 51-56 | 7·409 | 6·936 |
| 6-11 | 14·068 | 12·447 | 52-57 | 7·225 | 6·774 |
| 7-12 | 14·111 | 12·498 | 53-58 | 7·039 | 6·609 |
| 8-13 | 14·089 | 12·492 | 54-59 | 6·850 | 6·442 |
| 9-14 | 13·992 | 12·421 | 55-60 | 6·659 | 6·272 |
| 10-15 | 13·841 | 12·302 | 56-61 | 6·465 | 6·100 |
| 11-16 | 13·664 | 12·158 | 57-62 | 6·270 | 5·925 |
| 12-17 | 13·480 | 12·009 | 58-63 | 6·070 | 5·744 |
| 13-18 | 13·303 | 11·864 | 59-64 | 5·867 | 5·561 |
| 14-19 | 13·130 | 11·723 | 60-65 | 5·658 | 5·372 |
| 15-20 | 12·961 | 11·585 | 61-66 | 5·447 | 5·180 |
| 16-21 | 12·799 | 11·452 | 62-67 | 5·285 | 4·986 |
| 17-22 | 12·646 | 11·327 | 63-68 | 5·017 | 4·786 |
| 18-23 | 12·500 | 11·209 | 64-69 | 4·798 | 4·585 |
| 19-24 | 12·361 | 11·096 | 65-70 | 4·573 | 4·378 |
| 20-25 | 12·229 | 10·989 | 66-71 | 4·349 | 4·169 |
| 21-26 | 12·105 | 10·890 | 67-72 | 4·124 | 3·960 |
| 22-27 | 11·987 | 10·796 | 68-73 | 3·901 | 3·752 |
| 23-28 | 11·866 | 10·699 | 69-74 | 3·683 | 3·547 |
| 24-29 | 11·743 | 10·600 | 70-75 | 3·471 | 3·347 |
| 25-30 | 11·618 | 10·499 | 71-76 | 3·270 | 3·159 |
| 26-31 | 11·489 | 10·396 | 72-77 | 3·070 | 2·971 |
| 27-32 | 11·359 | 10·289 | 73-78 | 2·869 | 2·780 |
| 28-33 | 11·225 | 10·181 | 74-79 | 2·659 | 2·580 |
| 29-34 | 11·088 | 10·069 | 75-80 | 2·448 | 2·381 |
| 30-35 | 10·948 | 9·954 | 76-81 | 2·258 | 2·195 |
| 31-36 | 10·805 | 9·837 | 77-82 | 2·077 | 2·013 |
| 32-37 | 10·659 | 9·716 | 78-83 | 1·899 | 1·838 |
| 33-38 | 10·508 | 9·591 | 79-84 | 1·751 | 1·750 |
| 34-39 | 10·354 | 9·463 | 80-85 | 1·608 | 1·573 |
| 35-40 | 10·196 | 9·331 | 81-86 | 1·478 | 1·447 |
| 36-41 | 10·037 | 9·198 | 82-87 | 1·356 | 1·329 |
| 37-42 | 9·877 | 9·062 | 83-88 | 1·259 | 1·235 |
| 38-43 | 9·716 | 8·927 | 84-89 | 1·164 | 1·145 |
| 39-44 | 9·550 | 8·787 | 85-90 | 1·054 | 1·038 |
| 40-45 | 9·381 | 8·643 | 86-91 | ·902 | ·892 |
| 41-46 | 9·210 | 8·497 | 87-92 | ·738 | ·734 |
| 42-47 | 9·037 | 8·350 | 88-93 | .554 | ·547 |
| 43-48 | 8·862 | 8·200 | 89-94 | ·373 | ·369 |
| 44-49 | 8·683 | 8·046 | 90-95 | ·177 | ·175 |
| 45-50 | 8·503 | 7·891 | 91-96 | ·000 | ·000 |
| 46-51 | 8·326 | 7·737 | | | |

## TABLE IV.

*Showing the value of an Annuity on two joint lives, deduced from observations made at Northampton, the difference of ages being* 10 *years.—(Encyc. Metrop.)*

| Ages. | 4 per Cent. | 5 per Cent. |
|---|---|---|
| 1-11 | 10·782 | 9·544 |
| 2-12 | 12·438 | 11·010 |
| 3-13 | 13·019 | 11·528 |
| 4-14 | 13·374 | 11·850 |
| 5-15 | 13·479 | 11·954 |
| 6-16 | 13·578 | 12·052 |
| 7-17 | 13·599 | 12·083 |
| 8-18 | 13·569 | 12·070 |
| 9-19 | 13·482 | 12·006 |
| 10-20 | 13·355 | 11·906 |
| 11-21 | 13·217 | 11·797 |
| 12-22 | 13·078 | 11·686 |
| 13-23 | 12·934 | 11·570 |
| 14-24 | 12·784 | 11·450 |
| 15-25 | 12·630 | 11·324 |
| 16-26 | 12·470 | 11·193 |
| 17-27 | 12·311 | 11·063 |
| 18-28 | 12·158 | 10·939 |
| 19-29 | 12·013 | 10·820 |
| 20-30 | 11·873 | 10·707 |
| 21-31 | 11·742 | 10·600 |
| 22-32 | 11·615 | 10·498 |
| 23-33 | 11·485 | 10·393 |
| 24-34 | 11·352 | 10·285 |
| 25-35 | 11·217 | 10·175 |
| 26-36 | 11·078 | 10·062 |
| 27-37 | 10·936 | 9·946 |
| 28-38 | 10·791 | 9·826 |
| 29-39 | 10·642 | 9·703 |
| 30-40 | 10·490 | 9·576 |
| 31-41 | 10·336 | 9·448 |
| 32-42 | 10·182 | 9·320 |
| 33-43 | 10·027 | 9·190 |
| 34-44 | 9·869 | 9·058 |
| 35-45 | 9·706 | 8·921 |
| 36-46 | 9·540 | 8·781 |
| 37-47 | 9·370 | 8·636 |
| 38-48 | 9·195 | 8·487 |
| 39-49 | 9·015 | 8·333 |
| 40-50 | 8·834 | 8·177 |
| 41-51 | 8·658 | 8·025 |
| 42-52 | 8·483 | 7·875 |
| 43-53 | 8·308 | 7·724 |
| 44-54 | 8·130 | 7·569 |
| 45-55 | 7·948 | 7·411 |
| 46-56 | 7·763 | 7·249 |
| 47-57 | 7·574 | 7·084 |
| 48-58 | 7·382 | 6·915 |
| 49-59 | 7·186 | 6·742 |
| 50-60 | 6·989 | 6·568 |
| 51-61 | 6·795 | 6·395 |
| 52-62 | 6·600 | 6·222 |
| 53-63 | 6·399 | 6·042 |
| 54-64 | 6·196 | 5·860 |
| 55-65 | 5·986 | 5·671 |
| 56-66 | 5·774 | 5·479 |
| 57-67 | 5·559 | 5·283 |
| 58-68 | 5·341 | 5·084 |
| 59-69 | 5·121 | 4·883 |
| 60-70 | 4·900 | 4·680 |
| 61-71 | 4·679 | 4·476 |
| 62-72 | 4·458 | 4·272 |
| 63-73 | 4·236 | 4·066 |
| 64-74 | 4·019 | 3·864 |
| 65-75 | 3·806 | 3·665 |
| 66-76 | 3·606 | 3·477 |
| 67-77 | 3·405 | 3·289 |
| 68-78 | 3·199 | 3·095 |
| 69-79 | 2·979 | 2·887 |
| 70-80 | 2·757 | 2·675 |
| 71-81 | 2·542 | 2·470 |
| 72-82 | 2·334 | 2·271 |
| 73-83 | 2·141 | 2·085 |
| 74-84 | 1·991 | 1·941 |
| 75-85 | 1·856 | 1·811 |
| 76-86 | 1·739 | 1·699 |
| 77-87 | 1·633 | 1·597 |
| 78-88 | 1·546 | 1·514 |
| 79-89 | 1·427 | 1·400 |
| 80-90 | 1·278 | 1·255 |
| 81-91 | 1·078 | 1·061 |
| 82-92 | ·864 | ·852 |
| 83-93 | ·614 | ·606 |
| 84-94 | ·403 | ·398 |
| 85-95 | ·187 | ·185 |
| 86-96 | ·000 | ·000 |

### 3. *Of Assurances on Lives.*

This article has been already extended beyond its due limits; the following Table is therefore all that can be inserted on this subject.

*Terms of Assurance proposed by the Amicable Society, for assuring the sum of £100. upon the life of any healthy person from the age of* 8 *to* 72.

| Age. | For one year. | For 7 years. | For the whole life. | Age. | For one year. | For 7 years. | For the whole life. |
|---|---|---|---|---|---|---|---|
| | £. s. d. | £. s. d. | £. s. d. | | £. s. d. | £. s. d. | £. s. d. |
| 8 to 14 | 0 14 6 | 0 18 0 | 1 14 6 | 44 | 1 17 0 | 2 1 0 | 3 13 0 |
| 15 | 0 15 0 | 0 19 0 | 1 15 6 | 45 | 1 18 0 | 2 2 0 | 3 15 0 |
| 16 | 0 15 6 | 1 0 0 | 1 16 6 | 46 | 1 19 0 | 2 3 0 | 3 17 6 |
| 17 | 0 17 0 | 1 1 0 | 1 17 6 | 47 | 2 0 0 | 2 4 6 | 4 0 0 |
| 18 | 0 18 6 | 1 2 0 | 1 18 6 | 48 | 2 1 6 | 2 6 0 | 4 2 6 |
| 19 | 1 0 0 | 1 3 6 | 1 19 6 | 49 | 2 3 0 | 2 8 0 | 4 5 0 |
| 20 | 1 1 6 | 1 4 6 | 2 0 6 | 50 | 2 4 6 | 2 10 0 | 4 8 0 |
| 21 | 1 2 6 | 1 5 0 | 2 1 6 | 51 | 2 6 0 | 2 12 0 | 4 11 0 |
| 22 | 1 3 6 | 1 5 6 | 2 2 6 | 52 | 2 7 6 | 2 14 0 | 4 14 0 |
| 23 | 1 4 0 | 1 6 0 | 2 3 6 | 53 | 2 9 6 | 2 16 0 | 4 17 0 |
| 24 | 1 4 6 | 1 6 6 | 2 4 6 | 54 | 2 11 6 | 2 18 0 | 5 0 0 |
| 25 | 1 5 0 | 1 7 0 | 2 5 6 | 55 | 2 13 6 | 3 0 0 | 5 3 6 |
| 26 | 1 5 6 | 1 7 6 | 2 6 6 | 56 | 2 15 6 | 3 2 0 | 5 7 6 |
| 27 | 1 6 0 | 1 8 0 | 2 7 6 | 57 | 2 17 6 | 3 4 0 | 5 11 6 |
| 28 | 1 6 6 | 1 8 6 | 2 8 6 | 58 | 2 19 6 | 3 6 6 | 5 15 6 |
| 29 | 1 7 0 | 1 9 0 | 2 9 6 | 59 | 3 1 6 | 3 9 6 | 6 0 0 |
| 30 | 1 7 6 | 1 9 6 | 2 10 6 | 60 | 3 4 0 | 3 12 6 | 6 5 0 |
| 31 | 1 8 0 | 1 10 0 | 2 11 6 | 61 | 3 6 6 | 3 15 6 | 6 10 0 |
| 32 | 1 8 6 | 1 10 6 | 2 12 6 | 62 | 3 9 0 | 3 19 0 | 6 15 6 |
| 33 | 1 9 0 | 1 11 0 | 2 14 0 | 63 | 3 11 6 | 4 2 6 | 7 1 0 |
| 34 | 1 9 6 | 1 11 6 | 2 15 6 | 64 | 3 14 6 | 4 7 0 | 7 7 6 |
| 35 | 1 10 0 | 1 12 0 | 2 17 0 | 65 | 3 18 0 | 4 12 0 | 7 14 6 |
| 36 | 1 10 6 | 1 13 0 | 2 18 6 | 66 | 4 2 0 | 4 17 6 | 8 2 0 |
| 37 | 1 11 0 | 1 14 0 | 3 0 0 | 67 | 4 6 0 | 5 4 0 | 8 10 0 |
| 38 | 1 11 6 | 1 15 0 | 3 1 6 | 68 | 4 13 6 | 5 13 6 | 8 19 6 |
| 39 | 1 12 0 | 1 16 0 | 3 3 0 | 69 | 5 1 6 | 6 4 0 | 9 9 0 |
| 40 | 1 13 0 | 1 17 0 | 3 5 0 | 70 | 5 10 6 | 6 17 6 | 9 19 6 |
| 41 | 1 14 0 | 1 18 0 | 3 7 0 | 71 | 6 1 0 | 7 14 6 | 10 10 0 |
| 42 | 1 15 0 | 1 19 0 | 3 9 0 | 72 | 6 13 0 | 8 16 0 | 11 2 0 |
| 43 | 1 16 0 | 2 0 0 | 3 11 0 | | | | |

*Ex.* Let it be proposed to determine the annual payment to be made by a person aged 42, to insure £1000. payable at his decease.

| | £. s. |
|---|---|
| By Table, annual payment per cent. ......... | 3 9 |
| Multiply by ......... | 10 |
| | £34 10 |

**ANOMALISTIC** *Year.—See Earth* Elements of.

**ANOMALY,** in *Astronomy.—(Maddy, Playfair.)*

Given the mean anomaly ($m$), to find the true ($v$), (usually called Kepler's Problem.)

*1st Method.*—If the eccentricity ($e$) be *very* small,

$$\tan.\ \tfrac{1}{2}\, v = \frac{1+e}{1-e}.\ \tan.\ \tfrac{1}{2}\, m.$$

*2d Method.*—Let $u$ be the eccentric anomaly, and let the true and mean anomalies be measured from *aphelion;* then we have the following equations:—

$$m = u + e \sin.\ u.$$

$$\&\ \tan.\ \tfrac{1}{2}\, v = \sqrt{\frac{1-e}{1+e}}.\ \tan.\ \tfrac{1}{2}\, u.$$

Therefore, eliminating $u$ between these two equations, the relation between $m$ and $v$ may be found.

If the anomalies are measured from *perihelion,*

$$m = u - e \sin.\ u.$$

$$\&\ \tan.\ \tfrac{1}{2}\, v = \sqrt{\frac{1+e}{1-e}}.\ \tan.\ \tfrac{1}{2}\, u.$$

The following is the series for $v$ in terms of $m$.

$$v = m - \left(2\,e - \frac{1}{4}\,e^3 + \frac{5}{96}\,e^5\right) \sin.\ m + \left(\frac{5}{4}\,e^2 - \frac{11}{24}\,e^4 + \frac{17}{192}\,e^6\right)$$

$$\sin.\ 2\,m + \left(\frac{13}{12}\,e^3 - \frac{43}{64}\,e^5\right) \sin.\ 3\,m + \left(\frac{103}{96}\,e^4 - \frac{451}{480}\,e^6\right) \sin\ 4\ m$$

$$+\ \frac{1097}{960}\,e^5 \sin.\ 5\,m + \frac{1223}{960}\,e^6 \sin.\ 6\,m.$$

*Note.*—The constant coefficients must be reduced into degrees and minutes, by multiplying each of them by $57^0.29578$, the number of degrees in an arc equal to the radius.

*3d Method.*—To find the true anomaly in terms of the mean, in a series ascending by powers of $e$.

$$v = m + 2 \sin m.\ e + \frac{5}{4} \sin.\ 2\,m.\ e^2 + \&c.$$

And to find $m$ in terms of $v$,

$$m = v - (2\,c + e.\ \overline{1-c^2}) \sin.\ v + (e^2 + c\,e.\ \overline{1-c^2}) \sin.\ 2\,v - \&c.$$

$$\text{where } c = \frac{1-\sqrt{1-e^2}}{e}$$

The radius vector $r$ may also be expressed in terms of the mean anomaly, supposing the mean distance 1.

$$r = 1 + \frac{1}{2} e^2 - \left( e - \frac{3}{8} e^3 + \frac{5}{192} e^5 \right) \cos. m + \left( -\frac{1}{2} e^2 + \frac{1}{3} e^4 - \frac{1}{16} e^6 \right)$$

$$\cos. 2m - \left( \frac{3}{8} e^3 - \frac{45}{128} e^5 \right) \cos. 3m + \left( -\frac{1}{3} e^4 + \frac{2}{5} e^6 \right) \cos. 4m$$

$$- \left( \frac{125}{384} e^5 \right) \cos. 5m + \left( -\frac{27}{80} e^6 \right) \cos. 6m.$$

In the case of the sun, $e$ being small (viz. ·016814) its powers above the 3d. may be neglected, and in this case $y = (1^\circ.\ 55'.\ 26'', 35) \sin. m + (1'.\ 12'', 68) \sin. 2m + (1''.\ 05) \sin. 3m$.

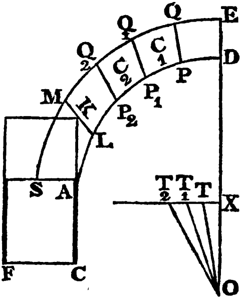
And $r = 1 + \frac{1}{2} e^2 - e \cos. m - \frac{1}{2} e^2 \cos. 2m$.

When $m$ is computed from the apogee instead of the perigee, the signs of the terms involving the *odd* multiples of $m$ must be changed.

ARCHES, *Equilibrium* of.—(*Whewell, Playfair.*)

1. In an arch which is in equilibrium, the weights of the voussoirs are as the differences of the tangents of the angles which their joints make with the vertical.

Hence if O T be in the line of the joint P Q or parallel to it, $OT_1$ parallel to $P_1Q_1$, &c., and $TT_1$ be horizontal, the weights of the voussoirs $C_1$, $C_2$ &c. will be as the portions $TT_1$, $T_1T_2$ &c.



*Cor.* 1. If the arch is a circle, the weights of the voussoirs are as the differences of the tangents of the arches, reckoned from the crown. This is nothing more than the general proposition above, applied to a particular case.

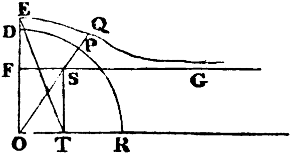
*Note.*—As the stones themselves cannot always be made in the proportion thus required; the wedges, of which they make parts, are supposed to be extended upward by courses of masonry. The whole mass included between the planes of the joints produced, as far as that masonry extends, is understood to make up the weight of the voussoirs.

*Cor.* 2. The *horizontal* pressure is represented by O X, and is the same at each joint.

*Cor.* 3. The pressures at the joints are represented by O T, O $T_1$ &c. and are therefore as the secants of the $\angle^{s.}$ which the joints make with the vertical. If $\theta$ be the $\angle$ of any joint with the vertical, and H the horizontal pressure, H sec. $\theta$ is the pressure at that joint.

*Cor.* 4. The line X $T_2$ will represent the whole weight of the mass between D E and $P_2$ $Q_2$, and similarly for any other joint; hence H tan. $\theta$ is the weight of any portion.

2. The intrados being a circle, with the joints in the direction of the radii, to find the extrados, so that the voussoirs may be in equilibrium.



Let P be any point of the intrados, O its centre, put D O P $= \theta$, O D $=$ O P $= l$, O Q $= r$, O E $= k$, then

$$r^2 = l^2 + (k^2 - l^2) \sec.^2 \theta.$$

Hence we have the following construction. Make O R horizontal, R F $=$ O E, F G horizontal. Let O P meet F G in S, draw S T vertical, and take O Q $=$ E T; the *locus* of Q will be the extrados.

*Cor.* 1. The extrados has F G for an asymptote.

*Cor.* 2. To find the equation to the extrados. Let O be the origin of the coordinates, $x$ and $y$ corresponding coordinates to the point Q, O D $= l$, O F $= a$; then the Equation to the curve is

$$x = \frac{y\sqrt{(l^2 + a^2 - y^2)}}{\sqrt{(y^2 - a^2)}}.$$

The extrados, in the case of a circular arch, is therefore a curve of the 4th order, very much resembling the conchoid of Nicomedes. It has an asymptote F G and also a point of contrary flexure, so that it coincides very nearly with the curve in which a road is usually carried over a bridge.

3. In an elliptic arch, or one of which the intrados is a semi-ellipse, if 2 $a$ be the span of the arch or the major axis of the ellipse, and $b$ the height of the arch or the semi-conjugate axis; then if from any point in

the curve a perpendicular $x$ be let fall on the longer axis, and W be the weight of the key-stone, the weight V of any voussoir is

$$\frac{a^3 W}{x^3 \sqrt{\left(a^2 + \frac{a^2 - b^2}{a^2} x^2\right)}} .$$

4. If the weights of the voussoirs are all equal, the arch of equilibrium is a catenarian curve, the same that a chain of uniform thickness would assume, if hanging freely; the horizontal distance of the points of suspension being equal to the span of the arch, and the depth of the lowest points of the chain being equal to the greatest height of the arch.

The equation to the catenary, if $x$ and $y$ be the corresponding coordinates from the vertex along the axis or vertical line, is

$$y = a + \text{h, l.}\ \frac{a + x + \sqrt{2\,a\,x + x^2}}{a}.$$

The constant quantity $a$ may be determined by experiment; for the chain being suspended; let a tangent be drawn to any point of the curve, and produced till it meet the axis; then as the subtangent is to the ordinate, so is the length of the chain, between the given point and the vertex, to the quantity $a$. When $a$ is found, the curve can be constructed.

5. The pressure of an arch on the piers or abutments which support it, may be estimated by considering the parts of the arch, which rest immediately on the abutments to a certain height, as parts of the abutments themselves; and the remainder of the arch as a wedge; tending to separate the abutments from one another.

Thus the part A L M S (see above Fig.) which would remain in its place though there were no pressure from above, may be regarded as a part of the pier, and L M E D &c., the remainder of the arch, as a wedge tending to overthrow the pier by its pressure on the plane M L. On these suppositions the thickness of the piers, so that their weight shall enable them to resist this pressure, may be determined.

Let the $\angle$ which M L makes with the vertical $= \theta$, twice the area M L D E $= a^2$, C K $= h$, and F C $= x$, then

$$x = -\frac{a^2}{2\,h\,\cos.^2\theta} + a\sqrt{\left(\frac{2}{\sin.\,2\,\theta} + \frac{a^2}{4\,h^2\cos.^4\theta}\right)}.$$

In the above demonstration, the hypothesis is that the pier A F, if the weight of the arch were too great to be sustained, would fall, by turning round F as a fulcrum. Now this is not what would happen; the part of the abutment behind S M would be thrust out in the horizontal direction, till the arch had room to fall; it is therefore against the masonry immediately behind the part A M, and chiefly in a horizontal direction, that the force is exerted.

**ARCHIMEDES'** *Spiral.*—See *Spiral.*

**ARCS** *Circular to find length of, in terms of the radius.—( Vince. )*

TABLE,

*For finding the length of Circular Arcs to Radius Unity.*

| Deg. | Length. | Deg. | Length. | Min. | Length. | Sec. | Length. |
|---|---|---|---|---|---|---|---|
| 1 | 0,0174533 | 60 | 1,0471976 | 1 | 0,0002909 | 1 | 0,0000048 |
| 2 | 0,0349066 | 70 | 1,2217305 | 2 | 0,0005818 | 2 | 0,0000097 |
| 3 | 0,0523599 | 80 | 1,3962634 | 3 | 0,0008727 | 3 | 0,0000145 |
| 4 | 0,0698132 | 90 | 1,5707963 | 4 | 0,0011636 | 4 | 0,0000194 |
| 5 | 0,0872665 | 100 | 1,7453293 | 5 | 0,0014544 | 5 | 0,0000242 |
| 6 | 0,1047198 | 120 | 2,0943951 | 6 | 0,0017453 | 6 | 0,0000291 |
| 7 | 0,1221730 | 150 | 2,6179939 | 7 | 0,0020362 | 7 | 0,0000339 |
| 8 | 0,1396263 | 180 | 3,1415927 | 8 | 0,0023271 | 8 | 0,0000388 |
| 9 | 0,1570796 | 210 | 3,6651914 | 9 | 0,0026180 | 9 | 0,0000436 |
| 10 | 0,1745329 | 240 | 4,1887902 | 10 | 0,0029089 | 10 | 0,0000485 |
| 20 | 0,3490659 | 270 | 4,7123890 | 20 | 0,0058178 | 20 | 0,0000970 |
| 30 | 0,5235988 | 300 | 5,2359878 | 30 | 0,0087266 | 30 | 0,0001454 |
| 40 | 0,6981317 | 330 | 5,7595865 | 40 | 0,0116355 | 40 | 0,0001939 |
| 50 | 0,8726646 | 360 | 6,2831853 | 50 | 0,0145444 | 50 | 0,0002424 |

Circular arc = radius = $57^{0}$. 2957795 = $57^{0}$. 17′. 44″, 8.

*Ex.* What is the length of a circular arc of $37^{0}$. 42′. 58″ ?

$30^{0}$ —— 0·5235988
$7^{0}$ —— 0·1221730
40′ —— 0·0116355
2′ —— 0·0005818
50″ —— 0·0002424
8″ —— 0·0000388

Length required 0·6582703

**ARCS** *Semi-diurnal.—(Vince.)*

## TABLES *of Semi-diurnal Arcs.*

**Latitude and Declination of the *same* kind.**

| Declination. | LATITUDE. | | | | | | |
|---|---|---|---|---|---|---|---|
| | 50° | 51° | 52° | 53° | 54° | 55° | 56° |
| D. | H. M. | H. M. | H. M. | H. M. | H. M. | H. M. | H. M. |
| 1 | 6. 8 | 6. 8 | 6. 9 | 6. 9 | 6. 9 | 6. 9 | 6. 10 |
| 2 | 6. 13 | 6. 13 | 6. 14 | 6. 14 | 6. 15 | 6. 15 | 6. 16 |
| 3 | 6. 18 | 6. 18 | 6. 19 | 6. 19 | 6. 20 | 6. 21 | 6. 22 |
| 4 | 6. 22 | 6. 22 | 6. 24 | 6. 25 | 6. 26 | 6. 27 | 6. 28 |
| 5 | 6. 27 | 6. 27 | 6. 29 | 6. 30 | 6. 31 | 6. 32 | 6. 34 |
| 6 | 6. 32 | 6. 33 | 6. 34 | 6. 36 | 6. 37 | 6. 38 | 6. 40 |
| 7 | 6. 37 | 6. 38 | 6. 40 | 6. 41 | 6. 43 | 6. 44 | 6. 46 |
| 8 | 6. 42 | 6. 43 | 6. 45 | 6. 47 | 6. 48 | 6. 50 | 6. 52 |
| 9 | 6. 47 | 6. 48 | 6. 50 | 6. 52 | 6. 54 | 6. 56 | 6. 58 |
| 10 | 6. 52 | 6. 54 | 6. 56 | 6. 58 | 7. 0 | 7. 2 | 7. 5 |
| 11 | 6. 57 | 6. 59 | 7. 1 | 7. 3 | 7. 6 | 7. 8 | 7. 11 |
| 12 | 7. 2 | 7. 4 | 7. 7 | 7. 9 | 7. 12 | 7. 15 | 7. 18 |
| 13 | 7. 7 | 7. 10 | 7. 12 | 7. 15 | 7. 18 | 7. 21 | 7. 24 |
| 14 | 7. 13 | 7. 15 | 7. 18 | 7. 21 | 7. 24 | 7. 28 | 7. 31 |
| 15 | 7. 18 | 7. 21 | 7. 24 | 7. 27 | 7. 31 | 7. 34 | 7. 39 |
| 16 | 7. 24 | 7. 27 | 7. 30 | 7. 33 | 7. 37 | 7. 41 | 7. 45 |
| 17 | 7. 29 | 7. 33 | 7. 36 | 7. 40 | 7. 44 | 7. 48 | 7. 52 |
| 18 | 7. 35 | 7. 38 | 7. 42 | 7. 46 | 7. 51 | 7. 55 | 8. 0 |
| 19 | 7. 41 | 7. 45 | 7. 49 | 7. 53 | 7. 58 | 8. 2 | 8. 7 |
| 20 | 7. 47 | 7. 51 | 7. 55 | 8. 0 | 8. 5 | 8. 10 | 8. 15 |
| 21 | 7. 53 | 7. 57 | 8. 2 | 8. 7 | 8. 12 | 8. 18 | 8. 24 |
| 22 | 7. 59 | 8. 4 | 8. 9 | 8. 14 | 8. 20 | 8. 26 | 8. 32 |
| 23 | 8. 6 | 8. 11 | 8. 16 | 8. 22 | 8. 28 | 8. 34 | 8. 41 |
| 24 | 8. 12 | 8. 18 | 8. 24 | 8. 30 | 8. 36 | 8. 43 | 8. 51 |
| 25 | 8. 19 | 8. 25 | 8. 31 | 8. 38 | 8. 45 | 8. 53 | 9. 1 |
| 26 | 8. 27 | 8. 33 | 8. 39 | 8. 47 | 8. 54 | 9. 2 | 9. 11 |
| 27 | 8. 34 | 8. 41 | 8. 48 | 8. 56 | 9. 4 | 9. 13 | 9. 23 |
| 28 | 8. 42 | 8. 49 | 8. 57 | 9. 5 | 9. 14 | 9. 24 | 9. 35 |
| 29 | 8. 50 | 8. 58 | 9. 6 | 9. 14 | 9. 25 | 9. 36 | 9. 49 |
| 30 | 8. 59 | 9. 8 | 9. 17 | 2. 26 | 9. 38 | 9. 50 | 10. 4 |
| 31 | 9. 9 | 9. 18 | 9. 28 | 9. 38 | 9. 51 | 10. 5 | 10. 22 |
| 32 | 9. 19 | 9. 28 | 9. 39 | 9. 52 | 10. 6 | 10. 23 | 10. 44 |

Latitude and Declination of *different* kinds.

| Declination. | LATITUDE. | | | | | | |
|---|---|---|---|---|---|---|---|
| | 50° | 51° | 52° | 53° | 54° | 55° | 56° |
| D. | H. M. | H. M. | H. M. | H. M. | H. M. | H. M. | H. M. |
| 1 | 5. 59 | 5. 58 | 5. 58 | 5. 58 | 5. 58 | 5. 58 | 5. 58 |
| 2 | 5. 54 | 5. 53 | 5. 53 | 5. 53 | 5. 53 | 5. 52 | 5, 52 |
| 3 | 5. 49 | 5. 49 | 5. 48 | 5. 48 | 5. 47 | 5. 47 | 5. 46 |
| 4 | 5. 44 | 5. 44 | 5. 43 | 5. 42 | 5. 42 | 5. 41 | 5. 40 |
| 5 | 5. 39 | 5. 39 | 5. 38 | 5. 37 | 5. 36 | 5. 35 | 5. 34 |
| 6 | 5. 35 | 5. 34 | 5. 33 | 5. 31 | 5. 30 | 5. 29 | 5. 28 |
| 7 | 5. 30 | 5. 29 | 5. 27 | 5. 26 | 5. 25 | 5. 23 | 5. 22 |
| 8 | 5. 25 | 5. 23 | 5. 22 | 5. 21 | 5. 19 | 5. 17 | 5. 16 |
| 9 | 5. 20 | 5. 18 | 5. 17 | 5. 16 | 5. 13 | 5. 12 | 5. 10 |
| 10 | 5. 15 | 5. 13 | 5. 11 | 5. 10 | 5. 8 | 5. 5 | 5. 3 |
| 11 | 5. 10 | 5. 8 | 5. 6 | 5. 4 | 5. 2 | 4. 59 | 4. 57 |
| 12 | 5. 5 | 5. 3 | 5. 0 | 4. 58 | 4. 56 | 4. 53 | 4. 51 |
| 13 | 5. 0 | 4. 57 | 4. 55 | 4. 52 | 4. 50 | 4. 47 | 4. 44 |
| 14 | 4. 54 | 4. 52 | 4. 49 | 4. 47 | 4. 44 | 4. 41 | 4. 37 |
| 15 | 4. 49 | 4. 46 | 4. 44 | 4. 41 | 4. 37 | 4. 34 | 4. 31 |
| 16 | 4. 45 | 4. 41 | 4. 38 | 4. 34 | 4. 31 | 4. 27 | 4. 24 |
| 17 | 4. 38 | 4. 35 | 4. 32 | 4. 28 | 4. 23 | 4. 21 | 4. 17 |
| 18 | 4. 33 | 4. 29 | 4. 26 | 4. 22 | 4. 18 | 4. 14 | 4. 9 |
| 19 | 4. 27 | 4. 23 | 4. 19 | 4. 15 | 4. 11 | 4. 7 | 4. 2 |
| 20 | 4. 21 | 4. 17 | 4. 13 | 4. 9 | 4. 4 | 3. 59 | 3. 54 |
| 21 | 4. 15 | 4. 11 | 4. 6 | 4. 2 | 3. 57 | 3. 52 | 3. 46 |
| 22 | 4. 9 | 4. 4 | 4. 0 | 3. 55 | 3. 50 | 3. 44 | 3. 38 |
| 23 | 4. 3 | 3. 58 | 3. 53 | 3. 47 | 3. 42 | 3. 36 | 3. 29 |
| 24 | 3. 56 | 3. 51 | 3. 46 | 3. 40 | 3. 34 | 3. 27 | 3. 20 |
| 25 | 3. 49 | 3. 44 | 3. 38 | 3. 32 | 3. 25 | 3. 18 | 3. 11 |
| 26 | 3. 42 | 3. 37 | 3. 30 | 3. 24 | 3. 17 | 3. 9 | 3. 1 |
| 27 | 3. 35 | 3. 29 | 3. 22 | 3. 15 | 3. 8 | 2. 59 | 2. 50 |
| 28 | 3. 28 | 3. 21 | 3. 14 | 3. 6 | 2. 58 | 2. 49 | 2. 38 |
| 29 | 3. 20 | 3. 12 | 3. 5 | 2. 56 | 2. 47 | 2. 37 | 2. 26 |
| 30 | 3. 11 | 3. 4 | 2. 55 | 2. 46 | 2. 36 | 2. 25 | 2. 13 |
| 31 | 3. 3 | 2. 54 | 2. 45 | 2. 35 | 2. 24 | 2. 12 | 1. 57 |
| 32 | 2. 53 | 2. 24 | 2. 44 | 2. 23 | 2. 11 | 1. 57 | 1. 40 |

## *Explanation of the Tables.*

The first is a Table of semi-diurnal arcs, when the latitude of the place and the declination of the body are of the same kind; the 2d. when the latitude and declination are of different kinds. The first column of each

Table contains the declination of the body from 1° to 32°, and at the top of each succeeding column is set down the latitude of the place from 50° to 56° both inclusive.

For the *sun*, the arc gives the time of its setting, and if it be subtracted from twelve o'clock, you get the time of its rising.

For a *star*, add and subtract the equation to and from the time at which the star passes the meridian, and you have the time of its setting and rising.

The time so given is the hour when the centre of the sun appears in the horizon, the eye being at the surface of the earth; thereby taking into consideration the effect of refraction.

*Example.*—In latitude 52°. 12′., and declination of the sun 23°. 28′., what is the time of its rising and setting?

| | | h. | m. |
|---|---|---|---|
| Latitude 52°. declination 23°. .......................... | arc | 8. | 16 |
| ——— 53. ——— .......................... | arc | 8. | 22 |
| 1 | | | 6 |

Hence 1° : 12′ :: 6*m* : 1*m*, to be added to 8*h*. 16*m*.

| | | h. | m. |
|---|---|---|---|
| Latitude 52°, declination 23° .......................... | arc | 8. | 16 |
| ——— 24 .......................... | arc | 8. | 24 |
| 1 | | | 8 |

Hence 1° : 28′ :: 8*m* : 4*m*, to be added also to 8*h*. 16*m*.

Therefore the semi-diurnal arc = 8*h*. 16*m*. + 1*m*. + 4*m*. = 8*h*. 21*m*. the time of setting; and 3*h*. 39*m*. = time of rising.

AREAS *of Curves*, whose Equations are given.

Let $x$ and $y$ be the abscissa and ordinate of the curve, then

$$\text{Area} = \text{fl. } y\,dx.$$

*Ex.* 1.—Area of a triangle = base × ½ perpendicular.

2. Area of the common parabola = $\frac{2}{3}\,xy$ = $\frac{2}{3}$ of the circumscribing rectangle. Or if the general equation is $a^{n-1}x = y^n$, area $= \frac{n}{n+1} \times xy$.

3. Area of circle whose radius = 1 is 3. 14159 &c. or if rad. = $r$, and $\pi = 3.\ 14159$ &c. area $= \pi\, r^2$; or in terms of circumference $C = C.\ \frac{r}{2}$.

4. Area of ellipse, if $a$ and $b$ = ½ ax. maj. and min., $= \pi.\ a\,b$.

5. Area of cycloid = 3 times area of the generating circle.

6. In the hyperbola the area between the asymptotes = fl. $\frac{m^2 dx}{x}$, (assuming $yx = m^2$), the hyperbola being equi-lateral; ∴ area = $m^2$ log. $x$ + C; and assuming it = $o$ when $x = m$, we shall have $m^2$ log. $\frac{x}{m}$ as the general expression for the area.

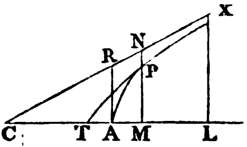
If $m = 1$, the areas are the hyperbolic logarithms of the corresponding abscissæ; and hence the origin of the term *hyperbolic* as applied to logarithms.

*For Areas of Spirals.—See Spiral.*

ARITHMETICAL *Progression.—See Progression.*

ASSURANCE *on Lives.—See Annuities.*

ASYMPTÒTES, *to draw.*



Find the value of $\frac{y\,dx}{dy}$ = subtangent M T; ∴ A T = $\frac{y\,dx}{dy} - x$ is known. Now suppose $x$ to become infinite, and T to move on to C; then if A C be finite the curve admits an asymptote. Next find the ratio of T M : M P, which, if we again suppose $x$ infinite, gives us the ratio of C L : L $x$; then by similar △s C L : L $x$ :: C A : A R, of which proportion the three first terms are known, and ∴ A R can be determined. Join C R, and produce it indefinitely, and C R is the asymptote required.

*Ex.* 1.—To draw an asymptote to the common hyperbola.

Here A T = $\frac{2ax + x^2}{a + x} - x$ (when $x$ is infinite) = $a$ = A C. Again T M : M P :: $\frac{2ax + x^2}{a + x} : \frac{b}{a}\sqrt{2ax + x^2}$ :: (when $x$ is infinite) $x : \frac{bx}{a}$ :: C L : L $x$ :: C A ($a$) : A R, ∴ A R = $b$; Hence from A draw A R = $b$; take C the centre, and join C R, and produce it indefinitely, and C R $x$ is the asymptote.

*Ex.* 2.—Let the equation be $y^3 = ax^2 + x^3$.

Proceed just as before, and we get C L = $x$, L $x$ = $x$; A C = $\frac{a}{3}$, ∴ $x : x :: \frac{a}{3}$ : A R = $\frac{a}{3}$ ∴ &c.

## ATMOSPHERE.

*Atmospheric air, properties of.*

1. Fluidity, elasticity, expansibility, and gravity.

*Atmospheric air, composition of.*

2. Nitrogen 79 parts, oxygen 21, and about 1 part in 1000 of carbonic acid gas. It also contains about 1 per cent. of water in the state of elastic vapour. If the calculation be made by weight, there will be, in every 100 measures of atmospheric air, 23¼ of oxygen, and 76¾ of nitrogen.

*Atmospheric air, specific gravity of.*

3. Specific gravity of air : that of water :: 1 : 832 or 833, when reduced to the pressure of 30 inches of the barometer, and the mean temperature of 55°. of the thermometer. 100 cubic inches of air at the surface of the sea, when the thermometer is at 60°, weigh 30½ grains.

*Atmospheric air, rarefaction and condensation of.*

4. The ratio of the spaces occupied by a given quantity of air in its greatest state of rarefaction, is to the same under the highest degree of condensation, as 550.000 to 1.

*Atmosphere, weight or pressure of.*

5. The pressure of the atmosphere in its mean state is equal to a column of quicksilver of an equal base and 30 inches high, or to a column of water of 34 feet in height. Hence its weight on every square inch is nearly equal to 15lbs. Mr Cotes computed that the pressure of this ambient fluid on the whole surface of the earth is equivalent to that of a globe of lead of 60 miles in diameter; and admitting the surface of a man's body to be about 15 square feet, he must sustain 32,400 lbs, or nearly 14½ tons weight. But since the variation in the height of the mercurial column may occupy a range of 3 inches, every square inch base on any body may at one time be pressed more than it is at others by a weight equal to three cubic inches of mercury. Hence it may be easily shewn that the difference in the weight of air, sustained by our bodies, in different states of the asmosphere, is often near a ton and a half.

*Atmosphere homogeneous, height of.*

6. Let $H =$ height of homogeneous atmosphere, $\delta$ its uniform density, $b$ the height of the barometer in feet, and D the density of the mercury, then

$$H = \frac{b\,D}{\delta}.$$

At a medium $\delta : D :: 1^2_9 : 13600$; and $b$ at a mean $=$ 30 inches $= 2\frac{1}{2}$ feet,

$$\therefore\ H = \frac{2\frac{1}{2} \times 13600}{1^2} = 27918 \text{ feet}, = \text{rather more than } 5\tfrac{1}{4} \text{ miles}.$$

*Atmosphere, density of.*

7. The density of the air is in proportion to the force which compresses it, or to its elasticity, or inversely as the spaces within which the same quantity of it is contained.

8. If altitudes be taken from the earth's surface in arithmetical progression, the density of the air decreases in geometrical progression.

9. Given the altitude above the earth's surface, to find the density of the air; and conversely.

Let $y =$ density at the distance $x$ from the earth's surface, $\delta$ the density at the surface, and $h$ the height of the homogeneous atmosphere, then

$$y = \delta \times e^{\frac{-x}{h}}, \text{ or by Art. 6, } y = \delta \times e^{-\frac{\delta x}{b\,\mathrm{D}}}.$$

Or conversely, having given the density to find the altitude, we have

$$x = h \times \text{hyp. log. } \frac{\delta}{y}; \text{ or in common logs. nearly } x = 1000 \times \text{log. } \frac{\delta}{y}.$$

In the above formulæ $\delta$ and $y$ denote the atmospherical pressures at the surface and altitude $x$, for which we may substitute M and $m$, the altitudes of the mercury in the barometer at those distances; we shall then have

$$x = 1000 \times \text{log. } \frac{\mathrm{M}}{m}.$$

This gives only the approximate height; for the correct formula—*see Barometer.*

10. If, instead of supposing gravity constant, we assume it to vary inversely as the $n^{th}$ power of the distance, we shall have, putting the earth's radius $= r$,

$$y = \delta . \frac{1}{e^{\frac{r}{(n-1)h} \cdot \frac{(r+x)^{n-1} - r^{n-1}}{(r+x)^{n-1}}}},$$

which is a general Equation, expressing the relation between the altitude and density.

*Cor.* If F varies as $\frac{1}{\mathrm{D}^2}$, $y = \delta . \frac{1}{e^{\frac{r}{h} \cdot \left(\frac{1}{r} - \frac{1}{r+x}\right)}}$;

hence if $r + x$ increase in harmonical progression, $\frac{1}{r+x}$ is in arithmetic, and $\therefore$, the densities themselves will decrease in geometric.

11. TABLE *exhibiting the comparative density of the air at the several corresponding heights.*

| Height in miles. | Rarity. | Height. | Rarity. |
|---|---|---|---|
| 0 | 1 | 35 | 1024 |
| 3½ | 2 | 42 | 4096 |
| 7 | 4 | 49 | 16384 |
| 14 | 16 | 56 | 65536 |
| 21 | 64 | 63 | 262144 |
| 28 | 256 | 70 | 1048576 |

And by pursuing the calculation, it might easily be shown that a cubic inch of the air we breathe would be so much rarified at the height of 500 miles, that it would fill a sphere equal in diameter to the orbit of Saturn.

*Atmosphere, refractive and reflective powers of.*

12. The altitude above the earth's surface at which the atmosphere begins to have any sensible effect on the rays of light to refract them $=$ 77.25 miles; and the altitude at which reflection begins $= 39.64$ miles, $=$ about half the altitude at which refraction begins.—(*Vince.*)

How much farther than this the atmosphere may extend, it is impossible to ascertain; it must, however, at all events, be limited in its extent by the centrifugal force of the earth, and the attraction of the moon.

For terrestrial refraction, and the refraction of the heavenly bodies—*see Refraction.*

*Atmosphere, motion of.*

13. To determine the velocity with which atmospheric air will rush into a vacuum, let $h =$ height of homogeneous atmosphere, and $v$ the required velocity, $g = 32\frac{1}{6}$ feet, then

$$v = \sqrt{2gh} = 8\sqrt{h} \text{ nearly}, = \text{at a medium } 1339 \text{ feet.}$$

14. To find the velocity with which air rushes into a medium rarer than itself, put V $=$ velocity with which it rushes into a vacuum, D the natural density of the air, and $\delta$ the density of the air contained in the vessel into which it is supposed to run; then

$$v = V\sqrt{\frac{D - \delta}{D}}.$$

15. To find the time in which air will fill a vacuum of given dimensions, put C $=$ capacity of the vessel in cubic feet, A the area of the section of the orifice, $h =$ height of homogeneous atmosphere; then

$$t = \frac{C}{4A\sqrt{h}}.$$

*Atmosphere, law of repulsion in the particles of.*

16. In general if the particles of a fluid repel each other with forces varying inversely as the $n^{th}$ power of their distances or as $\frac{1}{\delta n}$, & $d$ represent the density of any part, and $c$ the compressive force upon it; then

$$c \text{ varies as } d^{\frac{n+2}{3}} \text{ or varies as } \frac{1}{\delta^{n+2}}.$$

It appears by experiment, that the compressive force of atmospheric air varies as the density, $\therefore \frac{n+2}{3} = 1$ or $n = 1$; consequently the particles of air repel each other with forces which vary inversely as their distances.

*Cor.* This fluid will be elastic, if $n + 2$ be positive.

*Atmosphere, temperature of.*

17. Various formulæ for the mean temperature of any place at the level of the sea.

*Playfair's formula.*

$t = 58^{0} + 27^{0} \times \cos 2$ latitude.—Fahrenheit.

When 2 latitude is greater than 90°, cos 2 latitude is negative.

*Leslie's formula.*

$t = \cos^{2}$ lat. $\times 29^{0}$.—Centigrade.

*Daubisson's formula.*

$t = 27^{0} \times \cos^{2}$ lat.—Centigrade.

*Brewster's formula.*

For the old world, $t = 81\frac{1}{2}^{0} \times \cos$ lat.—Fahrenheit.
For the new, $t = 81\frac{1}{2}^{0} \times \cos^{2}$ lat. $\times 1.13$.

*Atkinson's formula.*

Deduced from Humboldt's observations in the new world.—*(See Mem. Astron. Soc.)*

$t = 97^{0}, 08 \times \cos^{\frac{3}{2}}$ lat.—$10^{0}, 53$.—Fahrenheit.

TABLE *of mean temperature at the level of the sea in different latitudes, calculated from Leslie's formula.*

| Lat. | Cent. | Fahr. |
|---|---|---|
| 0° | 29° | 84.2 |
| 5 | 28.78 | 83.8 |
| 10 | 28.13 | 82.6 |
| 15 | 27.06 | 80.7 |
| 20 | 25.61 | 78.1 |
| 25 | 23.82 | 74.9 |
| 30 | 21.75 | 71.1 |
| 35 | 19.46 | 67.0 |
| 40 | 17.01 | 62.6 |
| 45 | 14.50 | 58.1 |
| 50 | 11.98 | 53.6 |
| 51 | 11.49 | 52.7 |
| 52 | 10.99 | 51.8 |
| 53 | 10.50 | 50.9 |
| 54 | 10.02 | 50.0 |
| 55 | 9.54 | 49.2 |
| 56 | 9.07 | 48.3 |
| 57 | 8.60 | 47.5 |
| 58 | 8.14 | 46.6 |
| 59 | 7.69 | 45.8 |
| 60 | 7.25 | 45.0 |
| 65 | 5.18 | 41.3 |
| 70 | 3.39 | 38.1 |
| 75 | 1.94 | 35.5 |
| 80 | 0.86 | 33.6 |
| 85 | 0.22 | 32.4 |
| 90 | 0.0 | 32.0 |

Mean temperature of London, as observed at the apartments of the Royal Society for 20 years, from 1790 to 1809, = 50° 94. The greatest annual temperature during that time was 53°. 2, the least 48°. 5.

18. In ascending from the level of the sea, this mean temperature decreases nearly uniformly, though accurately the decrease seems somewhat slower as we ascend. Playfair calculates the diminution of heat at the rate of 1° for 270 feet nearly, when not far from the surface of the earth. Leslie allows 300 feet at the earth's surface; and at 1, 2, 3, 4, and 5 miles altitude, 295, 277, 252, 223, and 192 feet respectively, for every degree of Fahrenheit.

Hence to find the mean temperature at any height $h$ above the level of the sea, we must subtract from the formulæ in the last Art. $\frac{h}{270}$ according to Playfair, $\frac{h}{300}$ according to Leslie, and $\frac{h}{251 + \frac{h}{200}}$, according to Atkinson.

19. The temperature of profuse fountains gives very accurately the mean temperature of any place; and by this method the altitude of any place above the level of the sea may be nearly ascertained. Thus suppose $t$ = temperature of the spring (Fahrenheit), T = mean temperature due to that parallel, found by the above Table or formulæ, then

$(T - t) \times 300$ = height above the level of the sea in feet. If the altitude be very considerable, 300 is too large a multiplier, and a correction must be applied thus: Let $h$ = height found by the above rule, then

$$\frac{h \times (T - t)^2}{48600} = \text{correction to be subtracted from } h.$$

According to Atkinson *(see Mem. Astron. Soc.)* the height in feet due to any given depression of the thermometer $n$, is

$$h = \left\{251.3 + \frac{3}{2}(n-1)\right\} n.$$

$$\text{and } n = \frac{h}{251 + \frac{h}{200}} \text{ nearly.}$$

which two formulæ apply to both hemispheres.

20. To find the mean temperature of any day, under any parallel, and with any elevation.

Let $\lambda$ be the mean longitude of the sun, computed from the 1st of aries for any day of the year, the mean temperature of which is $y$; then in these latitudes.

$$y = 58^0 + 37^0 \cos 2 \text{ lat.} - \frac{h}{270} + 15^0 \times \sin(\lambda - 30^0)$$

21. On ascending into the atmosphere, there is a certain height in every latitude, where the mean temperature is below 32°; the curve joining all these points, is called the line of perpetual congelation; to find its height in any latitude.

$$H = 7642 + 7933. \cos. 2 L. \quad \textit{(Playfair.)}$$

TABLE *of the height of the curve of congelation in different latitudes, as computed by Leslie.*

| Lat. | Ht. of curve in feet. | Lat. | Ht. of curve in feet. |
|---|---|---|---|
| 0° | 15207 | 54 | 5290 |
| 5 | 15095 | 55 | 5034 |
| 10 | 14764 | 56 | 4782 |
| 15 | 14220 | 57 | 4534 |
| 20 | 13478 | 58 | 4291 |
| 25 | 12557 | 59 | 4052 |
| 30 | 11484 | 60 | 3818 |
| 35 | 10287 | 65 | 2722 |
| 40 | 9001 | 70 | 1778 |
| 45 | 7671 | 75 | 1016 |
| 50 | 6334 | 80 | 457 |
| 51 | 6070 | 85 | 117 |
| 52 | 5808 | 90 | 00 |
| 53 | 5548 | | |

In our latitudes, the altitude of the point of congelation may be found with sufficient precision by multiplying the mean temperature—32° by 300, and correcting as in Art. 19.

We will conclude this Article with the following short Tables and observations :—

TABLE *exhibiting the different gradations of the mean annual temperature in Western Europe and North America, continuing the scale to the Equator.—(Humboldt.)*

| Lat. | Old World. | New World. | Difference. |
|---|---|---|---|
| 0° | 81°. 5 | 81°. 5 | 0 |
| 20 | 77. 9 | 77. 9 | 0 |
| 30 | 70. 7 | 67. 1 | 3.6 |
| 40 | 63. 5 | 54. 5 | 9 |
| 50 | 50. 9 | 38. 3 | 12.6 |
| 60 | 41. 0 | 25. 0 | 16 |
| 70 | 33. 0 | 0. 0 | 33 |

The difference of mean temperature between summer and winter (reckoning each to consist of three months), is nothing at the equator, and constantly increases as we approach the pole, as shown in the following Table :—

| | Lat. | Mean temperat. of winter. | Mean temperat. of summer. | Differ. |
|---|---|---|---|---|
| Algiers | 37° | 61°. 5 | 80°. 2 | 18°.7 |
| Buda | 47½ | 31. 0 | 70. 5 | 26.5 |
| Upsal | 60 | 25. 0 | 60. 2 | 35.2 |

The following Table of mean annual temperature, drawn up principally by M. de. Humboldt, is worth the attention of meteorologists. Those cities, to which an asterisk is attached, are singularly situated with respect to climate, either by their elevation above the level of the ocean, or by circumstances independent of the latitude :—

| | Lat. | Temp. |
|---|---|---|
| Melville Island | 74°. 47′ | 1. 33° |
| Umeo | 63. 50 | 33. 25 |
| Petersburgh | 59. 56 | 38. 84 |
| Upsala | 59. 51 | 41. 90 |
| Stockholm | 59. 20 | 42. 26 |
| Copenhagen | 55. 41 | 45. 68 |
| Berlin | 52. 31 | 46. 58 |
| London | 51. 31 | 50. 36 |
| Paris | 48. 50 | 51. 26 |
| Vienna | 48. 13 | 50. 54 |

| | Lat. | Temp. |
|---|---|---|
| Geneva * | 46. 12 | 50. 18 |
| Milan | 45. 28 | 55. 76 |
| Marseilles | 43. 17 | 57. 74 |
| Toulon* | 43. 3 | 63. 50 |
| Rome | 41. 53 | 60. 26 |
| Naples | 40. 50 | 64. 40 |
| Madrid* | 40. 25 | 59. 00 |
| Havannah | 23. 10 | 78. 08 |
| Mexico* | 19. 25 | 62. 60 |
| Vera Cruz* | 19. 11 | 77. 72 |
| La Plata* | 2. 24 | 74. 66 |
| Equator at the level of the sea | 0. 0 | 80. 60 |
| Quito* | 0. 14 | 59. 00 |

From a general and extensive review of the various experimental data respecting the temperatures observed at different places on the earth's surface, the Editor of the Annales de Chimie deduces the following consequences.—(*Ann. de Chimie*, xxvii. 432.)

*In no place on the earth's surface, nor at any season*, will a thermometer raised 2 or 3 metres above the soil, and sheltered from all reverberation, attain the 37° of Reaumur, or 46° centigrade, or 114°. 8 Fahrenheit.

*On the open sea*, it will never attain 25° Reaumur, or 31° centigrade, or 87°. 8 Fahrenheit.

*The greatest degree of cold* ever observed on our globe in the air, is 40° Reaumur, or 50° centigrade below Zero, (58° Fahrenheit.)

*The temperature of the water of the ocean in any latitude*, or at any season, never rises above 24° Reaumur, or 30° centigrade, (86° Fahrenheit.)

**AXIS**, *to find the angle at which a curve cuts.*—(*Higman.*)

Find the value of $\frac{dy}{dx}$ in the given curve, take $y = o$, and we shall get the tangent of the angle required.

*Ex.* Let the Equation be $y = \frac{x}{a}. \sqrt{a^2 - x^2}$.

Here $\frac{dy}{dx} = \frac{\sqrt{a^2 - x^2}}{a} - \frac{x^2}{a \sqrt{a^2 - x^2}}$; now $y = o$, when $x = o$, and when $x = a$, and the values of $\frac{dy}{dx}$ are 1 and infinite respectively; $\therefore$ the curve cuts the axis at an angle of 45° at the origin, and at right angles when $x = a$.

**AXIS**, *rotation of bodies about.*—*See Rotation.*

Azimuth

# B

**BALANCE.** *(Playfair.)*

The balance, when well constructed, must have the following properties. (1.) It should rest in a horizontal position, when loaded with equal weights. (2.) It should have great sensibility, i. e. the addition of a small weight in either scale should disturb the equilibrium, and make the beam incline sensibly from the horizontal position. (3.) It should have great stability, i. e. when disturbed, it should quickly return to a state of rest.

That the first requisite may be obtained, the beam must have equal arms; and the centre of suspension must be higher than the centre of gravity. Were these centres to coincide, the sensibility would be the greatest possible, but the other two requisites of level and stability would be entirely lost.

The 2d requisite is the sensibility of the balance. If $a$ be the length of the arm of the balance, and $b$ the distance between the centre of suspension and the centre of gravity, P the load in either scale, and W the weight of the beam, the sensibility of the balance is as $\frac{a}{b\,(2\,P + W)}$; it is $\therefore$ greater, the greater the length of the arm, the less the distance between the two centres, and the less the weight with which the balance is loaded.

Lastly, the stability is proportional to $(2\,P + W)\,b$. The diminution of $b$ $\therefore$, while it increases the sensibility, lessens the stability of the balance. The lengthening of $a$ will, however, increase the former of these quantities, without diminishing the latter.

Hence the merit of balances depends upon the quantities $a$, $b$, and W.

**BALLOON.**—*See Æronautics.*

**BALLS** *iron and leaden, weight of.—See Shot.*

**BAROMETER.**

1. *Barometer, scale of.*

The usual scale of the Barometer is 31 very dry, or hard frost; 30. 5. settled fair or frost; 30 fair or frost; 29. 5. changeable; 29 rain or snow; 28 5. much rain or snow; 28 stormy.—*(Young's Nat. Phil.)*

2. *Barometer, measurement of heights by.*

*Professor Robison's formula in feet, without logarithms.*

Let $f$ = mean temperature of air at the two stations; $d$ = difference of Barometric heights in *tenths* of an inch; $m$ = mean Barometric heights; $\delta$ = difference of mercurial temperatures; then;

$$\text{Height} = \frac{30 \times \left(87 + 0.21\ (f - 32^\circ)\right) \times d}{m} \mp \delta \times 2.83,$$

— when the attached thermometer is highest at the lower station, and v, v.

*Sir G. Shuckburgh's formula in fathoms.*

Let $l$ = difference of logarithms of the heights of Barometer in inches; $d$ = difference of mercurial temperatures; $f$ as before; then

$$\text{Height} = (10000l \mp 0.440d) \times (1 + \overline{f - 32^\circ} \times .00244),$$

— when the attached thermometer is highest at the lower station, and v, v.

*Playfair's formula in fathoms, which does not differ much from La Place's.*

Let $b$ and $\beta$ be the height of the Barometer at the *lowest* and *highest* stations, $t$ and $t'$ the temperatures of the air (Fahr.) at those stations, $q$ and $q'$ the temperatures of the mercury in the two stations; then

$$\text{Height} = 10000 \left\{ 1 + .00244.\ \left(\frac{t + t'}{2} - 32^\circ\right) \right\} \log.\ \frac{b}{\beta\left(1 + \frac{q - q'}{10000}\right)}$$

*Formula Encyc. Metrop.*

The height in *feet* is

$$60347 \left(1 + \frac{t + t'}{900}\right) \log.\ \frac{b}{\beta\left(1 + \frac{q - q'}{9742}\right)}$$

where $t$ and $t'$ denote the number of degrees *above* the freezing point of Fahrenheit. This formula differs very little from the last.

3. *Barometer, correction of observed heights in.*

When the mercury in the tube of a Barometer sinks, and the surface of that in the basin rises; to determine the correction.

Let $a$ = the section of the tube, and $b$ = that of the basin, supposed cylindrical; then apparent diminution of height : the real diminution :: $b - a$ : $b$. In the best Barometers there is a contrivance for bringing the mercury in the basin always to the same level, which obviates the necessity of this correction.

*Barometer, correction of observed heights in, as far as regards a change of temperature.*

Given the temperature of the mercury in a Barometer, measured by the attached thermometer; to reduce the observed height to what it would have been at any other temperature, as for instance 32°.

Let $b$ = observed height of Barometer, $f$ = temperature; then true height at temperature 32° = (see Art. 2) $b \times \left(1 - \frac{f - 32^{\circ}}{10000}\right)$.

4. *Barometer, range of.*

Annual range of Barometer does not exceed from $\frac{1}{5}$ to $\frac{1}{3}$ an inch in the torrid zone; about two inches at Liverpool, the same at St Petersburg; at Melville Island, as observed by Capt. Parry, $1\frac{8}{10}$. The extreme variation scarcely any where exceeds 3 inches, viz. from 28 to 31 inches. In the apartments of the Royal Society (the barometer being 81 feet above low water), during a period of 22 years, viz. from 1800 to 1821, both inclusive, the mean height was 29.86; the greatest height 30.77; the least height 28.18; and consequently the greatest range 2.59; the mean annual range during the same period was 1.92. The barometer was once observed at Middlewick, as high as 31.00. Greatest height ever observed by Sir G. Shuckburgh, in London, was 30.957. In these climates, the barometer is generally lowest at noon and at midnight. The mean height is greatest at the Equinoxes, but greater in summer than in winter.

5. *Barometer, mean height of.*

*Mean height of the Barometer in various places, from Erxleben, and others.—(Young's Nat. Phil.)*

| | |
|---|---|
| Upsal | 30. 15 |
| S. Carolina | 30. 09 |
| Mean level of the sea. Fleuriau | 30. 095 |
| Atlantic. Burckhardt. | 30. 09 |
| Mediterranean. Do. | 30. 04 |
| Mean in England and Italy. Shuckburgh | 30. 04 |
| Mean level of the sea as usually estimated | 30. 00 |
| Fort St George | 30. 00 |
| Columbo | 29. 98 |
| Dover | 29. 90 |
| London R. S. | 29. 89 |
| 81 feet above the level of low water. The mean of any year scarcely differing 0.5. | |
| Leyden | 29. 84 |
| Kendal | 29. 80 |
| Padua | 29. 80 |

| | |
|---|---|
| Panama | 29. 80 |
| Porto Bello | 29. 80 |
| Liverpool | 29. 74 |
| Turin | 29. 62 |
| Petersburg | 29. 57 |
| Gottingen | 29. 37 |
| Paris | 29. 31 |
| Basle | 28. 82 |
| Nuremberg | 28. 69 |
| Zurich | 28. 29 |
| Clausthal | 27. 89 |
| Chur | 27. 71 |
| M. St Gothard | 23. 05 |
| Quito | 21. 37 |

We shall close this article with the following Proposition :—

If a Barometer tube be in part only filled with mercury, and then its open end be immersed in a basin of the same fluid, the mercury will sink below the point called the standard altitude, or the point at which it would have stood if no air had been left in ; and the standard altitude will be to the depression below that altitude, as the space occupied by the air after the immersion, to the space occupied before.

This Proposition may be applied to the solution of two problems ; for we may either give the quantity of air left in before immersion, to find the altitude of the mercury after immersion ; or we may give the altitude of the mercury after immersion, to find the quantity of air left in before.

*Ex.* Let 5 inches of air be left in a tube of 35 inches before inversion, to find the altitude of the mercury after.

Let $x =$ depression below the standard altitude — then $30 : x :: x + 5 : 5$, $\therefore x = 10$.

BARS *Iron, to find the weight of.—(Gregory.)*

The following is an approximate rule for finding the weight of cast iron bars :—

Take $\frac{7}{144}$ of the product of the breadth and thickness, each in eighths of an inch ; the result is the weight of one foot in length, in avoirdupois pounds.

Hence an inch square cast iron bar would require 9 feet, or 108 inches in length for ¼ cwt. For *wrought iron* square bars, allow 100 inches in length of an inch square bar to ¼ cwt.

**BELLOWS** *Hydrostatical.—See Fluids pressure of.*

## BINOMIAL THEOREM.

This series, in its most simple form, is as follows :—

$$(a+b)^n = a^n + n a^{n-1} b + n.\frac{n-1}{2} a^{n-2} b^2 + n.\frac{n-1}{2}.\frac{n-2}{3} a^{n-3} b^3 + n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4} a^{n-4} b^4 + \text{\&c.}$$

where $n$ is a whole number or fraction, positive or negative.

If $b$ be negative, the odd powers of $b$ will be also negative.

*Cor.* 1. The $n^{th}$ term of the series is $n.\frac{n-1}{2}.\frac{n-2}{3}\ldots\frac{n-\overline{n-2}}{n-1}.a^{n-\overline{n-1}}b^{n-1}$.

*Cor.* 2. If $n$ be a positive whole number, the series will consist of $n+1$ terms, but in every other case, the number of terms will be unlimited.

*Cor.* 3. If $n$ be a whole positive number, the whole sum of the indices $= .n\,\overline{n+1}$.

*Cor.* 4. If $n$ be a whole positive number, and $b$ also positive, the sum of the coefficients of $(a+b)^n = 2^n$ ; but if $b$ be negative, the sum of the coefficients $= o$ ; this appears by expanding the series, and making $a = b$.

*Cor.* 5. If we call the index $\frac{m}{n}$, and put $\frac{b}{a} = Q$, and let A, B, C, D, &c. represent the 1st, 2d, 3d, &c. terms of the series, with their proper signs, we shall have $(a+b)^{\frac{m}{n}} =$

$$a^{\frac{m}{n}} + \frac{m}{n} AQ + \frac{m-n}{2n}.BQ + \frac{m-2n}{3n}.CQ + \frac{m-3n}{4n}, DQ + \text{\&c.}$$

This is the most convenient practical form in the case of fractional or negative indices.

*Ex.* 1. $(a^2+x^2)^{\frac{1}{2}} = a + \frac{x^2}{2a} - \frac{x^4}{8a^3} + \frac{x^6}{16a^5} - \text{\&c.}$

2. $(1-x)^{\frac{1}{4}} = 1 - \frac{x}{4} - \frac{3x^2}{4.8} - \frac{3.7.x^3}{4.8.12} - \text{\&c.}$

3. $(a-x)^{\frac{1}{3}} = a^{\frac{1}{3}} - \frac{x}{3a^{\frac{2}{3}}} - \frac{x^2}{9a^{\frac{5}{3}}} - \frac{5x^3}{81a^{\frac{8}{3}}} - \text{\&c.}$

4. $(a+x)^{-2} = \frac{1}{a^2} - \frac{2x}{a^3} + \frac{8x^2}{a^4} - \frac{4x^3}{a^5} + \&c.$

5. $\frac{1}{2az+z^2} = \frac{1}{2az} - \frac{1}{4a^2} + \frac{z}{8a^3} - \&c.$

In expanding a trinomial, quadrinomial, mutinomial, consider every term, except the 1st., as the 2d term of a binomial, and then proceed according to the rule.

*Ex.* 1. $(a+b+c)^2 = (a+\overline{b+c})^2 = a^2 + 2a.\overline{b+c} + (b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc.$

2. $(a+b+c)^3 = (a+\overline{b+c})^3 = a^3 + 3a^2.\overline{b+c} + 3a.(b+c)^2 + (b+c)^3.$

3. $(a+b+c+d+\&c.)^n = a^n + na^{n-1}(b+c+d+\&c.) + n.\frac{n-1}{2}a^{n-2}(b+c+d+\&c.)^2 + n.\frac{n-1}{2}.\frac{n-2}{3}.a^{n-3}.(b+c+d+\&c.)^3 + \&c.$ but *see Demoivre's Analyt. p.* 87.

BISSEXTILE.—*See Calendar.*

BOILING *point of various liquids.—See Heat.*

BRIDGE.—*See Arches equilibrium of.*

BRIDGES.

*List of a few of the most remarkable modern Bridges, with the date of their erection, the lengths of the chord and versed sine of the centre arch in feet, &c. &c.*

STONE BRIDGES.

| *Place.* | *Date.* | *No. Arches.* | *Chord.* | *Ver. sin.* | *Curve.* |
|---|---|---|---|---|---|
| Avignon, Rhone | 1188 | 18 | 110¾ | 45¾ | circular. |
| Brioude, Allier | 1454 | — | 183 | 70¼ | circular. |
| This is the largest stone arch in existence. | | | | | |
| Florence, Arno | 1569 | 1 | 95¼ | 14¾ | elliptical. |
| The Rialto, Venice | 1591 | 1 | 96¾ | 20½ | |
| Grenoble, Drac | 1611 | — | 150 | 62¼ | circular. |
| Orleans, Loire | —— | 9 | 106½ | 29¾ | false ellipse. |
| Pont Royal, Seine | 1685 | 5 | 82 | —— | |
| Neuilly | 1774 | 5 | 128 | 32 | false ellipse. |
| Nantes, Seine | 1765 | 3 | 128 | 38¼ | elliptical. |

| *Place.* | *Date.* | *No. Arches.* | *Chord.* | *Ver. sin.* | *Curve.* |
|---|---|---|---|---|---|
| Maxence, Oise | 1785 | — | 76¾ | 6¼ | |
| Pont de la Concorde, Paris | 1791 | — | 93¾ | 9¾ | circular. |
| Saumur, Loire | 1770 | 12 | 60 | 21 | elliptical. |
| Bridge of Jena, Paris | 1815 | — | 91½ | 10¾ | circular. |
| Ulm, Danube | 1806 | 1 | 181¼ | 22¼ | circular. |
| Burton, Trent | 1200 | 34 | 1545 long, longest in Britain. | | |
| London Bridge | 1176 | 20 | 70½ | 22¾ | circular. |
| Llanwrst, Conway | 1600 | 3 | 58 | 17 | circular. |
| Pont y Pryd, Taaf | 1752 | 1 | 140 | 35 | circular. |
| Blackfriars | 1771 | — | 100 | 41½ | elliptical. |
| Waterloo | 1818 | 9 | 120 | 32 | elliptical. |
| Westminster | 1750 | 15 | 76 | — | circular. |

IRON BRIDGES.

| | *Chord.* | *Ver. sin.* | | *Chord.* | *Ver. sin.* |
|---|---|---|---|---|---|
| Colbrook Dale | 100 | 45 | Boston | 85 | 5½ |
| Sunderland | 240 | 30 | Southwark | 240 | 24 |
| Buildwas | 130 | 27 | Bonar | 150 | 20 |
| Bristol | 100 | 15 | | | |

SUSPENSION BRIDGES.

| | *Chord.* | | *Chord.* |
|---|---|---|---|
| Menai Bridge | 560 | Middleton, Tees | 70 |
| Its suspended weight 490 tons. | | Proposed Bridge at Runcorn | 1000 |
| Berwick | 432 | | |
| Dryburgh | 261 | | |

# C

CABLES *strength of.—See Cords.*

CALENDAR.

The civil year consists of 365 days; the real tropical year of 365*d.* 5*h.* 48*m.* 51,6*s.* The excess therefore of the tropical year amounts to nearly 24 hours, or one day, in four years. Hence the necessity of intercalating a day every fourth year, effected by making February contain 29 days. This correction was first applied by Julius Cæsar, and the year on which it fell was called by him Bissextile, by us Leap year. As it occurs every 4th year, and every 100th year was a leap year in the Julian account, it follows that every year divisible by four is a leap year. This correction is evidently too great by nearly twelve minutes, which would amount to one day in about 129 years. By the omission of this second correction, an error crept into the calendar, which was first amended by Pope Gregory, in 1582, who wishing to bring the vernal equinox to the 21st of March, the day on which it happened in the year 325, when the council of Nice was held, suppressed 10 days. The correction of the stile did not take place in England till 1752, at which time a suppression of 11 days became necessary. This is called by us the new stile. To correct the error in future, three intercalary days are omitted every 400 years. Thus the centenary years 1700, 1800, 1900, which ought to have been leap years, were ordered not to be so; and the same in 2100, 2200, 2300, and so on for succeeding centuries. The error of the calendar, as at present constituted, will not amount to one day in less than 4237 years.

CAPILLARY *Tubes.—(Playfair.)*

Glass tubes so called of which the diameter is less than $\frac{1}{10}$ of an inch.

1. The height to which water rises, and mercury sinks, in capillary tubes, varies inversely as the diameter of the tubes.

If the bore is $\frac{1}{100}$th of an inch, the rise is 5,3 inches.

2. If a capillary tube, composed of two cylinders of different bores, be immersed in water, first with the widest part downward, and afterwards with the narrowest, the water will rise in both cases to the same height.

3. If two plates of glass be kept parallel and near to one another, and if their ends be immersed in water, the water will ascend between them to half the height it would rise to in a tube having its diameter equal to the distance of the plates.

When the plates make an angle with one another, if they be immersed with the line of their intersection vertical, the water will ascend between them and form an hyperbola.

4. To find the diameter of a capillary tube, put into the tube some mercury, whose weight in grains $= w$, and let it occupy a length of the tube $= l$, then

$$\text{Diameter} = \sqrt{\frac{w}{l}} \times ,09123 \text{ in inches.}$$

**CATENARY** *Equations, &c. to.*

Let $x$, $y$, and $z$ be the abscissa, ordinate, and curve, then the Equations to the curve are,

$$d y = \frac{a\, d x}{z}.$$

$$d x = \frac{z\, d z}{\sqrt{a^2 + z^2}}.$$

$$z = \sqrt{2\, a\, x + x^2}.$$

$$y = a \times h.\, l.\ \frac{z + x}{z - x}.$$

$$z = \frac{a}{2}\left(\varepsilon^{\frac{y}{a}} - \varepsilon^{-\frac{y}{a}}.\right)$$

$$y = a \times h.\, l.\ \frac{x + a + \sqrt{x^2 + 2\, a\, x}}{a}.$$

$$\text{Subtangent} = \frac{z\, y}{a}.$$

$$\text{Area} = y \sqrt{a^2 + z^2} - a\, z.$$

$$\text{Surface} = 2\, \pi \left( y\, z + a z - a \sqrt{a^2 + z^2} \right)$$

$$\text{Content of solid} = \pi \left(\overline{2\, a^2 + y^2} \times \overline{a + x} - 2\, a \times \overline{y\, z + a^2}\right).$$

**CAUSTICS.**—*( Coddington.)*

I. *Caustics produced by reflection.*

1. Given a point, from which a thin pencil of rays proceeding fall on a curved reflector, to determine their intersections after reflection.

Let the incident ray ......................... $= u$
the reflected ray ......................... $= v$
$\angle$ of incidence ................................ $= \phi$
Perpendicular on the tangent ............ $= p$
Principal focal distance of reflector ... $= f$

then we have the following equations,

$$du + dv = o.$$

$$v = \frac{uf.\cos\phi}{u - f\cos\phi}; \text{ or } \frac{1}{u} + \frac{1}{v} = \frac{1}{f\cos\phi}.$$

$$\text{also } v = \frac{du}{\frac{2dp}{p} - \frac{du}{u}} = \frac{du}{d\log.\frac{p^2}{u}}.$$

2. Given the radiant point and the reflecting surface, to find the caustic.

Let $p$ and $u$ be the perpendicular and radius vector of the reflecting curve, $p'$ and $u'$ = do. of the caustic, the rest as before, then

$$u'^2 = (u + v)_2 - \frac{4p^2 v}{u}.$$

$$\text{and } p' = 2p\sqrt{1 - \frac{p^2}{u^2}}.$$

For $v$ put its value $\frac{uf\cos.\phi}{u - f\cos\phi}$ or $\frac{du}{d\log\frac{p^2}{u}}$ and for $p$ the proper function of $u$ given by the equation to the original curve, let $u$ be then eliminated, and we shall have an equation in $u'$ and $p'$, which will be that of the caustic.

*Ex.* Let the reflecting curve be the log. spiral.

$$\text{Here } p = mu,\ v = \frac{du}{\frac{2dp}{p} - \frac{du}{u}} = u;$$

$p' = 2mu'\sqrt{1 - m^2}$; $u'^2 = 4u^2 - 4\frac{m^2u^2u}{u} = 4u^2(1 - m^2)$; hence $p' = mu'$; the caustic is therefore another log. spiral.

An equation in rectangular coordinates may also be obtained, but the method is too long for insertion here.

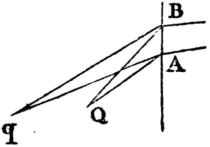
There are some simple cases in which it is easy to determine the nature of the caustic by geometrical investigation; for instance, when the reflecting curve is a circular arc, and parallel rays are incident in the plane of the circle; or when the focus of incident rays is in the circumference of the circle, the caustic in either case may be proved geometrically to be an epicycloid. When the reflecting curve is a common cycloid, and the rays are incident parallel to its axis, the caustic is also a common cycloid.

II. *Caustics produced by refraction.*

1. Required the focus of a thin pencil of rays, after being refracted obliquely at a plane surface.

Let Q and $q$ be the foci of incident and refracted rays, $\theta$ and $\theta'$ the ∠s. of incidence and refraction, then

$$q\,A : Q\,A :: \frac{\sin\theta}{\cos^2\theta} : \frac{\sin\theta'}{\cos^2\theta'}.$$



2. Required the same at a curved surface.

Let the incident ray ........................ $= u$
the refracted ray ........................ $= v$
∠ of incidence ................................ $= \phi$
∠ of refraction ................................ $= \phi'$
Radius of curvature of the surface at
the point of incidence .................... $= r$

then

$$v = \frac{u\,r\cos\phi'.\,\tan\phi}{u\tan\phi - (u + r\cos\phi)\tan\phi'}.$$

$$\text{and } \frac{1}{v} = \frac{1}{r\cos\phi'}, - \frac{(u + r\cos\phi)\sin\phi'}{u\,r\cos\phi'^2.\,\tan\phi}.$$

When $u$ is infinite, or the incident rays are parallel,

$$v = \frac{r\cos\phi'.\,\tan.\,\phi}{\tan\phi - \tan\phi'} = \frac{r\cos\phi'^2.\,\sin\phi}{\sin(\phi - \phi')}.$$

When $\phi$ is a right angle, or $u$ a tangent to the surface,

$$v = r\cos\phi'.$$

When $v$ is infinite, or the refracted rays parallel,

$$u = \frac{r\cos\phi\tan\phi'}{\tan\phi' - \tan\phi} = r\cos\phi^2.\ \frac{\sin\phi'}{\sin(\phi' - \phi)}.$$

For further information on this subject, *see Coddington's Optics.*

## CENTRAL FORCES.

1. *Of the motion of bodies in circular orbits.*

Let V = velocity of a body in a circle, R = radius, P = periodic time, F = accelerating force, $\pi = 3.14159$ &c. then

$$F = \frac{V^2}{R},\ \text{or} = \frac{4\,\pi^2\,R}{P^2}.$$

*Cor.* 1. Hence $V = \sqrt{F \times R}$, or $= \frac{2 \pi R}{P}$;

$$\text{and } P = 2 \pi \sqrt{\frac{R}{F}}, \text{ or } = \frac{2 \pi R}{V}.$$

*Ex.* 1. If F varies as $\frac{1}{R_2}$ or $= \frac{\phi}{R^2}$ ($\phi$ = absolute force),

$V = \sqrt{\frac{\phi}{R}}$, and $P = \frac{2 \pi R^{\frac{3}{2}}}{\phi^{\frac{1}{2}}}$. And in general if F varies as $\frac{1}{R^{2n-1}} = \frac{\phi}{R^{2n-1}}$; $V = \frac{\phi^{\frac{1}{2}}}{R^{n-1}}$, and $P = \frac{2 \pi R^n}{\phi^{\frac{1}{2}}}$.

*Ex.* 2. If the body revolve at the earth's surface $F = g = 32\frac{1}{6}$ feet, and R = the earth's radius, $\therefore V = \sqrt{g r}$, and $P = 2 \pi \sqrt{\frac{R}{g}}$. If the body revolve at any other distance $x$ from the earth's centre, $V = R \sqrt{\frac{g}{x}}$, and $P = \frac{2 \pi}{R} \sqrt{\frac{x^3}{g}}$.

*Cor.* 2. The same formulæ are applicable to the centrifugal force; Hence if $v$ = velocity of the earth round its axis, and $p$ = time of its revolving round its axis, centrifugal force at the equator $= \frac{v^2}{R}$, or $= \frac{4 \pi^2 R}{p^2}$.

*Ex.* Centrifugal force at the Equator : centripetal $:: \frac{4 \pi^2 R}{p^2} : g :: \frac{R}{p^2} : \frac{g}{4 \pi^2}$.

2. *Of the centripetal force of bodies revolving in any trajectories.*

Let P V = chord of curvature passing through the centre of force, $y$ = radius vector, $p$ = perpendicular on the tangent, $a$ = area described dat. temp. then

$$F = \frac{V^2}{\frac{1}{2} P V}, \text{ or } = \frac{4 a^2 \, dp}{p^3 \, dy}.$$

*Ex.* 1. If a body revolve in an ellipse, (force tending to the centre)

$$F = \frac{4 a^2 \times C P}{A C^2 \times C B^2}.$$

*Ex.* 2. If bodies revolve in the conic sections the force tending to the focus, $F = \frac{8as}{Ly^2}$, where $L =$ lat. rect.

*Cor.* The space through which a body P must fall, the force at P continuing uniform, to acquire the velocity in the curve $= \frac{PV}{4}$. If the curve be a circle, space $= \frac{R}{2}$.

3. *Of the linear velocity of bodies revolving in trajectories round a centre of force.*

$$\text{Here } V = \sqrt{F \times \tfrac{1}{4} PV}, \text{ or } = \frac{2a}{p}.$$

And velocity (V) in any point of a curve : velocity ($v$) of a body revolving in a circle at the same distance :: $\sqrt{PV} : \sqrt{p\ v}$ :: $\sqrt{\frac{dy}{y}} : \sqrt{\frac{dp}{p}}$.

*Ex.* 1. In an ellipse (the centre of force being in the centre), $V = \phi^{\frac{1}{2}} \times CD$. Also $V : v :: CD : CP$.

*Ex.* 2. In conic sections, having the centre of force in the focus, $V = \sqrt{\frac{\phi L}{2}} \times \frac{1}{SY}$; or, by substitution, we have in the parabola $V = \sqrt{\frac{2\phi}{SP}}$. In ellipse and hyperbola, $V = \sqrt{\frac{\phi \times PH}{AC.\,SP}}$.

*Ex.* 3. In the conic sections (force in the focus) $V : v :: \sqrt{HP} : \sqrt{AC}$. In the parabola, this ratio becomes that of $\sqrt{2} : 1$; in ellipse, that of $\sqrt{2} - : 1$; in the hyperbola, that of $\sqrt{2} + : 1$.

*Ex.* 4. In the ellipse velocity at any distance SP : velocity at the mean distance :: $\sqrt{HP} : \sqrt{SP}$.

4. *Of the angular velocities of bodies revolving in trajectories.*

Let $a =$ area described dat. temp. $y =$ distance, then

$$\text{Angular velocity varies as } \frac{a}{y^2}.$$

*Ex.* 1. In the conic sections (force in the focus) angular velocity varies as $\frac{L^{\frac{1}{2}} \times \phi^{\frac{1}{2}}}{y^2}$.

*Ex.* 2. Angular velocity in a conic section : do. in a circle at the same distance :: $(\frac{1}{2} L)^{\frac{1}{2}} : (SP)^{\frac{1}{2}}$.

*Ex.* 3. Angular velocity in ellipse : mean angular velocity :: $\frac{1}{SP^2} : \frac{1}{AC.CB}$.

*Ex.* 4. Angular velocity at mean distance : mean angular velocity :: CB : CA.

Let $\alpha$ be the angular velocity in any curve, then the rate at which it decreases, or

$$d\,\alpha \text{ varies as } \frac{\sqrt{y^2 - p^2}}{p\,y^4}.$$

*Ex.* 1. In a parabola, the decrement of the angular velocity is a maximum, when $y = \frac{8a}{7}$ $(a = \frac{1}{4}\text{ L. R.})$

*Ex.* 2. In the ellipse, the decrement is a max. when $3y^2 - 7ay + 4b^2 = o$.

5. *Of the paracentric velocity in any curve.*

$$\text{Par. velocity varies as } \frac{\sqrt{y^2 - p^2}}{p\,y}.$$

*Ex.* 1. In parabola, par. velocity is a maximum when $y = 2a$.

*Ex.* 2. In ellipse, $y = \frac{b^2}{a} = \frac{1}{2}$ L.

6. *Of the centrifugal force of bodies revolving in trajectories.*

Let $a$ = twice area described in 1″, then

$$\text{Centrifugal force} = \frac{a^2}{y^3}.$$

And centripetal force : centrifugal :: $2\,SP^3 : SY^3 \times PV$, or :: $\frac{y^3}{p^3} : \frac{dy}{dp}$.

*Ex.* 1. In an ellipse (force in the centre), centripetal : centrifugal force :: $CP^4 : AC^2 \times CB^2$.

*Ex.* 2. In conic sections (force in the focus) centrifugal force $= \frac{L \times \phi}{2\,SP^3}$; and centripetal : centrifugal force :: SP : $\frac{1}{2}$ L.

7. *Of the periodic times of bodies revolving in trajectories.*

Let A = whole area, $a$ = area dat. temp., then

$$\text{P. T.} = \frac{A}{a}.$$

*Ex.* 1. In ellipses (force in the centre), the periodic times $= \frac{2\pi}{\varphi^{\frac{1}{2}}}$, and are therefore equal in all ellipses.

*Ex.* 2. In an ellipse (force in the focus). $\text{PT} = \frac{2\pi\,\text{AC}^{\frac{3}{2}}}{\sqrt{\varphi}}$.

CENTRE *of Gravity.—( Vince, Playfair.)*

1. To find the centre of gravity of two given bodies, divide the distance between them in the inverse ratio of their quantities of matter, and the point so determined is the centre of gravity.

2. To find the centre of gravity of any number of bodies placed in the same straight line.

Let the bodies be A, B, C, D, &c. and their distances from a given point in the straight line be $a$, $b$, $c$, $d$, &c. then the distance of their centre of gravity from this point is

$$\frac{A\,a + B\,b + C\,c + D\,d, \,\&c.}{A + B + C + D, \,\&c.}.$$

3. In general, the distance of the centre of gravity of any system of bodies from a given plane, is equal to the sum of the products of all the masses, into their distances from the plane, divided by the sum of the masses.

*Cor.* If any of the bodies in this and the last Art. lie on the other side of the point or plane, their distances must be reckoned negative.

4. Any number of bodies being given in position, to find their centre of gravity.

The bodies must be referred to three planes given in position, cutting one another at right angles, one of them horizontal, and of course the other two vertical. Let the bodies be A, B, C, D, their distances from the given horizontal plane, $a, b, c, d$; their distances from one of the vertical planes $a', b', c', d'$, and from the other $a'', b'', c'', d''$; then if we take $x = \frac{A\,a + B\,b + C\,c + D\,d}{A + B + C + D}$, the centre of gravity of the system is in a horizontal plane at the distance $x$ from the given horizontal plane. Again take $x' = \frac{A\,a' + B\,b' + C\,c' + D\,d'}{A + B + C + D}$ and the centre of gravity is in a plane parallel to the first of the two vertical planes, and distant from it by the line $x'$. Lastly, take in the intersection of these planes a point distant from the second vertical plane by a quantity $x'' = \frac{A\,a'' + B\,b'' + C\,c'' + D\,d''}{A + B + C + D}$; and this point will be the centre of gravity of the given bodies, as is evident from the last Art.

5. If a body be placed upon a horizontal plane, and a line drawn from its centre of gravity perpendicular to that plane, the body will be sustained or not, according as the perpendicular falls within or without the base.

6. If a body be suspended by a point, it will not remain at rest till the centre of gravity is in the line which is drawn through that point perpendicular to the horizon.

*Cor.* Hence to find the centre of gravity of any plane mechanically, suspend it by a given point in or near its perimeter, and when it is at rest, draw across it a vertical line passing through that point. Suspend it in like manner by another point, and draw a vertical line as before. The intersection of these lines is the centre of gravity of the plane.

7. If any momenta be communicated to the parts of a system, its centre of gravity will move in the same manner that a body equal to the sum of the bodies in the system would move, were it placed in that centre, and the same momenta communicated to it in the same directions.

8. In any machine kept in equilibrium by the action of two weights, if an indefinitely small motion be given to it, the centre of gravity of the weights will neither ascend nor descend.

9. Formulæ for finding the centre of gravity of a body considered as an area, solid, surface, or curve.

Let $x$, $y$, and $z$, represent the abscissa, ordinate, and curve, $D$ = distance of vertex from the centre of gravity; then

For an area, $D = \frac{\text{fl.}\ y\,x\,dx}{\text{fl.}\ y\,dx}$.

For a solid, $D = \frac{\text{fl.}\ y^2\,x\,dx}{\text{fl.}\ y^2\,dx}$.

For a surface, $D = \frac{\text{fl.}\ y\,x\,dz}{\text{fl.}\ y\,dz}$.

For a curve line, $D = \frac{\text{fl.}\ x\,dz}{z}$.

*Ex.* 1. In a triangle and conical surface, let $a$ be the line from the vertex bisecting the base, then $D = \frac{2a}{3}$.

2. In parabola, $D = \frac{3}{5}$ altitude.

3. In a ½ circle distance from centre $= \frac{4r}{3\pi}$.

4. In a cycloid, $D = \frac{7a}{12}$.

5. In a sector of a circle, distance from centre $= \frac{2}{3}\ \frac{\text{rad. chord}}{\text{arc}}$.

6. In a circular arc, distance from centre $= \frac{\text{rad.} \times \text{chord}}{\text{arc}}$.

7. In cones and all regular pyramids, $D =$ ¾ altitude.

8. In a paraboloid, $D =$ ⅔ altitude.

9. In ½ sphere and ½ spheroid $D = \frac{5r}{8}$.

10. In the surface of a ½ sphere, $D = \frac{r}{2}$.

CENTRE *of Gyration.*

Let A, B, C, &c. be the bodies, or the particles of which the body is composed, S the point round which the particles revolve, D = distance of the centre of gyration from the axis, then

$$D = \sqrt{\left(\frac{A \times SA^2 + B \times SB^2 + C \times SC^2 + \&c}{A + B + C \times \&c.}\right)}$$

Or if $ds$ be the differential of the body at the distance $x$ from the axis,

$$D = \sqrt{\left(\frac{\text{fl. } x^2\, ds}{s}\right)}.$$

*Ex.* 1. In a straight line, $D = \frac{\text{Length}}{\sqrt{3}}$. $\sqrt{3} = 1{,}733$

2. In a circle revolving in its own plane, round its centre, or in a cylinder, $D = \frac{r}{\sqrt{2}}$. $\sqrt{2} = 1{,}414$

3. In the periphery of a circle revolving about its diameter, $D = \frac{r}{\sqrt{2}}$

4. In the plane of a circle revolving round its diameter, $D = \frac{r}{2}$.

5. In a sphere revolving round its diameter $D = r\sqrt{\frac{2}{5}}$. $= r \times$ ~~.63245~~ .81650

6. In the surface of a sphere, $D = r\sqrt{\frac{2}{3}}$. $r \times .63245$

7. In a cone about its axis, $D = r\sqrt{\frac{3}{10}}$.

## CENTRE *of Oscillation.*

1. Let D = distance of the point of suspension from the centre of oscillation, $\delta$ = distance from the centre of gravity, then

$$D = \frac{A + SA^2 + B + SB^2 + C + SC^2 + \&c.}{(A + B + C + \&c.) \times \delta}.$$

Or if $ds$ be the differential of the body at the distance $x$ from the axis,

$$D = \frac{\text{fl. } x^2\, ds}{s \times \delta}.$$

2. If S be the point of suspension, G the centre of gravity, O the centre of oscillation,

$$GO \text{ varies as } \frac{1}{SG}.$$

*Cor.* If O be made the point of suspension, S will be the centre of oscillation; or the centre of oscillation and the point of suspension are convertible.

3. If R be the centre of gyration,

$$SG : SR :: SR : SO.$$

*Ex.* 1. In a straight line, $D = \frac{2L}{3}$.

2. In an isosceles triangle vibrating flat ways, $D = \frac{3}{4}$ alt.

3. In a circle flat ways, $D = \frac{5}{4}r$.

4. In a parabola flat ways, $D = \frac{5}{7}$ alt.

5. In a sphere, $D = a + \frac{2r^2}{5a}$ ($a$ = distance of the point of suspension from the centre of the sphere.)

6. In a cone, $D = \frac{4}{5}\text{ axis} + \frac{(\text{rad. of base})}{5\text{ axis}}$.

7. In a circle vibrating edgeways, $D = \frac{3}{2}r$.

8. In a sector of a circle edgeways, $D = \frac{3\text{ arc} \times \text{rad.}}{4\text{ chord}}$.

9. In a rectangle edgeways, suspended by one angle, $D = \frac{2}{3}$ diagonal

10. In a parabola edgeways, suspended by the vertex, $D = \frac{5}{7}$ axis $+ \frac{1}{3}$ parameter.

To find the centre of oscillation practically, suspend the body freely by the point of suspension, and make it vibrate in small arcs, counting the vibrations it makes in any given time, as one minute. Call the number in a minute $n$, then will the distance of the centre of oscillation be $\frac{140850}{n^2}$ inches. For a still more accurate method—*see Captain Kater's Paper in the Phil. Trans. for* 1818.

CENTRE *of Percussion.*

When the percutient body revolves about a fixed point, the centre of percussion is the same as the centre of oscillation. But when the body moves with a parallel motion, the centre of percussion is the same as the centre of gravity.

CENTRE *of Pressure.*

Centre of pressure of a fluid against a plane, is that point against which a force being applied equal and contrary to the whole pressure, it will sustain it, so as that the body pressed on will not incline to either side. This, according to some writers, is the same as the centre of percussion, supposing the axis of suspension to be at the intersection of this plane with the surface of the fluid; while others assert, that though the *distance* of this intersection from the centre of pressure is the same as that of the centre of percussion, yet that they do not in general lie in the same line, and consequently are not the same point. The centre of pressure upon any plane *parallel* to the horizon, or upon any plane where the pressure is uniform, is the same as the centre of gravity of that plane.

CENTRIFUGAL *Force.—See Central Forces.*

CENTRIPETAL *Force.—See Central Forces.*

CERES.

This planet was discovered by M. Piazzi, of Palermo, Jan. 1, 1801. For its elements, &c.—*see Planets, elements of.*

CHANCES, *doctrine of.—(Wood.)*

1. If an event may take place in $n$ different ways, and each of these be equally likely to happen, the *probability* that it will take place in a specified way is $\frac{1}{n}$, certainty being represented by unity.

2. If an event may happen in $a$ ways, and fail in $b$ ways, any of these being equally probable, the chance of its happening is $\frac{a}{a+b}$; and the chance of its failing is $\frac{b}{a+b}$.

*Ex.* 1. The probability of throwing an ace with a single die in one trial is $\frac{1}{6}$; the probability of not throwing an ace is $\frac{5}{6}$; and the probability of throwing an ace or a deuce is $\frac{2}{6}$.

*Ex.* 2. If $n$ balls $a$, $b$, $c$, $d$, &c. be thrown promiscuously into a bag, and if two balls be drawn out, the probability that these will be $a$ and $b$ is $\frac{2}{n(n-1)}$.

3. If two events be independent of each other, and the probability that one will happen be $\frac{1}{m}$, and the probability that the other will happen $\frac{1}{n}$; the probability that they will both happen is $\frac{1}{mn}$.

*Cor.* 1. The probability that both do not happen is $\frac{mn-1}{mn}$.

*Cor.* 2. The probability that they will both fail is $\frac{(m-1).(n-1)}{mn}$.

*Cor.* 3. The probability that one will happen, and the other fail, is $\frac{m+n-2}{mn}$.

*Cor.* 4. If there be any number of independent events, and the probabilities of their happening be $\frac{1}{m}$, $\frac{1}{n}$, $\frac{1}{r}$ &c. respectively, the probability that they will all happen is $\frac{1}{mnr}$ &c. When $m = n = r$ &c. the probability is $\frac{1}{m^v}$, $v$ being the number of events.

*Ex.* 1. The probability of throwing an ace and then a deuce with one die is $\frac{1}{36}$.

*Ex.* 2. If 6 white and 5 black balls be thrown promiscuously into a bag, the probability that a person will draw out first a white, and then a black ball, is $\frac{3}{11}$. And the probability of drawing a white ball, and then two black balls is $\frac{4}{33}$.

*Ex.* 3. The probability of throwing an ace with a single die in two trials is $\frac{11}{36}$.

4. If the probability of an event's happening in one trial be $\frac{a}{a+b}$, the probability of its happening $t$ times exactly in $n$ trials is

$$\frac{n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \ldots\ldots \frac{n-t+1}{t} a^{t} b^{n-t}}{(a+b)^n}.$$

*Cor.* 1. The probability of the event's failing exactly $t$ times in $n$ trials is

$$\frac{n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \ldots\ldots \frac{n-t+1}{t} a^{n-t} b^{t}}{(a+b)^n}.$$

*Cor.* 2. The probability of the event's happening *at least* $t$ times in $n$ trials, is

$$\frac{a^n + n a^{n-1} b + n \cdot \frac{n-1}{2} a^{n-2} b^2 \ldots\ldots \text{to } n-t+1 \text{ terms.}}{(a+b)^n}$$

5. In astronomical or other observations, let $a$, $b$, $c$, $d$, &c. be the differences between the mean of the observations, and the observations themselves; $n$ the number of observations; $\pi = 3.14159$ &c.; then the mean error, or the greatest *probable* error is

$$\frac{\sqrt{(a^2 + b^2 + c^2 + \&c.)}}{n \sqrt{\pi}}.\text{—}(\textit{La Place.})$$

6. Let $n$ be the number of times an event has happened, where $n$ is very large, then the chance that the same event will occur again is $\frac{n+1}{n+2}$. Thus supposing 5000 years the greatest antiquity to which history goes back; then the probability that the sun will rise to-morrow is 1826214 to 1.—*(La Place.)*

## CHRONOLOGY.

*A short Chronological* TABLE *of remarkable discoveries and inventions, and of the most eminent Mathematicians and Philosophers.*

| | B. C. |
|---|---|
| First eclipse of the moon on record, observed at Babylon | 720 |
| Thales predicts an eclipse | 600 |
| Anaximander, globes and maps | 600 |
| Anaxagoras, eclipse—Pythagoras, astron. | 530 |
| Plato, geom.—Meton, Metonic cycle | 430 |
| Aristotle, Eudoxus | 360 |
| Obliquity of ecliptic first observed | 359 |
| A transit of the Moon over Mars observed | 357 |
| Euclid, geom. | 300 |
| Papirius Cursor, first sun dial at Rome | 293 |
| Dionysius, Astron. Æra | 285 |
| Apollonius, Archimedes, Aristarchus, Eratosthenes, about | 270 |
| Hipparchus, the father of Astronomy | 162 |
| | A. D. |
| Ptolemy, *Almagest*, born | 69 |
| Diophantus, analysis | 280 |
| Pappus and Theon | 380 |
| Proclus, Diocles, about | 500 |
| Figures employed by the Arabs | 813 |
| A conjunction of all the planets observed, Sept. 16 | 1186 |
| Alphonso, Astron. tables—Bacon R. | 1250 |
| Figures employed in England | 1253 |
| Mariner's compass said to be used at Venice | 1260 |
| A clock at Westminster Hall | 1288 |
| Spina invented spectacles at Pisa | 1299 |
| Windmills invented | 1299 |
| Gunpowder invented | 1330 |
| Decimal arithmetic introduced | 1402 |
| Printing invented by Faust | 1441 |
| Made public by Gutenberg | 1458 |
| Regiomontanus or Muller, astron. | 1460 |
| Watches made at Nuremberg | 1477 |
| First voyage round the world by Magellan | 1522 |
| Variation of the compass by Cabot | 1540 |
| Copernicus, Cardan, Vieta, about | 1550 |
| Dip of the magnetic needle observed | 1576 |
| Telescopes discovered by Jansen | 1590 |

| | |
|---|---|
| Tycho Brahe, Bacon, Galileo, Kepler, Des Cartes | 1600 |
| Thermometers invented by Drebel | 1610 |
| Napier, logarithms | 1614 |
| Vernier's index made known | 1631 |
| Cassini observes a transit of Mercury | 1636 |
| A transit of Venus first observed by Horrox | 1639 |
| Barometers by Torricelli | 1643 |
| Pendulum applied to clocks by Huygens | 1649 |
| Cavalerius, Fermat, Pascal, Wallis, Hevelius | 1650 |
| Air pump by Otto Guericke | 1653 |
| Royal Society established in London | 1662 |
| Foundation of the Royal Observatory at Greenwich | 1675 |
| Micrometer of Kircher | 1677 |
| Newtonian Philosophy published | 1686 |
| Savery had invented steam engines | 1696 |
| Bernouilli J., Barrow, Hooke, Leibnitz, Reaumur, Flamstead, Picard, Cotes, Taylor, Halley, | 1650 to 1700 |
| Aberration of light by Bradley | 1727 |
| Achromatic glasses invented | 1729 |
| Franklin, identity of lightning and electricity | 1747 |
| Harrison, time pieces | 1750 |
| Clairaut, Maclaurin, De Moivre, Simpson, Bouguer, Bernouillis, Dollond, Maupertuis | 1750 |
| New stile introduced into Britain | 1752 |
| Galvanism | 1791 |
| Telegraph invented by the French | 1794 |
| D'Alembert, Euler, Landen, Lalande, Maskelyne, Waring, &c. | from 1750 to 1800 |

*For a List of the most remarkable Æras—see Æra.*

CIRCLE *Equations to.*

1. Let $x$ and $y$ be rectangular coordinates; then if the origin be at the centre,

$$y = \sqrt{r^2 - x^2}.$$

If at the extremity of the diameter,

$$y = \sqrt{2rx - x^2}.$$

And in general if $x'$, $y'$ be the coordinates to the centre, the equation is, when the axes are rectangular,

$$(x - x')^2 + (y - y')^2 = r^2.$$

Hence every equation of two dimensions of the form $Ax^2 + Ay^2 +$

$B x + C y + D = o$, where the coefficients of $x_2$ and $y_2$ are the same, and the term involving $x y$ is wanting, is an equation to the circle; as for example $2 y^2 + 2x_2 - 4 y - 4 x + 1 = o$.

2. When the circle is considered as a spiral, let $a$ = distance of the centre of the polar coordinates from the centre of the circle, $y$ = rad. vect. $p$ = perpendicular on the tangent; then

$$p = \frac{r_2 - a^2 + y^2}{2 r}.$$

When the pole is in the circumference,

$$p = \frac{y^2}{2 r}.$$

CISSOID *of Diocles, Equations, &c. to.*

$$y^2 = \frac{x^3}{2 a - x}$$

Or, when considered as a spiral,

$$\varrho = \frac{2 a \sin._2 \theta}{\cos \theta}.$$

Subtangent $= \dfrac{2 x. (2 a - x)}{6 a - 2 x}$.

Area $= \dfrac{3 \pi a^2}{2} = 3$ area of the generating semicircle.

Content of solid $= - \pi \left( \dfrac{x^3}{3} + a x_2 + 4 a^2 x - 8 a_3 \log. \dfrac{2 a}{2 a - x} \right)$

which is infinite when $x = 2 a$.

CLEPSYDRA.—*See Fluids, discharge of.*

CLIMATE.—*See Atmosphere.*

CLOCK, *to correct going of.*—*See Pendulum.*

CLUSTERS *of Stars.*—*See Nebulæ.*

COHESION, *or Attraction of Cohesion.*—*See Elastic Bodies, equilibrium of.*

COINAGE.—*See Money.*

COLD *Artificial.*—*See Frigorific Mixtures.*

COLLISION *of Bodies.*—*( Wood, Whewell. )*

### I. *Of the impact of perfectly hard bodies.*

1. Let A and B be the quantities of matter contained in two perfectly

hard bodies, $a$ and $b$ their velocities before impact, $v$ the common velocity after impact, then

$$v = \frac{A\,a \pm B\,b}{A + B}.$$

$+$ or $-$, according as they move in the same or opposite directions before impact.

*Cor.* 1. When the bodies are equal, $v = \frac{a \pm b}{2}$.

*Cor.* 2. When B is at rest or $b = o$, $v = \frac{A\,a}{A + B}$.

2. In the direct impact of two perfectly hard bodies A and B, if $g$ = velocity gained by B, and $l$ = velocity lost by A after impact;

$$g = \frac{A\,.\,\overline{a \mp b}}{A + B}.$$

$$\text{and } l = \frac{B\,.\,\overline{a \mp b}}{A + B}.$$

$-$ or $+$, according as they move in the same or opposite directions.

*Cor.* $g + l = a \mp b.$

II. *Of the impact of perfectly elastic bodies.*

1. Let $r$ = relative velocity of two bodies = $a \mp b$, according as they move in the same or opposite directions, $g$ = velocity gained by B in the direction of A's motion, $l$ = velocity lost by A in that direction; then in the direct impact of two perfectly elastic bodies,

$$g = \frac{2\,A\,r}{A + B}.$$

$$\text{and } l = \frac{2\,B\,r}{A + B}.$$

*Cor.* 1. If A = B, B's velocity after impact = $a$, and A's velocity = $\pm\, b$, i. e. the bodies interchange velocities.

*Cor.* 2. In the congress of perfectly elastic bodies, the relative velocity after impact = the relative velocity before.

*Cor.* 3. If $a'$ and $b'$ be the velocities after impact,

$$A\,a^2 + B\,b^2 = A\,a'^2 + B\,b'^2.$$

*Cor.* 4. If there be a row of elastic bodies A, B, C, D, &c. at rest, and a motion communicated to A, and thence to B, C, D, &c.; then, if the bodies are equal, they will all remain at rest after impact, except the last, which will move off with a velocity equal to that with which the first moved. If they decrease in magnitude, they will all move in the direction of the first motion, the velocity of each succeeding body being greater than that communicated to the preceding; and the contrary, if the bodies increase in magnitude.

*Cor.* 5. The velocity thus communicated from A through B to C, when B is greater than one of the two A & C, and less than the other, exceeds the velocity which would be communicated immediately from A and C; and is a maximum when B is a mean proportional between A and C.

*Cor.* 6. If there be $n$ bodies in geometrical progression, whose common ratio is $r$,

$$\text{Velocity of first} : \text{velocity of last} :: (1+r)^{n-1} : 2^{n-1}$$

*Cor.* If the number of mean proportionals interposed between two given bodies A and X be increased without limit, the ratio of A's velocity to that communicated to X approximates to the ratio of $\sqrt{X} : \sqrt{A}$.

III. *Of the impact of imperfectly elastic bodies.*

1. In the direct impact of two imperfectly elastic bodies A and B, if the compressing force be to the force of elasticity $:: 1 : e$,

$$g = \frac{\overline{1+e} \,.\, A r}{A+B},$$

$$\text{and } l = \frac{\overline{1+e} \,.\, B r}{A+B}.$$

*Cor.* 1. The relative velocity before impact : do. after $:: 1 : e$.

*Cor.* 2. In imperfectly elastic bodies $A a^2 + B b^2$ is greater than $A a'^2 + B b'^2$—*(See last Art. Cor. 3.)*

*Cor.* 3. If there be $n$ imperfectly elastic bodies in geometrical progression, whose common ratio is $r$,

$$\text{Velocity of first} : \text{velocity of last} :: (1+r)^{n-1} : (1+e)^{n-1}.$$

IV. *Of the impact of bodies against immoveable planes.*

1. When a perfectly hard body impinges obliquely on a perfectly hard immoveable plane, after impact it will move along the plane, and velocity before impact : do. after :: radius : sin. of incidence.

2. If the body be perfectly elastic, the ∠ of incidence = ∠ of reflection; and velocity before incidence = velocity after.

3. If the body be imperfectly elastic, the velocity before incidence : velocity after reflection :: sin. of ∠ of reflection : sin. ∠ of incidence;

and compressing force : force of elasticity :: tan. ∠ reflection : tan. ∠ of incidence.

4. If an imperfectly elastic body fall from a given distance $a$ upon an immoveable plane, the whole space described, from the beginning to the end of the motion, is

$$\frac{1+e^2}{1-e^2} \times a.$$

And the whole time of the body's motion is

$$\frac{1+e}{1-e} \times \sqrt{\frac{2a}{g}} \quad (g = 32\tfrac{1}{6} \text{ feet.})$$

**COMBINATIONS.**—*See Permutations.*

**COMPASS**, *points of.*

*To reduce points of the compass to degrees reckoned from the meridian, and conversely.*

| N. E. Quadrant. | S. E. Quadrant. | Points. | D. M. | S. W. Quadrant. | N. W. Quadrant. |
|---|---|---|---|---|---|
| N. | S. | 0 | 0. 0. | S. | N. |
| N. by E. | S. by E. | 1 | 11. 15. | S. by W. | N. by W. |
| N. N. E. | S. S. E. | 2 | 22. 30. | S. S. W. | N. N. W. |
| N. E. by N. | S. E. by S. | 3 | 33. 45. | S. W. by S. | N. W. by N. |
| N. E. | S. E. | 4 | 45. 0. | S. W. | N. W. |
| N. E. by E. | S. E. by E. | 5 | 56. 15. | S. W. by W. | N. W. by W. |
| E. N. E. | E. S. E. | 6 | 67. 30. | W. S. W. | W. N. W. |
| E. by N. | E. by S. | 7 | 78. 45. | W. by S. | W. by N. |
| E. | E. | 8 | 90. 0. | W. | W. |

**COMPASS**, *variation and dip of.—See Variation.*

**CONCHOID** *of Nicomedes, Equations to, &c.*

$$(a+y)^2 \times (b^2 - y^2) = x^2 y^2.$$

Or, referred to the centre of revolution of its generating line $\varrho$, the equation is

$$\varrho = \frac{a}{\cos.\,\theta} + b.$$

$$\text{Area} = \tfrac{1}{2}\, b \times \left(\text{arc of quadrant} - \text{arc rad. } b \;\&\; \text{sin. } y + \frac{y\sqrt{b^2-y^2}}{2}\right)$$

$$-\,\frac{ab}{2} \times \text{h. l. } \frac{b-\sqrt{b^2-y^2}}{b+\sqrt{b^2-y^2}}.$$

Content of the whole solid, formed by a revolution round the asymptote, $= \pi b^2 \times \left(\frac{1}{3}\pi a \times \frac{2b}{3}\right)$.

**CONDENSER.**—*See Pump condensing.*

**CONDITION,** *Equations of.—See Equation.*

**CONE.**—*Equation to the section of a right Cone.—(Francœur.)*

Let the vertical angle of the cone $= \beta$, the angle which the cutting plane makes with the side $= \alpha$, and the distance of this plane from the vertex $= c$, then the equation to the section is

$$y^2 = \frac{\sin.\alpha}{\cos.^2\frac{\beta}{2}}\left(c\,x \sin.\beta - x^2 \sin.(\alpha+\beta)\right).$$

*Cor.* 1. If $\alpha + \beta$ be less than 180°, or the plane cut both sides of the cone, the section is an ellipse.

*Cor.* 2. If $\alpha + \beta = 180°$, or the plane be parallel to the side, the section is a parabola.

*Cor.* 3. If $\alpha + \beta$ be greater than 180°, or the plane cut the opposite cones, the section is an hyperbola.

*Cor.* 4. The ½ major and ½ minor axes of the ellipse and hyperbola are

$$\frac{c\sin.\beta}{2\sin.(\alpha+\beta)},\ \text{and}\ \frac{c\sin.\beta}{2\cos.\frac{\beta}{2}\sin.(a+\beta)}\cdot\sqrt{\sin.\alpha.\sin.(\alpha+\beta)}.$$

*Cor.* 5. The lat. rect. of the parabola $= 4\,c\sin.^2\frac{\beta}{2}$.

*Cor.* 6. The parallel and subcontrary sections of an oblique cone are circles.

**CONGELATION.**—*See Heat.*

**CONGELATION,** *point of perpetual.—See Atmosphere.*

**CONIC SECTIONS,** *properties of.*

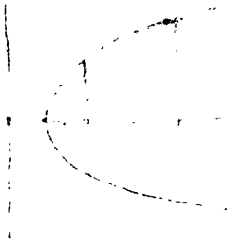
PARABOLA.

Latus rectum or $L = 4\,S\,A$.

$T\,N = 2\,A\,N$.

$S\,Y^2 = S\,P.\,S\,A$; i. e. $p = \sqrt{a\,r}$.

$Q\,v^2 = 4\,S\,P.\,P\,v$.



$S\,P = \frac{2\,A\,S}{1 + \cos.\,\theta}$ or $= \frac{2\,A\,S}{2\cos.^2\frac{\theta}{2}}$, where $\theta = \angle$ traced out by rad. vect. S P.

Ch. curv. $= 4\,S\,P$.

Diam. curv. $= \frac{4\,S\,P^{\frac{3}{2}}}{\sqrt{S\,A}}$.

area $= \frac{2}{3}xy = \frac{2}{3}$ of the circumscribing rectangle
page 58

Equation to the curve $y^2 = a\,x\;(a = L)$.

*Note.*—The general equation to a parabolic curve is $a^{n-1}y = x^n$.

If $n = 3$, it is called the cubical parabola.

If $n = \frac{3}{2}$, it is called the semi-cubical parabola.

area in general equation $\frac{n}{n+1} \times xy$
page 58

### ELLIPSE.

Rectification of Parabola p. 74 t

$S\,P + P\,H = 2\,A\,C$.

$A\,S.\,S\,M = B\,C^2$.

$L = \frac{2\,B\,C^2}{A\,C}$.

$S\,Y^2 = B\,C^2.\,\frac{S\,P}{P\,H}$; i. e. $p = b\sqrt{\frac{r}{2\,a - r}}$.

$S\,P.\,P\,H = C\,D^2$.

$A\,C^2 + C\,B^2 = C\,P^2 + C\,D^2$.

$A\,C.\,C\,B = C\,D.\,P\,F$, or if the perpendicular P F be called P,

$$P = \frac{a\,b}{\sqrt{a^2 + b^2 - r^2}}.$$

$$Q\,v^2 = \frac{P\,v.\,v\,G \times C\,D^2}{C\,P^2}.$$

$B\,C = a\sqrt{1 - e^2}$, where $e$ = eccentricity $= \frac{S\,C}{A\,C}$.

$$C\,P = \frac{b}{\sqrt{(1 - e^2\cos.^2\theta)}} = \sqrt{\frac{1 - e^2}{1 - e^2\cos.^2\theta}}.$$

$$P = \frac{b^2}{a}.\,\frac{1}{1 + e\cos.\,\theta} = \frac{a\,(1 - e^2)}{1 + e\cos.\,\theta}.$$

Ch. curv. through centre $= \frac{2\,C\,D^2}{C\,P}$.

Ch. curv. through focus $= \frac{2\,CD^2}{AC}$.

Diameter curv. $= \frac{2\,CD^2}{PF}$, or $= L \times \frac{SP^3}{SY^3}$.

Equation to the curve, when referred to its principal diameters.

$$\frac{y^2}{b^2} + \frac{x^2}{a^2} = 1.$$

And when the coordinates originate at the vertex,

$y^2 = \frac{b^2}{a^2}\,(2\,a\,x - x^2)$.

area [illegible]

Or $y^2 = \frac{b^2}{a^2}\,(a^2 - x^2)$ when the origin is at the centre.

### HYPERBOLA.

$HP - SP = 2\,AC$.

$AS.\ SM = BC^2$.

$L = \frac{2\,BC^2}{AC}$.

[illegible]

$SY^2 = BC^2.\ \frac{SP}{PH}$, i. e. $p = b\sqrt{\frac{r}{2a+r}}$.

$SP.\ PH = CD^2$.

$AC^2 - CB^2 = CP^2 - CD^2$.

$AC.\ CB = CD.\ PF$, or $P = \frac{ab}{\sqrt{\rho^2 - (a^2 - b^2)}}$.

$Qv^2 = \frac{Pv.\ vG \times CD^2}{CP^2}$.

$BC = a\sqrt{(e^2 - 1.)}$

$CP = \frac{b}{\sqrt{(e^2 \cos.^2\theta - 1)}} = a\sqrt{\frac{e^2 - 1}{e^2 \cos.^2\theta - 1}}$.

$SP = \frac{b^2}{a} \cdot \frac{1}{1 + e\cos.\theta} = \frac{a\,(e^2 - 1)}{1 + e\cos.\theta}$.

Ch. curv. through centre $= \frac{2\,CD^2}{CP}$.

Ch. curv. through focus $= \frac{2\,CD^2}{AC}$.

Diam. curv. $= \frac{2\,CD^2}{PF}$ or $= L.\ \frac{SP^3}{SY^3}$.

Equation to the curve, when referred to its principal diameters,

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = -1.$$

$$\text{or } \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1.$$

And when the coordinates originate at the vertex, $y^2 = \frac{b^2}{a^2}\ (2\,a\,x + x^2)$.

Or $y^2 = \frac{b^2}{a^2}\ (x^2 - a^2)$, when the origin is at the centre.

Equation to the hyperbola, when referred to its asymptotes, is $x\,y = \frac{a^2 + b^2}{4}$, where $y$ is parallel to the other *a*symptote.

If the hyperbola is equilateral, $x\,y = \frac{a^2}{2}$.

The general equation to an hyperbolic curve is $y\,x^{\,n} = a^{\,n+1}$

*Note.*—The general Equation to the Conic Sections, referred to their axes is

$y^2 = m\,x + n\,x^2$, where $m$ is the latus rectum, and the conic section is a parabola, ellipse, or hyperbola, according as $n = o$, or is negative, or positive.

CONTACT *of Curves.—(Higman.)*

Let there be two curves, whose equations are $y = f\,(x)$, and $y' = \varphi\,(x')$, and suppose them (1) to have a point in common, so that when $x = x'$, $y = y'$ : (2) that $x = x'$, $y = y'$, and $\frac{d\,y}{d\,x} = \frac{d\,y'}{d\,x'}$ : (3) that besides the preceding conditions $\frac{d^2\,y}{d\,x^2} = \frac{d^2\,y'}{d\,x'^2}$ and so on; then will the distance between the curves be infinitely greater in the first case, than it is in the second; infinitely greater in the second than it is in the third; and so on continually.

CONTINUED *Fractions.—See Fractions.*

COORDINATES *Polar, to find the relation between.—(Higman.)*

If the relation between the rectangular coordinates $x$ and $y$ in any curve be given, that between the polar ones $\varrho$ and $\theta$ may be determined; and conversely.

For $x = \varrho$ cos. $\theta$, and $y = \varrho$ sin. $\theta$; substitute these values in the given equation, and the polar one will be found.

*Ex.* Let $y^2 = \frac{x^3}{a - x}$ By substitution

$$\varrho^2 \sin.^2 \theta = \frac{\varrho^3 \cos.^3 \theta}{a - \varrho \cos. \theta};$$

$$\therefore a \sin.^2 \theta = \varrho \cos. \theta \times (\sin.^2 \theta + \cos.^2 \theta) = \varrho \cos. \theta \times 1$$

$$\therefore \varrho = \frac{a \times \sin.^2 \theta}{\cos. \theta}.$$

CORDS, *strength of.—(Gregory.)*

The best mode of estimating the strength of a cord of hemp is to multiply by 200 the square of its number of inches in girth, and the product will express in pounds the practical strain it may be safely loaded with. For cables, multiply by 120, instead of 200. The ultimate strain is probably double this.

For the *utmost* strength that a cord will bear before it breaks, a good estimate will be found by taking $\frac{1}{5}$ of the square of the girth of the cord, to express the tons it will carry. This is about double the rule for practice just given; and is, even for an ulterior measure, too great for tarred cordage, which is always weaker than white.

In cables, the strength when twisted, is to the strength when the fibres are parallel, as about 3 to 4.

*The following is the breaking strain, by experiment, in the best bower cables at present employed in the British navy.—(Encyc. Metrop.)*

| Sizes, circum. in inches. | No. of threads in each. | Breaking strain. | | |
|---|---|---|---|---|
| | | TONS. | | |
| 23 | 2736 | 114 | 0 | 0 |
| 21 | 2268 | 89 | 0 | 0 |
| 18 | 1656 | 63 | 0 | 0 |
| 14½ | 1080 | 40 | 0 | 0 |

From the experiments of Mr. Labillardiere, it appears, that if we call the strength of flax 1000; that of the American aloe will be 596; of hemp 1390; of New Zealand flax 1996; and of silk 2894.—*(Young's Nat. Phil.)*

COSINES, *figure of.—See Figure.*

CUBATURES *of Solids.—See Solids.*

CUBE *Roots of Numbers.—See Involution.*

CURVATURE *radius of, in any curve, whose equation is given.*

Let $x$, $y$, and $z$ represent the abscissa, ordinate, and curve, then

$$\text{Rad.} = \frac{d\,z^3}{-d\,x\,d^2y}\ (d\,x\text{ being constant}) = \frac{d\,x^2}{-d_2\,y}\left(1 + \frac{d\,y^2}{d\,x^2}\right)^{\frac{3}{2}}$$

$$\text{Or Rad.} = \frac{d\,z^3}{d\,y\,d^2x}\ (d\,y\text{ being constant}) = \frac{d\,y^2}{d_2\,x}\left(1 + \frac{d\,x^2}{d\,y^2}\right)^{\frac{3}{2}}$$

*For the Curvature of Spirals—see Spiral.*

### CYCLE.

A circulation of time between the returns of the same event.

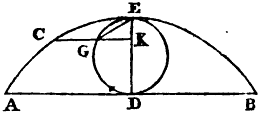
*Cycle* of the sun, a space of 28 years, in which time the days of the month return again to the same days of the week, and the sun's place to the same degrees of the Ecliptic on the same days, so as not to differ 1° in 100 years; and the leap years return again in respect to the days of the week on which the days of the months fall. To find it, add 9 to the given year of Christ, and divide the sum by 28, and the quotient is the number of cycles elapsed since his birth, and the remainder is the cycle for the given year; if nothing remain the cycle is 28.

*Cycle* of the moon, or golden number, a revolution of 19 years, in which time the conjunctions, oppositions, and all other aspects of the moon, return on the same days of the months as they did 19 years before, but about 1½ hours sooner. To find it, add 1 to the given year of Christ, and divide the sum by 19, and the quotient is the number of cycles elapsed from the birth of Christ, and the remainder is the cycle for the given year, or the golden number; and if nothing remain, 19 is the cycle.

*Cycle* of Indiction, a revolution of 15 years, but has no dependance on the motions of the heavenly bodies. It was used by the Romans for indicating the times of certain payments made by the subjects to the republic, established by Constantine, A. D. 312. To find it, subtract 312 from the given year, and divide by 15.

*Julian* period. From the multiplication of the Solar cycle of 28 years, into the Lunar of 19, and Indiction of 15, arises the Julian period of 7980 years, in which time they all return again in the same order. The Julian period, commencing before all the known epochs, is, as it were, a common receptacle of them all, and to which they may all be reduced *(see Æra.)* To find it, add to any year of Christ, 4713, and it gives the year of the Julian period; or subtract for any time *before* Christ.

**CYCLOID,** *principal properties of.*



1. Circ. arc E G = G C.

2. Tangent at C is parallel to the chord E G.

3. Cycloidal arc E C = 2 chord E G.

4. Area of cycloid = 3 times area of the generating circle.

5. Solid generated by the revolution of the cycloid about its base A B : its circumscribing cylinder :: 5 : 8.

6. Centre of gravity of the whole cycloid = $\frac{5}{8}$ of the axis from the vertex.

7. Rad. curv. at E = 2 D E.

8. Equations to the cycloid; put $a$ = diameter E D; $x$ and $y$ the co-ordinates E K, K C; $z$ = arc E C; $z'$ = arc E G; then

$$dz = a^{\frac{1}{2}} x^{-\frac{1}{2}} dx.$$

$$\text{and } y = z' + \sqrt{ax - x^2}.$$

For the oscillation of a body in a cycloid; *see Pendulum.*

# D

**DAMS.**—*See Fluids.*

**DATES.**—*See Chronology.*

**DAY** *of the week to find.*—*See Dominical Letter.*

**DAYS,** *length and increase of, &c.*

TABLE,

*Shewing, with sufficient accuracy for common purposes, the length and increase of the days in this country, at different seasons of the year, together with the beginning and end of twilight.*

| JANUARY. | | | | | FEBRUARY. | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Days. | Length of Day. | Day inc. | Day breaks | Twi. ends. | Days. | Length of Day. | Day inc. | Day breaks | Twi. ends. |
| 1 | 7. 50 | 0. 6 | 6. 0 | 6. 0 | 1 | 9. 4 | 1. 20 | 5. 31 | 6. 29 |
| 6 | 56 | 12 | 5. 58 | 2 | 6 | 20 | 36 | 24 | 37 |
| 11 | 8. 6 | 22 | 51 | 6 | 11 | 40 | 56 | 16 | 45 |
| 16 | 16 | 32 | 49 | 11 | 16 | 59 | 2. 14 | 7 | 51 |
| 21 | 30 | 46 | 44 | 16 | 21 | 10. 18 | 32 | 4. 58 | 7. 3 |
| 26 | 44 | 1. 0 | 38 | 22 | 26 | 38 | 52 | 49 | 12 |

### MARCH.

| Days. | Length of Day. | Day inc. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 10. 48 | 3. 4 | 4. 44 | 7. 17 |
| 6 | 11. 8 | 24 | 32 | 29 |
| 11 | 28 | 44 | 22 | 40 |
| 16 | 48 | 4. 4 | 12 | 50 |
| 21 | 12. 8 | 24 | 2 | 8. 1 |
| 26 | 26 | 42 | 3. 50 | 13 |

### APRIL.

| Days. | Length of Day. | Day inc. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 12. 50 | 5. 6 | 3. 33 | 8. 28 |
| 6 | 13. 10 | 26 | 21 | 40 |
| 11 | 30 | 46 | 8 | 58 |
| 16 | 50 | 6. 6 | 2. 54 | 9. 7 |
| 21 | 14. 8 | 24 | 40 | 21 |
| 26 | 26 | 42 | 26 | 35 |

### MAY.

| Days. | Length of Day. | Day inc. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 14. 44 | 7. 0 | 2. 7 | 9. 55 |
| 6 | 15. 2 | 18 | 1. 52 | 10. 10 |
| 11 | 18 | 34 | 30 | 33 |
| 16 | 34 | 50 | 7 | 56 |
| 21 | 48 | 8. 4 | 0. 32 | 11. 48 |
| 26 | 16. 0 | 16 | No real night. | |

### JUNE.

| Days. | Length of Day. | Day inc. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 16. 12 | 8. 28 | | |
| 6 | 22 | 38 | No real night | |
| 11 | 28 | 44 | but constant | |
| 16 | 32 | 48 | day or twi- | |
| 21 | 34 | 50 | light. | |
| 26 | 34 | 50 | | |

### JULY.

| Days. | Length of Day. | Day dec. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 16. 30 | 0. 4 | | |
| 6 | 24 | 10 | | |
| 11 | 16 | 18 | No real night | |
| 16 | 6 | 28 | | |
| 21 | 15. 56 | 38 | | |
| 26 | 42 | 52 | 0. 44 | 11. 14 |

### AUGUST.

| Days. | Length of Day. | Day dec. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 15. 24 | 1. 10 | 1. 22 | 10. 35 |
| 6 | 8 | 26 | 42 | 15 |
| 11 | 14. 50 | 44 | 2. 0 | 9. 57 |
| 16 | 34 | 2. 0 | 18 | 40 |
| 21 | 16 | 18 | 33 | 25 |
| 26 | 13. 56 | 38 | 48 | 10 |

### SEPTEMBER.

| Days. | Length of Day. | Day dec. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 13. 34 | 3. 0 | 3. 5 | 8. 54 |
| 6 | 16 | 18 | 19 | 40 |
| 11 | 12. 56 | 38 | 32 | 27 |
| 16 | 36 | 58 | 43 | 16 |
| 21 | 16 | 4. 18 | 54 | 5 |
| 26 | 11. 58 | 36 | 4. 5 | 7. 54 |

### OCTOBER.

| Days. | Length of Day. | Day dec. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 11. 38 | 4. 56 | 4. 17 | 7. 41 |
| 6 | 18 | 5. 16 | 23 | 31 |
| 11 | 10. 58 | 36 | 38 | 21 |
| 16 | 38 | 56 | 48 | 11 |
| 21 | 20 | 6. 14 | 57 | 2 |
| 26 | 0 | 34 | 5. 6 | 6. 53 |

### NOVEMBER.

| Days. | Length of Day. | Day dec. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 9. 38 | 6. 56 | 5. 15 | 6. 44 |
| 6 | 20 | 7. 14 | 22 | 37 |
| 11 | 4 | 30 | 29 | 30 |
| 16 | 8. 48 | 46 | 35 | 24 |
| 21 | 32 | 8. 2 | 42 | 18 |
| 26 | 20 | 14 | 48 | 12 |

### DECEMBER.

| Days. | Length of Day. | Day dec. | Day breaks | Twi. ends. |
|---|---|---|---|---|
| 1 | 8. 8 | 8. 26 | 5. 54 | 6. 6 |
| 6 | 7. 58 | 36 | 57 | 3 |
| 11 | 52 | 42 | 58 | 2 |
| 16 | 46 | 48 | 59 | 1 |
| 21 | 44 | 50 | 6. 0 | 0 |
| 26 | 46 | 0in. 2 | 5. 59 | 1 |

**DEGREE**, *decimal parts of.—See Time.*

**DEGREES**, *&c. converted into Time.—See Time.*

**DEGREES** *of Latitude and Longitude.*

**TABLE** *of the lengths of different degrees in fathoms, computed by Col. Lambton, for every three degrees from the Equator to the Pole.—(Phil. Trans.* 1818.*)*

| Lat. | Degrees on the Meridian. | Degrees on the Perpendicular. | Degrees of Longitude. |
|---|---|---|---|
| 0 | 60459,2 | 60848,0 | 60848,0 |
| 3 | 60460,8 | 60848,4 | 60765,0 |
| 6 | 60465,0 | 60850,1 | 60516,8 |
| 9 | 60473,5 | 60852,8 | 60103,6 |
| 12 | 60484,5 | 60856,5 | 59526,7 |
| 15 | 60498,4 | 60861,1 | 58787,3 |
| 18 | 60515,1 | 60866,7 | 57887,7 |
| 21 | 60534,3 | 60873,2 | 56830,0 |
| 24 | 60556,0 | 60880,5 | 55628,1 |
| 27 | 60579,8 | 60888,5 | 54252,0 |
| 30 | 60605,5 | 60897,1 | 52738,4 |
| 33 | 60632,7 | 60906,2 | 51080,2 |
| 36 | 60661,3 | 60915,8 | 49281,9 |
| 39 | 60690,8 | 60925,7 | 47348,2 |
| 42 | 60721,3 | 60935,7 | 45284,0 |
| 45 | 60751,8 | 60946,1 | 43095,4 |
| 48 | 60782,3 | 60956,4 | 40787,8 |
| 51 | 60812,5 | 60966,5 | 38367,5 |
| 54 | 60842,1 | 60976,5 | 35841,1 |
| 57 | 60870,7 | 60986,1 | 33215,4 |
| 60 | 60898,0 | 60995,2 | 30497,6 |
| 63 | 60923,7 | 61003,8 | 27695,2 |
| 66 | 60947,5 | 61011,8 | 24815,7 |
| 69 | 60969,1 | 61018,9 | 21867,2 |
| 72 | 60988,3 | 61025,6 | 18857,9 |
| 75 | 61005,1 | 61031,0 | 15796,0 |
| 78 | 61018,9 | 61035,8 | 12690,1 |
| 81 | 61029,9 | 61039,5 | 9518,7 |
| 84 | 61037,8 | 61042,1 | 6380,6 |
| 87 | 61042,6 | 61043,7 | 3194,8 |
| 90 | 61044,3 | 61044,3 | |

**DEGREE** *French.*

The French usually divide the circumference of the circle into 400°, each degree into 100′, and each minute into 100″. Hence if $n$ = number of French degrees, &c. the corresponding number of English = $n - \frac{n}{10}$; i. e. from the number we must subtract the same, after the decimal point has been removed one place to the left.

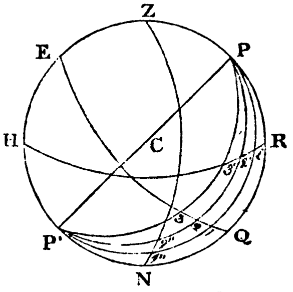
*Exs.* What number of degrees, minutes, &c. in the English scale correspond to 71°. 15′., and to 26°. 0735, in the French scale.

| | |
|---|---|
| 71.15 | 26.0735 |
| 7.115 | 2.60735 |
| 64.035 | 23.46615 |
| 60 | 60 |
| 2.100 | 27.96900 |
| 60 | 60 |
| 6.000 | 58.14000 Answer 23°. 27′. 58″. |
| 64°. 2′. 6″. | |

DEW.—*See Rain.*

DIALLING.

In all Dials universally, the style or gnomon is parallel to the earth's axis, and, on account of the great distance of the sun, may be imagined actually to coincide with it. In like manner the dial plate is parallel to, and supposed actually to coincide with, some great circle of the earth; and the hours may be conceived to be traced out by the shadow of the axis of the earth (here supposed hollow) upon one of these great circles. Hence there may be an infinite number of different kinds of dials, as they depend upon the position of the plane (on which the shadow of the earth's axis falls) with respect to the meridian and horizon. Thus if the shadow be received upon the Equator E Q, the dial is called an Equatorial Dial; if upon H R (a great circle of the earth in the plane of the horizon), a Horizontal one; if upon Z N, which is in the plane of the prime vertical, a North or South Dial, &c. &c. And in these three last cases, it is obvious that the shadow of the earth's axis, when the sun is on the meridian, or at 12 o'clock, will cut these several circles in Q, R, and N. At 1 o'clock, or when the ∠ Q P 1 is 15°, it will cut them at 1, 1′, 1″; at 2 o'clock, or when the ∠ at P is 30°, in 2, 2′, 2″, &c.; which are $\therefore$ the 12 o'clock, 1 o'clock, 2 o'clock, &c. marks.



### *Equatorial Dial.*

In this Dial, since the sun moves uniformly 15° per hour, the ∠s. at P, and consequently the arcs of the circle Q E, which measure them, will increase uniformly. Hence we have only to take from Q the arcs 15°, 30°, 45°, &c., and they will be 1 o'clock, 2 o'clock, 3 o'clock, &c., marks. This Dial, unless graduated on both sides, will only shew the hours for the six summer months, viz. from the vernal to the autumnal equinoxes.

### *Horizontal Dial.*

Here the arcs R 1′, 1′ 2′ &c. are not equal, but must be calculated by the resolution of the right ∠d. △s. P R 1′, P R 2′, &c., where R P 1′ = 15°, R P 2′ = 30°, &c., then we shall have

$$\tan. R\,1' = \sin. \text{lat.} \times \tan. 15^\circ.$$

$$\tan. R\,2' = \sin. \text{lat.} \times \tan. 2 \times 15^\circ.$$

&c. &c.

This Dial shews the hour throughout the year, whenever the sun is above the horizon. In order to fix a horizontal dial, find the time by the sun's alt. when it is at or near the solstices, and set a well regulated watch to that time; then when the watch shews 12 o'clock, at that instant set the dial to 12 o'clock, and it stands right.

### *Vertical North and South Dials.*

Here to find the arcs N 1″, N 2″, &c., we have in the right ∠d. △ P′ N 1″,

$$\tan. N\,1'' = \text{cosin. lat.} \times \tan. 15^\circ.$$

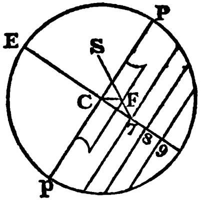
$$\tan. N\,2'' = \cos. \text{lat.} \times \tan. 2 \times 15^\circ.$$

&c. &c.

If P be the North Pole, this represents a South Dial. The construction for the Vertical North Dial is nearly the same. In this Dial the number of hours shewn in a day can never exceed twelve, which is the case at both the equinoxes; at any other season of the year, the number of hours shewn is less.

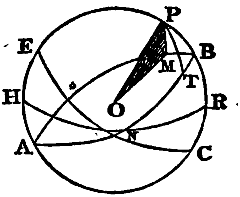
To find whether a wall be full south for a vertical south Dial, erect a gnomon perpendicular to it, and hang a plumb line from it; then when the watch shews 12, if the shadow of the gnomon coincide with the plumb line, the wall is full south.

### *Vertical East Dial.*



Here the plane of the Dial is in the meridian, and the gnomon a parallelogram perpendicular to it (as represented in the Fig.) and the shadows upon the plane will evidently be all parallel to the gnomon, and to one another. Moreover, at 6 o'clock, the sun, being due east, will be in the plane of the gnomon, and ∴ cast the shadow perpendicularly upon the Dial or on P*p*. To find the 7 o'clock mark, let S be the sun at that hour, and S F a ray proceeding from it cutting the Dial in 7; then in the plane right ∠d. △ C F 7, C 7, = C F × tan. ∠ C F 7 = height of style × tan. $15_0$. C 8 = height of style × tan. 2 × 15°. &c. Similarly may be constructed a vertical West Dial. The East Dial will not shew the hour after 12 o'clock at noon, nor the West Dial before.

### *General Problems.*



1. Given the latitude of the place, and the position of the plane of the Dial, both with respect to the meridian and horizon; it is required to find the elevation of the style, the distance of the sub-style from the meridian, and the arc intercepted between the meridian and any other given hour line.

Let B O A be the plane of the dial, given in position both with respect to the horizon H R, and the meridian P E A C; then in the right angled △ B N R, the ∠s. B N R, N B R are given, ∴ B R may be found; but P R = latitude, ∴ P B is known. Now let a plane pass through O P, and let it be turned about till it becomes perpendicular to B O A, and let it cut the circumference of B A in M, then P M is that meridian which is perpendicular to B O A, ∴ in the right angled △ P M B, P B and ∠ P B M are known, ∴ P M = elevation of the style, and M B, the distance of the substyle from the meridian, may be found. Draw P T, making an ∠ of 15° with P B; then will T be the 1 o'clock mark, and to find it we have P B, and B P T = 15°, and ∠ P B T = supplement of N B R, ∴ B T may be found, and so on for the other hours.

2. To determine the curve, traced out by the extremity of the shadow of a vertical gnomon on a horizontal plane.—*(Maddy.)*

Conceive a line A B to be the gnomon, A P the shadow, A N the direction of the meridian shadow. Draw P N perpendicular to A N, and let A N $= x$, P N $= y$, A B $= a$, $l =$ latitude of the place, $\delta =$ sun's declination; then

$$y^2 = \frac{(\cos.^2 l - \sin.^2 \delta).\, x^2 + 2a \sin. l \cos. l\, x + (\sin.^2 l - \sin.^2 \delta).\, aa.}{\sin.^2 \delta}$$

*Cor.* If $\cos. l = \sin. \delta$, or $l = 90^\circ - \delta$, the curve is a parabola, if $\cos. l$ is greater than $\sin. \delta$, or $l$ less than $90^\circ - \delta$, an hyperbola, if $\cos l$ is less than $\sin. \delta$, or $l$ greater than $90^\circ - \delta$, an ellipse.

## DIFFERENTIALS.

### TABLE I.

*Differentiation of Algebraic and Transcendental Functions; and of the higher orders of Differentials.*

| QUANTITY. | DIFFERENTIAL. |
|---|---|
| $ax$ ... ... ... ... ... | $a\,dx.$ |
| $ax + by - \frac{x}{c} + e$ ... | $a\,dx + b\,dy - \frac{dx}{c}.$ |
| $x^n$ ... ... ... ... ... | $nx^{n-1}dx.$ |
| $x^{\frac{n}{m}}$ ... ... ... ... ... | $\frac{n}{m}x^{\frac{n}{m}-1}dx.$ |
| $\left(a^m + x^m\right)^{\frac{1}{n}}$ ... ... | $\frac{1}{n}\left(a^m + x^m\right)^{\frac{1}{n}-1} \times mx^{m-1}dx.$ |
| $xy$ ... ... ... ... ... | $x\,dy + y\,dx.$ |
| $x^m y^n$ ... ... ... ... | $my^n x^{m-1}dx + nx^m y^{n-1}dy.$ |
| $\frac{x}{y}$ ... ... ... ... ... | $\frac{y\,dx - x\,dy}{y^2}.$ |
| Hyp. log. $x$ ... ... ... | $\frac{dx}{x}.$ |
| Hyp. log. $1 + x$ ... ... | $\frac{dx}{1+x}.$ |
| $e^x$ ... ... ... ... ... | $e^x\,dx \times$ h. l. $e.$ |
| $y^x$ ... ... ... ... ... | $y^x\,dx \times$ h. l. $y + xy^{x-1}dy.$ |

| | |
|---|---|
| Sin. $x$ … … … … | $dx \cos. x$. |
| Cos. $x$ … … … … | $-dx \sin. x$. |
| Sin. $mx$ … … … … | $m\,dx \cos. mx$. |
| Cos. $mx$ … … … … | $-m\,dx \sin. mx$. |
| Ver. sin. $x$ … … … | $dx \sin. x$. |
| Tan. $x$ … … … … | $\frac{dx}{\cos.^2 x}$. |
| Cot. $x$ … … … … | $-\frac{dx}{\sin.^2 x}$. |
| Sec. $x$ … … … … | $\frac{dx \sin. x}{\cos.^2 x} = dx \tan. x \times \sec. x$. |
| Cosec. $x$ … … … … | $-\frac{dx \cos. x}{\sin.^2 x}$. |
| $(\text{Sin.})^m x$ … … … | $m\,(\sin.)^{m-1} x\,dx \cos. x$. |
| $(\text{Cos.})^m x$ … … … | $-m\,(\cos.)^{m-1} x\,dx \sin. x$. |
| 2d. differential $x^2$ ($dx$ constant) … … | $2\,dx^2$. |
| 2d. diff. $x^2$ ($dx$ variable) … … … | $2\,dx^2 + 2\,x\,d^2x$. |
| 2d. diff. $y^2 = 2ax - x^2$ ($dx$ constant) | $y\,d^2y + dy^2 = -dx^2$. |
| 2d. diff. $2xy + ay^2 - bx^2 = 0$ … … | $dx\,dy + x\,d^2y + dy\,dx + a\,dy^2 + a\,y\,d^2y - b\,dx^2 = 0$. |

## TABLE II.

*Of Integrals, containing a few of the most usual forms that occur in the practical solution of Problems.*

| DIFFERENTIAL. | INTEGRAL. |
|---|---|
| $x^n dx$ … … … … | $\frac{x^{n+1}}{n+1}$. |
| $\frac{dx}{x^n}$ … … … … … | $-\frac{1}{(n-1)}\frac{1}{x^{n-1}}$. |
| $(a \pm x^n)^{m-1} x^{n-1} dx$ … | $\pm \frac{1}{mn}(a \pm x^n)^m$. |
| Ex. 1. $\frac{x\,dx}{\sqrt{a^2+x^2}}$ … | $\sqrt{a^2+x^2}$. |

Ex. 2. $\frac{x\,d\,x}{(a^2-x^2)^{\frac{3}{2}}}$ ... $\frac{1}{\sqrt{a^2-x^2}}$.

$x\,d\,y$ ... ... ... ... $x\,y -$ fl. $y\,d\,x$.

$x^m\,d\,x\,(a+b\,x^n)^p$ $m$, $n$, and $p$ any numbers whatever; integral may be found if $\frac{m+1}{n}$ or $\frac{m+1}{n}+p$ be a whole number.

Ex. 1. $\frac{x^m\,d\,x}{(a+b\,x)^p}$ ... $\frac{1}{b^{m+1}}\left(\frac{z^{m-p+1}}{m-p+1}-\frac{m\,a\,z^{m-p}}{m-p}+\frac{m\,(m-1)\,a^2\,z^{m-p-1}}{2\,(m-p-1)}\right.$ — &c. where $m$ is an integer, and $z = a+b\,x$.

Ex. 2. $\frac{a\,d\,x}{(a^2+x^2)^{\frac{3}{2}}}$ ... $\frac{x}{a\sqrt{a^2+x^2}}$.

$\frac{d\,x}{x}$ ... ... ... ... ... h. l. $x$.

$\frac{d\,x}{x \pm a}$ ... ... ... ... h. l. $(x \pm a)$.

$\frac{n\,x^{n-1}\,d\,x}{a^n+x^n}$ ... ... ... h. l. $(a^n+x^n)$.

$\frac{d\,x}{\sqrt{x^2 \pm a^2}}$ ... ... ... h. l. $\left(x+\sqrt{x^2 \pm a^2}\right)$

$\frac{d\,x}{\sqrt{x^2 \pm 2\,a\,x}}$ ... ... ... h. l. $\left(x \pm a+\sqrt{x^2 \pm 2\,a\,x}\right)$.

$\frac{2\,a\,d\,x}{a^2-x^2}$ ... ... ... ... h. l. $\frac{a+x}{a-x}$.

$\frac{2\,a\,d\,x}{x^2-a^2}$ ... ... ... ... h. l. $\frac{x-a}{x+a}$.

$\frac{2\,a\,d\,x}{x\sqrt{a^2+x^2}}$ ... ... ... h. l. $\frac{\sqrt{a^2+x^2}-a}{\sqrt{a^2+x^2}+a}$.

$\frac{2\,a\,d\,x}{x\sqrt{a^2-x^2}}$ ... ... h. l. $\frac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}$.

The last 9 forms of fluents may be found by a table of hyperbolic logarithms, or by a table of common logarithms, by multiplying the logarithm by 2.30258509, which will give the corresponding hyp. log.

$$\frac{a\,dy}{\sqrt{a^2-y^2}} \quad \ldots \ldots \ldots \quad \text{circ. arc rad. } a \text{ and sin. } y.$$

$$\frac{a\,dx}{\sqrt{2ax-x^2}} \quad \ldots \ldots \quad \text{circ. arc rad. } a \text{ and ver. sin. } x.$$

$$\frac{a^2\,dt}{a^2+t^2} \quad \ldots \ldots \ldots \ldots \quad \text{circ. arc rad. } a \text{ and tan. } t.$$

$$\frac{a^2\,ds}{s\sqrt{s^2-a^2}} \quad \ldots \ldots \ldots \quad \text{circ. arc rad. } a \text{ and sec. } s.$$

$$\frac{-a\,dx}{\sqrt{a^2-x^2}} \quad \ldots \ldots \ldots \quad \text{circ. arc rad. } a \text{ and cos. } x.$$

The five last forms may be found by a table exhibiting the length of circ. arcs for all degrees, &c. of the quadrant to rad. 1 *(see Arc)*; for by multiplying these arcs by $a$, we shall have their lengths to radius $a$. Thus, if the integral of $\frac{a\,dy}{\sqrt{a^2-y^2}}$ were required, when $y$ is the sin. 30°), we have the length of an arc of 30° to rad. 1 = 0.5235987; hence length to rad. $a$ is $a \times 0.5235987$ = integral required; and so for the rest.

$$\frac{x^{mn-1}\,dx}{(a \pm x^n)^{m+1}} \quad \ldots \ldots \quad \frac{1}{mna} \times \frac{x^{mn}}{(a \pm x^n)^m}.$$

$$\frac{(a \pm xn)^{m-1}\,dx}{x^{mn+1}} \quad \ldots \quad \frac{-1}{mna} \times \frac{(a \pm x^n)^m}{x^{mn}}.$$

$$\left(\frac{m\,dx}{x} + \frac{n\,dy}{y} + \frac{r\,dz}{z}\right) x^m y^n z^r \quad \ldots \ldots \quad x^m y^n z^r.$$

$$\frac{x^{-1}\,dx}{a \pm x^n} \quad \ldots \ldots \ldots \ldots \quad \frac{1}{na}\,\text{h. l. } \frac{x^n}{a \pm x^n}.$$

$$\frac{x^{\frac{1}{2}n-1}\,dx}{a-x^n} \quad \ldots \ldots \ldots \quad \frac{1}{n\sqrt{a}}\,\text{h. l. } \frac{\sqrt{a}+\sqrt{x^n}}{\sqrt{a}-\sqrt{x^n}}.$$

$$\frac{x^{\frac{1}{2}n-1}\,dx}{\sqrt{\pm a + x^n}} \quad \ldots \ldots \ldots \quad \frac{2}{n}\,\text{h. l. } \sqrt{x^n} + \sqrt{\pm a + x^n}.$$

$$\frac{x^{\frac{1}{2}n-1}\,dx}{\sqrt{a-x^n}} \;\ldots\;\ldots\;\ldots\; \frac{2}{n}\times \text{arc sin.} \sqrt{\frac{x^n}{a}}.$$

$$\frac{x^{-1}\,dx}{\sqrt{a\pm x^n}} \;\ldots\;\ldots\;\ldots\; \frac{1}{n\sqrt{a}} + \text{h. l.}\ \frac{\pm\sqrt{a\pm x^n}\pm\sqrt{a}}{\sqrt{a\pm x^n}+\sqrt{a}}.$$

$$\frac{x^{-1}\,dx}{\sqrt{-a+x^n}}. \;\ldots\;\ldots\;\ldots\; \frac{2}{n\sqrt{a}}\times \text{arc. sect.} \sqrt{\frac{x^n}{a}}.$$

$dx\sqrt{ax-x^2}$ … … ½ circ. seg. to diam. $a$ and ver. sin. $x$.

$$a^x x^n\,dx \;\ldots\;\ldots\;\ldots\; \frac{a^x}{k}\left(x^n - \frac{nx^{n-1}}{k} + \frac{n(n-1)x^{n-2}}{k^2} \ldots\ldots \pm \frac{1.\,2.\,3.\,\ldots\ldots\,n}{k^n}\right)$$

if $n$ be an integer and positive and $k =$ h. l. $x$.

$$\frac{dx}{\sin. x} \;\ldots\;\ldots\;\ldots\;\ldots\; \text{log. tan.}\ \frac{x}{2}.$$

$$\frac{dx}{\cos. x} \;\ldots\;\ldots\;\ldots\;\ldots\; \text{log. tan.}\ \left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$\sin. mx.\cos. nx.\,dx \;\ldots\; -\tfrac{1}{2}\left(\frac{\cos. (m+n)\,x}{m+n} + \frac{\cos. (m-n)\,x}{m-n}\right).$$

**DIP** *of the Horizon.—See Horizon.*

**DIP** *of the Magnetic Needle.—See Variation.*

**DISCHARGE** *of Fluids.—See Fluids discharge of.*

**DISCOUNT.**—*See Interest.*

**DISCOVERIES,** *dates of.—See Chronology.*

**DIVING BELL.**—*(Bland.)*

Having given the form of a Diving Bell, and the depth it has descended, to determine how high the water will have risen.

Let $x =$ abscissa, measured from the vertex, occupied by the air, $h =$ depth of the vertex below the surface of the water; M and $m$ the capacities of the whole bell and of that part occupied by the air, $a$ the altitude of the barometer; then

$$14a : 14a + h + x :: m : M.$$

from which proportion $x$ may be found.

Ex. Let the bell be a paraboloid, whose equation is $y^2 = 4cx$.

$$\text{Here } M = 2c\pi\, b^2 \;\&\; m = 2c\pi\, x^2.$$

$$\therefore\; x^3 + (14a + h)\,x^2 = 14ab^2.$$

from which equation $x$ may be found.

**DOMINICAL** *Letter to find.*

Rule.—Divide the centuries by 4, and take twice what remains from 6, then add the remainder to the odd years above the even centuries, and their fourth. Divide their sum by 7, and the remainder taken from 7 will leave the number answering to the letter required.

Thus to find the letter for 1826.

The centuries 18 divided by 4 leave 2, the double of which taken from 6 leaves 2, to which add the odd years 26 and their fourth part 6, their sum 34 divided by 7 leaves 6, which taken from 7 leaves 1, answering to A the first letter of the alphabet.

If it be leap year, the above rule gives the Dominical letter from the end of February to the end of the year; the letter immediately succeeding will be the Dominical letter for the beginning of the year, from January to the end of February.

Knowing the Dominical letter, we may, by the help of the following Table, find the day of the week answering to any given day of the month, and conversely.

TABLE.

| *Months.* | *Dominical Letters.* | | | | | | |
|---|---|---|---|---|---|---|---|
| January, October .................. | A | B | C | D | E | F | G |
| February, March, November | D | E | F | G | A | B | C |
| April, July ........................... | G | A | B | C | D | E | F |
| May ....................................... | B | C | D | E | F | G | A |
| June ...................................... | E | F | G | A | B | C | D |
| August ................................. | C | D | E | F | G | A | B |
| September, December .......... | F | G | A | B | C | D | E |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| | 29 | 30 | 31 | | | | |

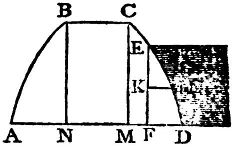
Here the first vertical column contains the several months in the year, and that part of the other columns immediately opposite contains the dominical letters: the under part contains the days of the month on which the Sundays happen; and hence the other days of the week are easily found.

DOUBLE *Stars.—See Stars.*

DYKE.—*(Bland.)*

A mound or obstacle opposed to the effort made by a fluid to spread itself.

1. Let A B C D be a vertical section of a dyke opposed to the stagnant fluid. Its parts are supposed to be so connected, as to yield to the pressure of the fluid, either by turning altogether round the point A, or by sliding along the horizontal base D A.



2. Supposing the dyke to yield by turning round A, to determine when there will be an equilibrium.

Let H K $= x$, E K $= y$, E F $= a$, A D $= b$, F D $= c$, $s =$ specific gravity of fluid, $s' =$ do. of dyke, $Q =$ the product of the area A B C D multiplied by the distance of A from the vertical passing through the centre of gravity of the area; then in the case of equilibrium,

$$\tfrac{1}{6}\, s a^2 = s' Q + s.\ \text{fl.}\ (b - c + x).\ y\, dx.$$

3. Supposing the dyke to yield by sliding along its horizontal base: to determine when there will be an equilibrium; neglecting the vertical pressure of the fluid.

The base being horizontal, the mass which it sustains is supported against the horizontal force of the fluid only by its adhesion to the base, and the resistance arising from friction. Supposing these resistances $=$ $n$ times the weight of the dyke ($n$ being determined by experiment); and let P $=$ the area of the section A B C D; then

$$P = \frac{s}{s'} \times \frac{a^2}{2n}.$$

4. If A B and C D be straight lines, i. e. if the sides of the dyke be rectilinear, and A D, B C horizontal; to determine the equation of equilibrium of Art. 2.

Let C M $= h$, M D $= e$, A N $= e'$, then $b^2 + \left(\frac{sc}{s'h}.\ a - e\right) b -$

$\frac{s\,(a^3 + c^2\,a)}{3\,s'\,h} + \frac{1}{3}\,(e^3 - e'^3) = o$ an equation which includes all the cases of rectilinear sloping banks.

*Cor.* If the slopes be $= o$, or the dykes vertical, $e = o$, $e' = o$, and $\sigma = o$,

$$\therefore\ b = \sqrt{\frac{s\,a^3}{3\,s'\,h}}.$$

5. If the sides be rectilinear, and A D, B C horizontal, to determine the equation of equilibrium of Art. 3.

$$\text{Here } b = \frac{s}{s'} \times \frac{a^2}{2\,h\,n} + \tfrac{1}{2}\,(e + e').$$

The preceding equations have been investigated on the supposition of a perfect connexion of all the parts of the dyke; they are ∴ only applicable to such as are constructed of masonry.

# E

## EARTH, *Elements of.*

The principal elements of the earth, according to the determination of La Place, in the last edition of his Systeme du Monde, are as follows :—

| | | |
|---|---|---|
| Equatorial Diameter | 7924 | English miles. |
| Polar do. | 7908 | ............... |
| Mean do. | 7916 | ............... |
| Mean circumference | 24869 | ............... |
| Mean length of a degree | 69.08 | ............... |
| Surface | 196862256 | square miles. |
| Solidity | 259726936416 | cubic miles. |

Density of earth about 5 times that of common water.

Mass of earth is $\frac{1}{337086}$ of the mass of the sun.

Weight of a body at Equator : do. at the Poles :: 1 : 1.00569.

Length of seconds Pendulum at Equator = 39.027 in.; do. at Poles = 39.197 in.

Centrifugal force at Equator is about $\frac{1}{289}$ of gravity, and ∴ if the rotatory motion of the earth were 17 times greater than it is, bodies at the Equator would have no weight.

Mean distance of earth from sun = 93,321,724 English miles.

Aphelion distance = 94,889,528 miles.

Perihelion do. = 91,753,920 miles.

Eccentricity = .0168, ½ ax. maj. being 1.

Time of sidereal revolution 365*d.* 6*h.* 9*m.* 11,5*s.*
Do. tropical ..................... 365*d.* 5*h.* 48*m.* 51,6*s*; being less than in the time of Hipparchus by 11,2.″

Mean velocity of earth in its orbit 59′. 10,7″ each day.

Velocity in its perihelion ......... 1°. 1′. 9,9.″

Do. in aphelion ...................... 57.′ 10,7″

Revolution about the line of the apsides, or anomalistic year, 365*d.* 6*h.* 14*m.* 2*s.*

Tropical revolution of apsides performed in 20,931 years.

Inclination of axis to Ecliptic 23°. 27′. 57″., which decreases at the rate of 52,1″. in a century, but this decrease can never exceed 2°. 42′.

Nutation of axis = 19,3″.

Precession of the equinoxes 50,1″ annually, or 1°. 23′. 30″. in a century. A complete revolution performed in 25868 years.

Length of sidereal day 23*h.* 56*m.* 4,1*s.*; and has not varied the hundredth part of a second since the time of Hipparchus.

The interval between the vernal and autumnal equinoxes is (on account of the excentricity of the earth's orbit and its unequal velocity therein) nearly eight days longer than the interval between the autumnal and vernal equinoxes. These intervals are at present nearly as follows:—

| | *d.* | *h.* | *m.* | | *d.* | *h.* | *m.* |
|---|---|---|---|---|---|---|---|
| From spring equinox to summer solstice ........................ | 92. | 21. | 45. | = | 185. | 35. | 20. |
| From summer solstice to autumnal equinox ........................ | 93. | 13. | 35. | | | | |
| From autumn equinox to winter solstice ........................ | 89. | 16. | 47. | = | 178. | 18. | 29. |
| From winter solstice to spring equinox ........................ | 89. | 1. | 42. | | | | |
| | | | | Difference ......... | 7. | 16. | 51. |

**EARTH,** *figure of.—(Playfair, Maddy.)*

1. To find the radius of curvature at any point of the terrestrial meridian, supposing the earth to be an oblate spheriod.

Let $a$ and $b$ be the Equatorial and Polar ½ axes, $r$ the rad. of curv. to the latitude $\lambda$, $c = a - b$ = compression, $m = 57^\circ.2957795$ the number of degrees in an arc = radius; then

$$r = a - 2c + 3c \sin. 2\lambda.$$

$$\text{or} = a - \frac{c}{2} - \frac{3c}{2} \cos. 2\lambda.$$

and if D = length of a degree in lat. $\lambda$, $r = m\,D$

$$\therefore D = \frac{a}{m}\left(1 - \frac{c}{2a} - \frac{3c}{2a} \cos. 2\lambda.\right)$$

*Cor.* 1. At the Equator $m\,D = a - 2c$; at the Pole $m\,D = a + c$; and in lat. 45°. $= a - \frac{1}{2}c$. Hence if E, P, and M = the degree at the Equator, Pole, and lat. 45°.; $M = \frac{1}{2}(P + E)$.

*Cor.* 2. The excess of a degree in any lat. above that at the Equator, or $D - E$, varies as $\sin.^2 \lambda$.

2. The lengths of two degrees of latitude, of which the middle points are in given latitudes, being known by admeasurement, the Equatorial and Polar diameters of the earth may be calculated from the following formulæ.

Let D and D′ be the given degrees (the least, or that nearest the Equator being D) $\lambda$ and $\lambda'$ the latitudes of their middle points, then

$$c = \frac{m.\,(D' - D.)}{3 \sin.\,(\lambda' + \lambda) \times \sin.\,(\lambda' - \lambda)}.$$

and the compression, or ellipticity of the earth

$$= \frac{c}{a} = \frac{D' - D}{3\,D. \sin.\,(\lambda' + \lambda) \times \sin.\,(\lambda' - \lambda)}.$$

from which two equations $a$ and $c$, and consequently $a$ and $b$, may be found.

The following are the five arcs, which have been measured with the greatest care :—

| *Latitude.* | *Degrees in Fathoms.* | *Country.* | *By whom.* |
|---|---|---|---|
| 0°. 0′. 0″. | 60480.2 ... | Peru ....... | Condamine, &c. |
| 11. 0. 0. | 60486.6 ... | India ...... | Major Lambton. |
| 45. 0. 0. | 60759.4 ... | France ... | Cassini, &c. |
| 52. 2. 2. | 60826.6 ... | England... | Colonel Mudge. |
| 66. 20. 10. | 60952.4 ... | Lapland... | Swanberg, &c. |

By combining these in pairs, and taking the mean, we get the following results.

$$a : b :: 312 : 311.$$

$$D = 69.044 - .3299 \times \cos. 2\,\lambda \text{ in miles,}$$

or $D = 60759.472 - 290.576 \times \cos. 2\,\lambda$ in fathoms, which expresses the degrees of the meridian in any latitude.

$$\frac{c}{a} = .0032 = \frac{1}{312.5}.$$

$$c = 12.680 \text{ miles.}$$

$$a = 3962.349 \text{ miles.}$$

$$b = 3949.669 \text{ miles.}$$

Hence circumference of elliptic meridian = 24855.84 miles ; do. of equator = 24896.16 miles ; ∴ difference = 40 miles nearly.

3. The figure of the earth may also be determined, by comparing a degree of the meridian with the degree of a great circle perpendicular to the meridian in the same latitude, by the following formulæ.

Let Δ be the degree of the curve perpendicular to the meridian, the rest as before, then

$$c = \frac{m}{2}\,(\Delta - D) \times \frac{1}{\cos.^2 \lambda}.$$

$$\text{and } \frac{c}{a} = \frac{\Delta - D}{2\,\Delta \cos.^2 \lambda} \text{ nearly.}$$

4. To find the compression by means of a second's pendulum, considering the earth as a spheroid of equilibrium.

Let $p$ and $p'$ be the lengths of two pendulums oscillating seconds in latitudes $\lambda$ and $\lambda'$, $c$ the compression, the equatorial radius being unity; then

$$c = \frac{p - p'}{p \sin.^2 \lambda - p' \sin.^2 \lambda'}.$$

**5. Comparison of the figure of the earth, deduced from actual admeasurement of a degree in different latitudes, with that deduced from the theory of gravity.**

If a homogeneous fluid revolve on an axis, it will form itself into an oblate spheroid, of which the Polar ½ axis : radius of Equator :: attraction at Equator — centrifugal force at Equator : attraction at the Pole.

In the case of the earth, this ratio will be :: 229 : 230.

If the earth be not homogeneous, but composed of strata that increase in density towards the centre, the spheroid will have less oblateness than if it were homogeneous, and it is demonstrable that if the density increase so that it be infinite at the centre, the ellipticity $= \frac{1}{578}$, which is the case of the least ellipticity; $\frac{1}{230}$ is the case with the greatest.

Hence as the ellipticity of the earth has been shewn to be less than $\frac{1}{230}$ (viz. $\frac{1}{312}$), it is evident that if the earth is a spheroid of equilibrium, it is denser towards the interior. This has been indisputably proved to be the case by actual experiment.—*See Mountain, attraction of.*

But after all, whether the earth be a spheroid of equilibrium, whether the N. and S. ½ spheres be equal and similar to each other, and what is the ratio of an arc of the meridian, measured in a given latitude, to the whole meridian, are questions to which complete solutions have not yet been given.

6. TABLE of the ellipticities of the earth.

| *Authors.* | *Ellipticities.* | *Principles.* |
|---|---|---|
| Newton | $\frac{1}{230}$ | Theory of Gravity. |
| Playfair | $\frac{1}{312}$ | Mensuration of Arcs. |
| Lambton | $\frac{1}{310}$ | Do. |
| Sabine | $\frac{1}{312.6}$ to $\frac{1}{314.3}$ | Vibration of Pendulum. |
| Treisnecker | $\frac{1}{329}$ | Occultation of Stars. |
| La Place | $\frac{1}{334}$ | Precession and Nutation. |
| | $\frac{1}{305}$ | Theory of the Moon. |

Upon the whole, the ellipticity probably lies between $\frac{1}{307}$ and $\frac{1}{314}$. But Captain Sabine, from some very recent experiments on the length of the Pendulum *(see Pendulum)*, states the ellipticity at $\frac{1}{288.4}$. For Tables of Degrees of Latitude and Longitude, *see Degree.*

EARTH's *Surface, extent of.—(Encyc. Britt. Supplt.)*

The extent of the four great divisions of the world is as follows:—

| | | *Sq. Eng. Miles.* |
|---|---|---|
| Europe, with its Isles | | 3,432,000 |
| Africa, with Madagascar | | 11,420,000 |
| Continental Asia | 16,890,000 | 21,090,000 |
| Isles, including New Holland and Polynesia | 4,200,000 | |
| South America | 6,420,000 | 15,300,000 |
| North do. | 8,100,000 | |
| Islands | 160,000 | |
| Greenland (supposed) | 620,000 | |
| Total | | 51,242,000 |

The ocean, with all its inland bays and seas, covers an area of 145,000,000 square miles, or nearly $\frac{3}{4}$ of the surface of the globe. About $\frac{7}{12}$ of the great body of waters lie in the southern hemisphere; and $\frac{5}{12}$

in the northern. In the one the ocean : the land :: 7 : 5; and in the other :: 13 : 12.

If we suppose the mean depth of the ocean to be two miles, the cubic content will be 290,000,000 of cubic miles.

Comparative superficial extent of the frigid, temperate, and torrid zones, taking the whole area of the globe as unity.—(*Lacroix.*)

| | |
|---|---|
| The frigid zones occupy | $\frac{83}{1000}$. |
| The temperate zones | $\frac{519}{1000}$. |
| The torrid zone | $\frac{398}{1000}$. |

EARTH, *density of.—See Mountain, attraction of.*

EARTH, *internal temperature of.*—(*Encyc. Britt. Supp.*)

In descending below the surface of the earth, a considerable increase of temperature is observed, as the following examples prove.

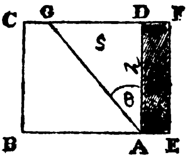
At Giromagny, in the Vosges, annual temperature at surface is 49°; at 110 yards depth, 53°. 6; at 336 yards, 65°. 8; at 472 yards, 74°. 6.

In Saxony, in four of the deepest mines, annual temperature at surface is 46°. 4; at 170 to 200 yards depth, 54°. 5; at 280 yards, 58°; at 360 yards, 62°. 6.

In the coal mine of Killingworth, the deepest in Britain, annual temperature at surface is 48°; at 300 yards, 70°; at 400 yards, 77°. In seven others of the deepest coal mines in Britain, a corresponding gradation was observed.

In these British mines, the increment of temperature is about 1° for 15 yards of descent. In the Vosges it is about 1° for 20 yards, and in Saxony 1° for 22 yards. Taking 20 yards as a mean, if the increase follows the same arithmetical ratio to a considerable depth, we should find the temperature of the Bath waters (116°) at 1320 yards below the surface; and that of boiling water at 3300 yards, or nearly two miles.

EARTH, *pressure of against walls.*—(*Gregory.*)



Let D A E F be the vertical section of a wall, behind which is placed a bank or terrace of earth, of which a prism, whose section is represented by D A G, would detach itself and fall down, were it not prevented by the wall. Then A G is called the *line of rupture*, or the natural slope, or natural declivity. In sandy or loose earth, the ∠ B A G seldom exceeds 30°; in stronger earth it becomes 37°; and in some favourable cases more than 45°.

1. If $h$ = A D, $x$ = A E, $\theta = \angle$ D A G, and S and $s$ represent the specific gravities of the wall and earth, the state of equilibrium is expressed by this equation,

$$\tfrac{1}{2}\, x^2.\ S = \tfrac{1}{6}\, h^2.\ s.\ \tan.^2 \tfrac{1}{2}\,\theta.$$

*Ex.* Suppose the wall to be 39.37 feet high, of brick, specific gravity 2000, and the bank of earth specific gravity 1428, and the natural slope 53°; then

$$\tfrac{1}{2}\, x^2.\ 2000 = \tfrac{1}{6} \times 39.37^2 \times 1428 \times \tan._2\ 26\tfrac{1}{2}^{\circ},$$

$$\therefore\ x = 9.6 \text{ feet} = \text{thickness of wall.}$$

The following practical results may be found useful.

*Values of* D G *for different materials.*

| | |
|---|---|
| Bank of vegetable earth | D G = .618 $h$. |
| Do. of sand | D G = .677 $h$. |
| Do. of vegetable earth mixed with small gravel | D G = .646 $h$. |
| Do. of rubbles | D G = .414 $h$. |
| Do. of vegetable earth mixed with large gravel | D G = .618 $h$. |

*Thickness of walls, both faces vertical.*

| | |
|---|---|
| 1. Wall brick, 109 lbs. per cubic foot, bank vegetable earth carefully laid course by course | D F = .16 $h$. |
| 2. Wall unhewn stones, 135 lbs. per cubic foot, earth as before | D F = .15 $h$. |
| 3. Wall brick, earth clay well rammed | D F = .17 $h$. |
| 4. Wall unhewn stones, earth as before | D F = .16 $h$. |
| 5. Wall of hewn freestone, 170 lbs. per cubic foot, bank vegetable earth | D F = .13 $h$. |
| 6. Do. bank clay | D F = .14 $h$. |
| 7. Bank of earth mixed with large gravel, wall of bricks | D F = .19 $h$. |
| Do. of unhewn stone | D F = .17 $h$. |
| Do. of hewn freestone | D F = .16 $h$. |
| 8. Bank of sand. | |
| Wall of bricks | D F = .33 $h$. |
| Do. of unhewn stones | D F = .30 $h$. |
| Do. of hewn freestone | D F = .26 $h$. |

When the earth of the bank is liable to be much saturated with water the proportional thicknesses of the walls must at least be doubled.

2. For walls with an interior slope, or a slope towards the bank, let the base of the slope be $\frac{1}{n}$ of the height, then

$$\mathrm{D\,F} = h\sqrt{\left(\frac{1}{3\,n^2} + m.\ \frac{s}{\mathrm{S}}\right)} - \frac{h}{n}$$

where $m = .0424$ for vegetable or clayey earth, mixed with large gravel; $m = .0464$ if the earth be mixed with small gravel; $m = .1528$ for sand; and $m = .166$ for semifluid earths.

*Ex.* Let the height of a wall be 20 feet, and $\frac{1}{20}$ of the height for the base of the slope, suppose also the specific gravity of the wall and bank to be 2600 and 1400, and the earth semifluid; then

$$\mathrm{D\,F} = 20\sqrt{\left(\frac{1}{1200} + .166 + \frac{14}{26}\right)} - \frac{20}{20}$$

$= 5$ feet, while the thickness of the wall at the bottom will be 6 feet.

**EASTER,** *to find it on any year.—(Delambre.)*

1. Divide the year proposed by 19 ............... Call remainder $a$.
2. Divide the same number by 4 .................. Call remainder $b$.
3. Divide it also by 7 ................................. Call remainder $c$.
4. Divide $(19\,a + \mathrm{M})$ by 30 ....................... Call remainder $d$.
5. Divide $(2\,b + 4\,c + 6\,d + \mathrm{N})$ by 7 .......... Call remainder $e$.
6. Then Easter day will fall either on $(22 + d + e)$ of March; or on $(d + e - 9)$ of April.

Values of M and N in the above calculation.

| | M. | N. |
|---|---|---|
| From 1700 to 1799 | 23 | 3 |
| 1800 to 1899 | 23 | 4 |
| 1900 to 1999 | 24 | 5 |

Exceptions to this rule:

1. If the computation give April 26, substitute the 19th.
2. If it give .................... April 25, substitute the 18th.

**ECCENTRICITY** *of a Planet's orbit.—(Woodhouse, Playfair.)*

Let $e$ be the eccentricity of the orbit, $g$ the greatest equation of the centre, found by observation, and put $\frac{g}{57^\circ.\ 29578} = h$, then

$$e = \tfrac{1}{2}\,h - \frac{11}{768}\,h^3 - \frac{587}{983040}\,h^5 - \&c.$$

In the earth's orbit $h$ is very small, $\therefore e = \frac{1}{2} h$ nearly.

The secular diminution $= 18''.79$, and $\therefore$ if this diminution continued uniform (which, however, we have not a right to suppose) the earth's orbit would become a circle in about 36300 years.

## ECHO.

That an echo may return one syllable as soon as it is pronounced, the reflecting surface should be 80 or 90 feet distant; for a dissyllabic echo 170 feet, &c. This is upon the supposition that sound proceeds at the rate of 1142 feet per second, and that the ear can distinguish the succession of two sounds or syllables, when the interval between them is $\frac{1}{7}$th of a second.—*(Playfair.)*

An echo in Woodstock Park repeats 17 syllables by day, and 20 by night. An echo on the north side of Shipley church in Sussex, repeats 21 syllables.—*(Young's Nat. Phil.)*

## ECLIPSES.—*(Woodhouse, Playfair.)*

### 1. *Eclipses of the Moon.*

1. The length of the earth's shadow varies, according to the distance of the sun and earth, between the limits of 212,896, and 220,238 semidiameters of the earth; its mean length being 216,531. And in general if $r$ be the earth's radius, $\frac{D}{2}$ the apparent semidiameter, and $p$ the horizontal parallax of the sun, the length of the shadow, reckoned from the earth's centre,

$$= \frac{r}{\sin.\left(\frac{D}{2} - p\right)} \text{ or } = \frac{r}{\sin.\ \frac{109\ D}{220}}.$$

2. Hence half the angle subtended at the earth's centre by the section of the shadow, at the distance of the moon, (if P be the horizontal parallax of the moon) is

$$P + p - \frac{D}{2}.$$

From this formula the apparent diameters of the earth's shadow may be computed for various distances of the sun and moon, as in the following Table.

| | | *Apparent diam. of earth's shadow.* |
|---|---|---|
| Sun in perigee ......... | Moon in apogee ... .............. | 1°. 15′. 24″.3036 |
| | at mean distance .................. | 1. 23. 2·31 |
| | in perigee .......................... | 1. 30. 40.3164 |

| | | *Apparent diam. of earth's shadow.* | | |
|---|---|---|---|---|
| Sun at mean distance | Moon in apogee | 1. | 15. | 56.8656 |
| | at mean distance | 1. | 23. | 34.872 |
| | in perigee | 1. | 31. | 12.8784 |
| Sun in apogee | Moon in apogee | 1. | 16. | 28.2936 |
| | at mean distance | 1. | 24. | 6.3 |
| | in perigee | 1. | 31. | 44.3064 |

3. The distance of the centres of the moon and of the earth's shadow, when the moon's disk just touches the shadow (if $d$ = moon's diameter) is

$$P + p - \frac{D}{2} + \frac{d}{2}.$$

*Cor.* If $P = 57'.\ 1''$, $p = 8'',8$, and $\frac{D}{2} = 16'.\ 1''.3$, we have the mean apparent ½ diameter of the earth's shadow $= 41'.\ 8''.5$, which is nearly three apparent ½ diameters of the moon. Hence since the moon in the space of an hour moves over a space nearly equal to its diameter, the moon may be entirely within the shadow, or a total eclipse may endure, about two hours.

4. The apparent ½ diameter of a section of the penumbra at the moon's orbit =

$$P + p + \frac{D}{2}.$$

And the distance of the moon's centre and of the centre of the shadow, when the moon first enters the penumbra, is

$$P + p + \frac{D}{2} + \frac{d}{2}.$$

5. To find the time, duration, and magnitude of a lunar eclipse.

Let $m$ = moon's motion in longitude,
$n$ = moon's motion in latitude,
$s$ = sun's (or the shadow's centre's) motion in longitude,
$\lambda$ = moon's latitude when in opposition,
$t$ = time from opposition,
$c$ = distance of moon and earth's shadow,

and let $\frac{n}{m-s} = \tan.\ \theta$.

$$\text{then } t = \frac{1}{n}\left\{-\lambda \sin.^2\theta \pm \sin.\theta\,\sqrt{(c^2 - \lambda^2 \cos.^2\theta)}\right\}$$

from which expression may be deduced values of the time, corresponding to any assigned values of $c$, as in the following instances.

(j) To determine the time at which the moon first enters the penumbra, for $c$ put $P + p + \frac{D}{2} + \frac{d}{2}$; $t$ has two values, and the second value will denote the time at which the moon quits the penumbra.

(ij) To determine the time at which the moon enters the umbra, put $c = P + p + \frac{d}{2} - \frac{D}{2}$.

(iij) To determine the time when the whole disk has just entered the shadow, we must deduct $d$ from the preceding value, and make $c = P + p - \frac{d}{2} - \frac{D}{2}$; and similarly for other phases.

(iiij) To find the middle of the eclipse, we have $t = -\lambda \frac{\sin.^2 \theta}{n}$, and in that case the distance of the centres ($c$) is $= \lambda \cos.\ \theta$.

(v) The nearest approach of the centres being known, the magnitude of the eclipse is easily ascertained. Thus on the supposition that $\lambda \cos.\ \theta$ is less than the distance $\left(P + p + \frac{d}{2} - \frac{D}{2}\right)$ at which the moon's limb just touches the shadow, some part of the moon's disk is eclipsed; and the portion of the diameter of the eclipsed part is

$$P + p + \frac{d}{2} - \frac{D}{2} - \lambda \cos.\ \theta.$$

The portion of the diameter of the non-eclipsed part is the moon's apparent diameter $d$, minus the preceding expression, and therefore is

$$\lambda \cos.\ \theta + \frac{d}{2} + \frac{D}{2} - P - p.$$

If this expression should be equal nothing, the eclipse would be *just* a total one. If the expression should be negative, the eclipse may be said to be *more than* a total one, since the upper boundary of the moon's disk would be below the upper boundary of the section of the shadow.

(vj) If in the expression

$$\frac{2}{n} \sin.\ \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}.$$

we substitute for $c$, $P + p + \frac{d}{2} - \frac{D}{2}$ we have the time from the moon's first entering to her finally quitting the shadow or *umbra*. And if in the same expression we substitute for $c$, $P + p + \frac{d}{2} + \frac{D}{2}$, we have the whole time of an eclipse, from the moon's first entering, till her finally quitting the penumbra.

6. Ecliptic limits. When the mean opposition is 12°. 36′ distant from the node, there can be no eclipse; and when it is less than 9°. distant from it, there must be an eclipse. Between these limits 12°. 36′ and 9°. the matter is uncertain, and must be decided by the calculation of the true place of the moon.

### II. *Eclipses of the Sun.*

1. Let $r$, R be the radii of the moon and earth, the rest as before; then the length of the moon's shadow

$$= \frac{r}{\sin.\left\{\left(\frac{D}{2} - p\frac{r}{R}\right)\frac{P}{P-p}\right\}}$$

By means of this formula, we have

| | *Length of shadow.* | *Moon's distance.* |
|---|---|---|
| Sun in apogee, moon in perigee ..................... | 59.730 | 55.902 |
| Sun in perigee, moon in apogee ..................... | 57.760 | 63.862 |

Hence in the latter case, the moon's shadow never reaches the earth, and the eclipse cannot any where be total.

The moon's mean motion about the centre of the earth is 33′ in an hour; and the shadow of the moon ∴ traverses the surface of the earth, when it falls on the surface perpendicularly, with a velocity of about 380 miles in a minute. When the shadow falls obliquely, its velocity appears greater in the inverse ratio of the sine of the obliquity.

The duration of a total eclipse in any given place cannot exceed 7*m.* 58*s.*

An annular eclipse may last 12*m.* 24*s.*

2. The apparent ½ diameter of the moon's shadow $= \frac{d-D}{2} + \frac{P}{P-p}$.

Hence when $d = D$ apparent ½ diameter $= o$, or the vertex of the conical shadow just reaches the earth. When $d$ is less than D, the expression is negative, in other words the shadow never reaches the earth.

In a similar manner may the formulæ for the penumbra of the earth be transformed and adapted to the case of the moon.

(iii) The solar ecliptic limits = 17°. 21′. 27″. If the conjunction happens nearer to the node than this, there may be an eclipse. If it be more distant, there can be none.

Solar eclipses are more difficult of computation than lunar ones; nor is it possible to enter here upon the methods that have been employed. We shall ∴ conclude this article with an account of the number of eclipses that may take place in a year.

### III. *Eclipses, number of.*

In the space of 18 years, there are usually about 70 eclipses, 29 of the moon, and 41 of the sun.

Seven is the greatest number of eclipses that can happen in a year, and two the least.

If there are seven, five must be of the sun, and two of the moon. If there are only two, they must be both of the sun; for in every year there are at least two eclipses of the sun.

There can never be more than three eclipses of the moon in a year; and in some years there are none at all.

Though the number of solar eclipses is greater than of lunar in the ratio of 3 to 2, yet more lunar than solar eclipses are visible in any particular place, because a lunar eclipse is visible to an entire hemisphere, and a solar is only visible to a part.

## ECLIPTIC, *obliquity of.—(Woodhouse, Vince.)*

The mean obliquity of the Ecliptic in January 1, 1827 = 23°. 27′. 43″.7. For the variations in the obliquity, *see Precession.* But besides these variations in the obliquity, arising from solar inequality and nutation, the former of which passes through all its changes in the period of half a year, and the latter in 9 years and 3¼ months, the obliquity of the Ecliptic has, as far back as observation goes, been diminishing from the action of the planets, particularly Venus and Jupiter. This diminution, called the secular diminution, is *at present* 52″ in a century. There is, however, a mean to the obliquity which it cannot pass, and round which it oscillates backwards and forwards. According to La Grange, the inclination will never vary more than 5°. 23′ from the year 1700.

Hence if we have given the mean obliquity for any time, and wish to find the true obliquity, we must correct the given mean obliquity by the secular diminution, the solar inequality, and the nutation. The analytical expression for the obliquity, including these corrections, is

$$E - \frac{0''.52 \times n}{365} + 0''.4345 \times \cos.\ 2\ \text{sun's longitude} + 9''.63 \times \cos.\ N$$

E being the mean obliquity at the beginning of the year, N the supplement of the node, and $n$ the number of days from the beginning of the year.

**ELASTIC** *bodies, equilibrium of.—(Whewell.)*

This subject may be comprised under three heads. (1.) Elasticity of Extension and Compression, as in the case of a string stretched by a force. (2.) Elasticity of Flexure, as when wires and laminæ of different metals and other substances exert a force to unbend themselves when forcibly bent. (3.) The Elasticity of Torsion, as when twisted threads of metal exert a force to untwist themselves. Our view of these several subjects must necessarily be very limited and imperfect.

### 1. *Elasticity of Extension.*

1. When an elastic string of given length is stretched by a given force, to find its length.

The increase of length is proportional to the tension. Let $\iota$ be the measure of the *extensibility* of the string, whose length at first is $a$; $t$ the force or weight with which the string is stretched, which of course measures the tension; then the increase of length $= a \iota t$, and the length $l$ when stretched will ∴ be

$$a + a \iota t, \text{ or } a\,(1 + \iota t)$$

We may determine $\iota$, if we know the original length of the string, and its length for any given value of $t$. It may be convenient to know it in terms of the force which will draw out the string to *double* its length. Let E be this force; hence

$$a\,(1 + \iota \mathrm{E}) = 2a, \text{ and } \iota = \frac{1}{\mathrm{E}}.$$

Hence the length of the string under a tension $t$ becomes

$$= a\left(1 + \frac{t}{\mathrm{E}}\right).$$

E may be expressed by a length of the given string, whose weight would draw the string $a$ to double its length. E is then called the *modulus of elasticity.*

2. A uniform elastic string hangs vertically, stretched by its own weight: to find its length.

The same notation being retained,

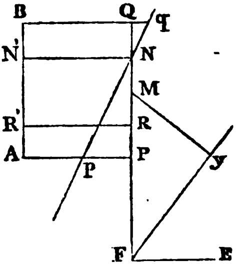
$$l = a + \frac{\iota a^2}{2} \text{ or } = a + \frac{a^2}{2\,\mathrm{E}}.$$

*Cor.* 1. If $a = \mathrm{E}$, $l = \frac{3\,\mathrm{E}}{2}$.

*Cor.* 2. Since $l = a\left(1 + \frac{\iota a}{2}\right)$, it appears that the weight of the string stretches it half as much, as if it were all collected at the lowest point.

### 2. *Elasticity and resistance of solid materials.*

Here we suppose that all solid bodies may be considered as made up of elastic fibres capable of extension and compression; and that the resistance to extension is proportional to the extension in each fibre.



When a solid body is acted on by any *force*, it may be partly extended and partly compressed. Thus let a mass A B Q P be acted upon by a force F compressing it in the direction E F. The surface P N Q may be brought into the direction $p$ N $q$; in this case all the fibres R R′ which are on one side of N are shortened; all those on the other side of N are lengthened. N N′ remains the same as in the natural state. N is called the *neutral point*; and the line which separates the parts of the body which are compressed from those which are elongated is called the *neutral line*.

1. When a rectangular prismatic mass is compressed by a force parallel to the direction of the axis: to find the neutral line.

Let $PM = MQ = a$, $MF = h$, $MN = n$, then

$$n = \frac{(2a)^2}{12h} = \frac{PQ^2}{12MF}.$$

*Cor.* 1. If $h = \frac{1}{3}a$, $n = a$, or the neutral point is in the surface, and the whole beam is compressed.

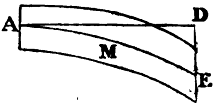
2. When a rectangular prism is acted upon by any force in any direction; to find the neutral point at any part.

Let a force $f$ act in the line $y$ F on a prism A B P Q, then the same notation being retained, we have as before

$$n = \frac{a^2}{3h}.$$

*Cor.* If the force act perpendicularly to the axis, $h$ is infinite, $n = o$, and the neutral point is in the axis.

3. When a rectangular prismatic beam is made to deviate a little from a straight line by the action of a given force perpendicular to it, to find the deflexion.



Since the force is perpendicular to the beam, and the beam is nearly a straight line, we may *(by Cor. last Art.)* suppose the neutral point coincident with the axis. Let A M E represent the axis bent by a force acting perpendicularly to A D its original position; and let F be a length of the beam equivalent to the force $f$, $l =$ length $b =$ breadth, and $a =$ thickness of the beam, E the modulus of elasticity, then the whole deflexion $\delta$

$$= \frac{F}{E} \cdot \frac{l^3}{a^3}.$$

or if for F we put its value $\frac{f}{2ab}$,

$$\delta = \frac{f l^3}{2 E a^3 b}.$$

*Cor.* 1. Hence for a given breadth and thickness, the deflexion is as the force and cube of the length; and for a given weight and length, the deflexion is inversely as the breadth and cube of the thickness.

*Cor.* 2. Let the direction of the tangent at E make an $\angle\, \theta$ with the tangent at A; then $\theta$ may be called the *angular deflexion*, and we have

$$\tan \theta = \frac{F}{E} \cdot \frac{3 l^2}{2 a^3} = \frac{3 f l^2}{4 a^3 b}.$$

The angular deflexion is as the force and square of the length.

4. When a rectangular prismatic beam in a horizontal position is bent by its own weight; (its thickness being vertical) to find the deflexion.

The same notation being retained, the whole deflexion

$$\frac{3 l^4}{8 E a^4}.$$

*Cor.* In this and the last Art. $\delta$ being observed, E may be found.

5. A rectangular prismatic beam is compressed by a given force acting in a direction parallel to the axis; to find the deflexion.

Let $a$ be $\frac{1}{2}$ the thickness of the beam, $l = \frac{1}{2}$ the length, $h =$ distance of the force from the axis; then if E be very large compared with F, we have the deflexion

$$= h \left(\sec. \frac{l \sqrt{3 F}}{a \sqrt{E}} - 1\right).$$

*Cor.* If the force act at the extremities of the axis, $h = o$, and there will be no deviation except

$$\frac{l^2}{a^2} = \frac{\pi^2 E}{12 F} = .8225 \frac{E}{F}.$$

Hence we may find the weights which columns of given materials will support. Thus, if in fir-wood the modulus E be 10,000,000 feet, a bar, an inch square, and 10 feet long, may begin to bend, when

$$F = .8225 \times \frac{1}{(120)^2} \times 10,000,000 = 571 \text{ feet.}$$

### 3. *Elasticity of Torsion.*

1. Let $f$ and $f'$ be the forces necessary to twist a metallic thread, from the position in which it would naturally hang, through the ∠s. $\theta$ and $\theta'$; then if $\theta$ and $\theta'$ be very small,

$$\frac{f}{f'} = \frac{\theta^2}{\theta'^2}.$$

On this principle depends the *Torsion Balance* of Coulomb, which has been employed for the purpose of measuring very small repulsive and attractive forces. In some cases the instrument was constructed with so much delicacy, that each degree of torsion required a force of only $\frac{1}{122400}$ of a grain.

*Height of the Modulus of Elasticity in thousands of feet.—(Encyclop. Brit. Supplem.)*

| | | | |
|---|---|---|---|
| Iron and steel | 10,000 | Fir wood | 10,000 |
| Copper | 5,700 | Elm | 8,000 |
| Brass | 5,000 | Beech | 8,000 |
| Silver | 3,240 | Oak | 5,060 |
| Tin | 2,250 | Box | 5,050 |
| Crown glass | 9,800 | Ice | ,850 |

The following Table is the result of experiments by Mr. Rennie, published in the first part of the Phil. Trans. for 1818.

Mr. Rennie found a cubical inch of the following bodies crushed by the following weights :—

| | *lbs. av.* |
|---|---|
| Elm | 1284 |
| American Pine | 1606 |
| White Deal | 1928 |
| English Oak | 3860 |

*Cubes of 1½ inch.*

| | *Sp. gr.* | |
|---|---|---|
| Chalk | | 1127 |
| Red Brick | 2168 | 1817 |
| Derby Grit | 2316 | 7070 |
| Portland | 2428 | 10284 |

| | *Sp. gr.* | |
|---|---|---|
| Craigleith White Freestone | 2452 | 12346 |
| Yorkshire Paving | 2507 | 12856 |
| White Statuary Marble | 2760 | 13632 |
| Bromley Fell Sandstone, near Leeds | 2506 | 13632 |
| Cornish Granite | 2662 | 14302 |
| Dundee Sandstone | 2530 | 14918 |
| Compact Limestone | 2584 | 17354 |
| Purbeck | 2599 | 20610 |
| Black Brabant Marble | 2697 | 20742 |
| Very Hard Freestone | 2528 | 21254 |

Cubes of different metals of ¼ inch were crushed by the following weights :—

| | *lbs. av.* | | *lbs. av.* |
|---|---|---|---|
| Cast Iron | 9773 | Wrought Copper | 6440 |
| Cast Copper | 7318 | Cast Tin | 966 |
| Fine Yellow Brass | 10304 | Cast Lead | 483 |

Bars of different metals six inches long, and ¼ inch square, were suspended by nippers, and broken by the following weights :—

| | *lbs. av.* | | *lbs. av.* |
|---|---|---|---|
| Cast Iron, horizontal | 1166 | Gun Metal | 2273 |
| Ditto, vertical | 1218 | Copper hammered | 2112 |
| Cast Steel | 8391 | Cast Copper | 1192 |
| Blistered Steel hammered | 8322 | Fine Yellow Brass | 1123 |
| Shear Steel do. | 7977 | Cast Tin | 296 |
| Swedish Iron do. | 4504 | Cast Lead | 114 |
| English Iron do. | 3492 | | |

ELASTIC *bodies, theory of.—See Collision.*

ELLIPSE, *principal properties of.—See Conic Sections.*

ELLIPTICITY *of the Earth.—See Earth, figure of.*

EMBANKMENT.—*See Dyke, and Earth pressure of.*

EPOCH.—*See Æra.*

EQUATIONS *of condition.—(Playfair, Maddy.)*

Any equation expressing the relation that obtains among the coefficients of another equation, is called an *Equation of condition.* These equations are used in determining by observation the constant coefficients in an assumed or given function of a variable quantity. Thus let us suppose that the form of the function is known from theory, but that the constant quantities that enter into it, are to be determined by observation; required, considering that every observation is liable to error, in what way these quantities may be most accurately determined.

Rule.—Substitute the quantities known by observation for $y$ and $x$, in the given formula (each observation being supposed to afford a value both of $x$ and $y$), and thus, as many equations of condition will be obtained, as there are observations. If these exceed the number of quantities to be found, or of the equations wanted, let there be composed from the addition of them into separate sums, as many equations as are necessary, each consisting of as many of the given equations as the question admits of. From the equations thus obtained, the quantities sought may be determined with the least probability of error.

Suppose the general formula to be

$$y = A \sin. x + B \sin. 2x,$$

and that from observation we have eight values of $x$ and $y$, viz.

| Values of $x$. | Values of $y$. |
|---|---|
| 140° | 73′.5 |
| 135 | 80.2 |
| 130 | 87.0 |
| 125 | 94.1 |
| 120 | 99.5 |
| 115 | 104.5 |
| 110 | 107.5 |
| 105 | 110.2 |

Hence,

$$\begin{array}{rcrcl}
.6428\ A & - & .9848\ B & = & 73.5 \\
.7071\ A & - & 1.0000\ B & = & 80.2 \\
.7660\ A & - & .9848\ B & = & 87.0 \\
.8191\ A & - & .9397\ B & = & 94.1 \\
.8660\ A & - & .8660\ B & = & 99.5 \\
.9063\ A & - & .7660\ B & = & 104.5 \\
.9397\ A & - & .6428\ B & = & 107.5 \\
.9660\ A & - & .5000\ B & = & 110.2
\end{array}$$

By adding the first four into one, and also the second four, we get

$$2.9350\ A - 3.9033\ B = 334.8, \text{ and}$$
$$3.6780\ A - 2.7748\ B = 421.7;$$

and therefore,

$$A = \frac{3.9033 \times 421.7 - 2.7748 \times 334.8}{3.678 \times 3.9033 - 2.935 \times 2.7748}.$$

$$\text{or } A = 1^{\circ}.55'.00.$$

In like manner, $B = 1'.2$; so that the equation becomes,

$$y = (1^{\circ}.\ 54'.2)\ \sin.\ x + (1'.2)\ \sin.\ 2\,x.$$

This is nearly the equation of the centre in the earth's orbit.

In this way all the elements of any of the planetary orbits may be determined *simultaneously*, or corrected if they are already nearly known. In the construction of Astronomical Tables, the number of equations combined has amounted to many hundreds.

In the example above, no method was to be followed, but that of dividing the original equations into two parcels or groups, from the sums of which the new equations were to be deduced. But when it happens in the given equations, that the terms involving the same unknown quantity have different signs, the best way is to order all the equations so that one of the unknown quantities, as A, shall have the same sign throughout; and then to add them together, for the first of the derivative equations. Let the same be done with B, C, &c. whatever be the number of the quantities sought. Thus, each of the unknown quantities will occur in one of the equations, with the greatest possible coefficient; and the coefficients of the same unknown quantity, in the different equations, will become by that means as unequal as they can be rendered, which contributes to make the divisor by which that quantity is to be found, as large, and itself of course, as accurate as the case will admit of.

*Ex.* Let the equations be

$$\begin{aligned} 3 - x + y - 2\,z &= o \\ 5 - 3\,x - 2\,y + 5\,z &= o \\ 21 - 4\,x - y - 4\,z &= o \\ 14 + x - 3\,y - 3\,z &= o \end{aligned}$$

changing the signs of the last equation, and adding,

$$15 - 9\,x + y + 2\,z = o$$

similarly for $y$, $37 - 5\,x - 7\,y \qquad = o$

for $z$, $33 - \ x - y - 14\,z = o$

From these equations $x = 2.486$

$y = 3.517$

$z = 1.928$

### *Second Method.*

$$\begin{aligned} \text{Let } m + a\,x + b\,y + c\,z + \&c. &= o, \\ m' + a'\,x + b'\,y + c'\,z + \&c. &= o, \\ m'' + a''\,x + b''\,y + c''\,z + \&c. &= o, \\ \&c. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots &= o, \end{aligned}$$

be the equations; multiply the first by $a$, the second by $a'$, and so on; then by addition,

$$ma + m'a' + \&c. + (a^2 + a'^2 + \&c.)\,x + (ab + a'b' + \&c.)\,y + (ac + a'c' + \&c.)\,z = 0,$$

Similarly

$$(mb + m'b' + \&c.) + (ab + a'b' + \&c.)\,x + (b^2 + b'^2 + \&c.)\,y + (bc + b'c' + \&c.)\,z + \&c. = 0,$$

$$(mc + m'c' + \&c.) + (ac + a'c' + \&c.)\,x + (bc + b'c' + \&c.)\,y + (c^2 + c'^2 + \&c.)\,z + \&c. = 0,$$

$$\&c. + \&c. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = 0.$$

By this means as many equations are formed as there are unknown quantities, and from them $x$, $y$, $z$, &c. may be determined.

The method applied to the example in the preceding article gives the reduced equations

$$-88 + 27x + 6y = 0,$$
$$-70 + 6x + 15z = 0,$$
$$-107 + y + 54z = 0.$$

From whence $x = 2.470$, $y = 3{,}551$, $z = 1.916$.

The above mode of reducing the linear equations, which is called the *Method of Least Squares*, was invented by Gauss.

**EQUATION** *of Payments.*

*Common rule.*

Let $p$ and $p'$ be the sums due at the end of the times $n$ and $n'$; $x$ = equated time

$$\text{then } x = \frac{pn + p'n'}{p + p'}.$$

i.e. equated time is found by multiplying each sum by the time at which it is due, and dividing by the sum of the payments.

This rule is erroneous in principle, being founded upon the supposition that the receiver gains *interest* upon the latter sum by receiving it before it is due; whereas in fact he ought only to gain the *discount*. In most questions, however, that occur in business, the error is so trifling, that the above rule will always be made use of as the most eligible method.

*Correct rule.*

Let $r$ = interest of £1. for one year, the rest as before, put $\dfrac{pr\,(n + n') + p' + p}{pr} = a$, and $\dfrac{prnn' + p'n' + pn}{pr} = b$, then

$$x = \frac{a \pm \sqrt{a^2 - 4b}}{2}.$$

EQUATION *of Time.*

The equation of time, relatively to its causes, depends on two circumstances; (1) the obliquity of the ecliptic; and (2) the unequal angular motion of the sun in its orbit. The equation of time, as arising from the first cause, would be the difference of the sun's longitude and its right ascension converted into time. In the first and third quadrants, apparent time would precede true; in the second and fourth quadrants, true time would precede apparent; and at the Tropics and Equinoxes, true and apparent time would coincide. Also upon this supposition, the equation would be a maximum at 4 points, viz. when the cosine of the sun's declination is a mean proportional between radius, and the cosine of the obliquity of the ecliptic.

The equation of time, as arising from the second cause, would be the difference between the true and mean anomaly. Hence true and apparent time would coincide at the higher and lower apsides. From the higher to the lower apside, apparent time would precede true; from the lower to the higher apside, true time would precede apparent. The equation, in this case, would be greater at two points than at any other, viz. when the earth's distance from the sun is a mean proportional between the ½ axes of its orbit. To find it, when both causes are considered together, let A be the sun's *true* right ascension, M his *mean* longitude, $\nu$ the equation of the Equinoxes in longitude; then $\nu \times$ cos. obliquity $=$ the equation of the Equinoxes in right ascension, and

$$\text{Equation of time} = \frac{A - M - \nu \times \text{cos. obliquity}}{15}.$$

which is to be added to apparent time if positive, and subtracted if negative.

As the sun's true right ascension is deduced from the true longitude and the apparent obliquity of the ecliptic, both of which vary from one age to another; hence tables of the equation of time, constructed for any one time, are not true for another. The following Table, therefore, taken from the Nautical Almanack for 1828, or leap year, though inapplicable when any very nice determinations of the time are required, may yet be useful for regulating common clocks or watches, as the error for the next half century will only amount to a few seconds.

## TABLE.

### *Equation of Time for every Day in the Year 1836.*

| Days. | Jan. | Feb. | Mar. | Apr. | May. | June | July. | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Add | Add | Add | Add | Sub. | Sub. | Add | Add | Sub. | Sub. | Sub. | Sub. |
| | m s | m s | m s | m s | m s | m s | m s | m s | m s | m s | m s | m s |
| 1 | 3 35 | 13 52 | 12 35 | 3 54 | 3 5 | 2 33 | 3 25 | 5 57 | 0 15 | 10 25 | 16 17 | 10 38 |
| 2 | 4 4 | 14 0 | 12 23 | 3 36 | 3 13 | 2 24 | 3 37 | 5 53 | 0 34 | 10 43 | 16 17 | 10 14 |
| 3 | 4 32 | 14 7 | 12 10 | 3 18 | 3 19 | 2 14 | 3 48 | 5 49 | 0 53 | 11 2 | 16 17 | 9 50 |
| 4 | 4 59 | 14 13 | 11 56 | 3 0 | 3 26 | 2 4 | 3 58 | 5 44 | 1 12 | 11 20 | 16 16 | 9 26 |
| 5 | 5 27 | 14 18 | 11 42 | 2 42 | 3 31 | 1 54 | 4 9 | 5 38 | 1 32 | 11 38 | 16 14 | 9 1 |
| 6 | 5 54 | 14 23 | 11 28 | 2 25 | 3 37 | 1 43 | 4 19 | 5 32 | 1 51 | 11 55 | 16 11 | 8 35 |
| 7 | 6 20 | 14 27 | 11 17 | 2 7 | 3 41 | 1 33 | 4 29 | 5 25 | 2 11 | 12 12 | 16 8 | 8 9 |
| 8 | 6 46 | 14 30 | 10 59 | 1 50 | 3 45 | 1 21 | 4 39 | 5 18 | 2 32 | 12 29 | 16 3 | 7 43 |
| 9 | 7 11 | 14 32 | 10 43 | 1 33 | 3 48 | 1 10 | 4 48 | 5 10 | 2 52 | 12 45 | 15 58 | 7 15 |
| 10 | 7 36 | 14 34 | 10 28 | 1 17 | 3 51 | 0 58 | 4 57 | 5 1 | 3 12 | 13 0 | 15 52 | 6 48 |
| 11 | 8 1 | 14 35 | 10 12 | 1 1 | 3 53 | 0 46 | 5 5 | 4 52 | 3 33 | 13 15 | 15 45 | 6 20 |
| 12 | 8 25 | 14 35 | 9 55 | 0 45 | 3 55 | 0 34 | 5 13 | 4 43 | 3 54 | 13 30 | 15 37 | 5 52 |
| 13 | 8 48 | 14 34 | 9 39 | 0 29 | 3 55 | 0 1 | 5 20 | 4 32 | 4 15 | 13 44 | 15 29 | 5 23 |
| 14 | 9 10 | 14 33 | 9 22 | 0 11 | 3 56 | 0 9 | 5 27 | 4 22 | 4 36 | 13 58 | 15 19 | 4 55 |
| | | | | Sub. | | Add | | | | | | |
| 15 | 9 32 | 14 30 | 9 5 | 0 1 | 3 56 | 0 4 | 5 34 | 4 10 | 4 57 | 14 11 | 15 9 | 4 25 |
| 16 | 9 54 | 14 28 | 8 47 | 0 16 | 3 55 | 0 17 | 5 40 | 3 58 | 5 18 | 14 24 | 14 58 | 3 56 |
| 17 | 10 15 | 14 24 | 8 30 | 0 30 | 3 54 | 0 30 | 5 45 | 3 46 | 5 39 | 14 36 | 14 46 | 3 27 |
| 18 | 10 35 | 14 19 | 8 12 | 0 44 | 3 52 | 0 43 | 5 50 | 3 33 | 6 0 | 14 47 | 14 34 | 2 57 |
| 19 | 10 54 | 14 14 | 7 54 | 0 58 | 3 49 | 0 56 | 5 54 | 3 20 | 6 21 | 14 58 | 14 20 | 2 27 |
| 20 | 11 12 | 14 9 | 7 36 | 1 11 | 3 46 | 1 9 | 5 58 | 3 6 | 6 42 | 15 8 | 14 6 | 1 58 |
| 21 | 11 30 | 14 2 | 7 18 | 1 24 | 3 43 | 1 22 | 6 1 | 2 51 | 7 3 | 15 18 | 13 51 | 1 28 |
| 22 | 11 47 | 13 55 | 7 0 | 1 36 | 3 39 | 1 35 | 6 4 | 2 37 | 7 24 | 15 27 | 13 35 | 0 58 |
| 23 | 12 3 | 13 47 | 6 41 | 1 48 | 3 34 | 1 47 | 6 6 | 2 21 | 7 45 | 15 35 | 13 18 | 0 28 |
| | | | | | | | | | | | | Add |
| 24 | 12 18 | 13 39 | 6 23 | 1 59 | 3 29 | 2 0 | 6 7 | 2 5 | 8 5 | 15 43 | 13 1 | 0 2 |
| 25 | 12 33 | 13 29 | 6 4 | 2 10 | 3 24 | 2 13 | 6 8 | 1 49 | 8 26 | 15 50 | 12 42 | 0 32 |
| 26 | 12 47 | 13 20 | 5 46 | 2 20 | 3 18 | 2 25 | 6 8 | 1 33 | 8 46 | 15 56 | 12 23 | 1 2 |
| 27 | 13 0 | 13 9 | 5 27 | 2 30 | 3 12 | 2 38 | 6 8 | 1 16 | 9 7 | 16 1 | 12 4 | 1 32 |
| 28 | 13 12 | 12 59 | 5 8 | 2 40 | 3 5 | 2 50 | 6 7 | 0 58 | 9 26 | 16 6 | 11 43 | 2 0 |
| 29 | 13 23 | 12 47 | 4 50 | 2 49 | 2 57 | 3 2 | 6 5 | 0 41 | 9 46 | 16 10 | 11 22 | 2 30 |
| 30 | 13 33 | | 4 31 | 2 57 | 2 50 | 3 14 | 6 3 | 0 23 | 10 6 | 16 13 | 11 0 | 3 0 |
| 31 | 13 43 | | 4 13 | | 2 41 | | 6 0 | 0 4 | | 16 15 | | 3 29 |

**The above Table contains the equation of time for leap year; but the equation may be found for other years as follows. For the first year after leap year take one-fourth of the difference between the equations for the given and preceding days, which is to be added to the equation for the given day, if at that time the equation is decreasing; but subtracted if it is increasing. In the second after leap year, take half the difference between the equations; and in the third, take three-fourths of the difference, and apply this correction in the same manner as before.**

*Note.*—The word *add* in the Table denotes that the equation of time, as there expressed, must be *added* to the apparent time, shewn by a Dial or other instrument, in order to give the mean or equated time. In those columns to which the word *sub* is prefixed, it implies that the equation of time must be *subtracted* from the apparent time, in order to give the true or correct time.

If it be proposed to convert mean time into apparent, this is done by a contrary process, by applying the equation of time to the mean time given, with its title or sign changed, viz. subtracting instead of adding, and adding instead of subtracting.

EQUILIBRIUM of *Floating Bodies.—(Playfair, Bland.)*

1. When the centre of gravity of a floating body is in the same vertical line with the centre of gravity of the fluid displaced, the body remains in equilibrium.

2. If in a floating body, of which the transverse section is the same from one end of the body to the other, $a$ be the length of the water line, $c^2$ the area of the section of the immersed part, $d$ the distance between the centre of gravity of the whole and the centre of gravity of the immersed part, and $i$ an indefinitely small inclination from the position of equilibrium, the momentum of the force tending to restore the equilibrium is

$$\left(\frac{a^3}{12\,c^2} - d\right) \text{W sin. } i.$$

If $\frac{a^3}{12\,c^2}$ is greater than $d$, the force tends to restore the body to its state of equilibrium, or the equilibrium is that of *stability*.

If $\frac{a^3}{12\,c^2} = d$, there is no force tending either to restore or destroy the equilibrium; or the equilibrium is that of *indifference*.

If $\frac{a^3}{12\,c^2}$ be less than $d$, the force becomes negative, and tends to overset the body; or the equilibrium is that of *instability*.

When W remains the same, the stability is proportional to $\left(\frac{a^3}{12\,c^2} - d\right)$ sin. $i$.

When the centre of gravity of the body is lower than the centre of gravity of the immersed part, $d$ is negative, and the quantity $\frac{a^3}{12\,c^2} - d$ is affirmative, whatever be the magnitude of $\frac{a^3}{12\,c^2}$.

If in the axis of the solid, or in the line passing through the two centres, there be taken a point distant from the centre of the immersed part by $= \frac{a^3}{12\, c^3}$, this point is called the *metacentre*; and the stability will be positive or negative or nothing, according as the metacentre is above, below, or coincident with the centre of gravity of the floating body.

3. If a rectangular parallelopiped float in a fluid, with its altitude $a$ perpendicular to the surface; if its breadth be $b$, and its specific gravity $n$, that of the fluid being 1, its stability will be as $\frac{b^2}{6} - n\,(1 - n)\; a^2$.

When it has no stability, $\frac{b^2}{6} - n\,(1 - n)\; a^2 = o$, and $a = \frac{b}{\sqrt{6\, n\,(1 - n)}}$, and $n = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{b^2}{6\, a^2}}$.

*Cor.* 1. When $\frac{b^2}{6\, a^2}$ is less than $\frac{1}{4}$, or when the height of the solid has a greater proportion to the base of the section than $\sqrt{2} : \sqrt{3}$, two values may assigned to the specific gravity of the body, which will cause it to float in the equilibrium of indifference.

*Cor.* 2. If $n = \frac{1}{2}$, as is nearly the case with fir, $a = b\sqrt{\frac{2}{3}} = \frac{5\, b}{6}$ nearly. The truth of this conclusion may be shewn by experiment.

EQUILIBRIUM *of an Elastic Body.—See Elastic Bodies equilibrium of.*

EQUILIBRIUM *of a Point.—See Forces composition of.*

EQUINOXES, *precession of.—See Precession.*

ERRORS *in Time, in Astronomy.—See Time.*

EVAPORATION.

Mean monthly evaporation from the surface of water, from the experiments of Dr Dobson, of Liverpool, in the years 1772, 1773, 1774, and 1775.—(*Phil. Trans., and Manchester Memoirs.*)

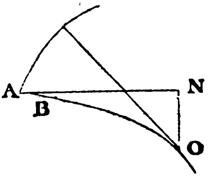
| | *Inches.* | | *Inches.* |
|---|---|---|---|
| January | 1.50 | July | 5.11 |
| February | 1.77 | August | 5.01 |
| March | 2.64 | September | 3.18 |
| April | 3.30 | October | 2.51 |
| May | 4.34 | November | 1.51 |
| June | 4.41 | December | 1.49 |

From some very accurate experiments made by Mr Dalton, the mean annual evaporation, over the whole surface of the globe, has been estimated at 35 inches; this gives 94,450 cubic miles for the water annually evaporated over the whole globe.—*see Rain.*

EVECTION.—*See Moon.*

EVOLUTES *of Curves.—(Higman.)*

To find the equation to the evolute.



Let $AN = \alpha$, and $NO = -\beta$, then may the relation between $\alpha$ and $\beta$ be found by eliminating $x$ and $y$ from the equations

$$y = f(x),$$

$$y - \beta = -\frac{1 + \frac{dy^2}{dx^2}}{\frac{d^2 y}{dx^2}},$$

$$x - \alpha = -\frac{dy}{dx}(y - \beta).$$

*Ex.* Required the evolute of the parabola.

$$\text{Here } y^2 = mx$$

$$y - \beta = \frac{4y^3}{m^2} + y.$$

$$x - \alpha = -\frac{2y^2}{m} - \frac{m}{2}.$$

Find values of $y$ and $x$ from the two last equations, substitute them in the first, and we shall have

$$\beta^2 = \frac{16}{27m}\left(\alpha - \frac{m}{2}\right)^3 = \frac{16}{27m}\alpha'^3, \text{ if } \alpha' = \alpha - \frac{m}{2}.$$

∴ the evolute is the semicubical parabola.

EVOLUTION.—*See Involution.*

EXPANSION *of liquids and solids by Heat.—See Heat.*

EXPANSION *of Water.—See Heat.*

EYE, *dimensions of, &c.—(Coddington.)*

The proportions of the spaces occupied by the three humours of the eye vary in different animals, as may be seen from the following Table, taken from *M. Cuvier's Anatomie Comparee*, which shews the parts of the *axis* lying in the several humours.

| | *Aqueous Humour.* | *Chrystalline.* | *Vitreous Humour.* |
|---|---|---|---|
| Man ..................... | $\frac{3}{22}$ | $\frac{4}{22}$ | $\frac{15}{22}$ |
| Dog ..................... | $\frac{5}{21}$ | $\frac{8}{21}$ | $\frac{8}{21}$ |
| Ox ..................... | $\frac{5}{37}$ | $\frac{14}{37}$ | $\frac{18}{37}$ |
| Sheep ..................... | $\frac{4}{17}$ | $\frac{11}{17}$ | $\frac{12}{17}$ |
| Horse ..................... | $\frac{9}{43}$ | $\frac{16}{43}$ | $\frac{18}{43}$ |
| Owl ..................... | $\frac{8}{27}$ | $\frac{11}{27}$ | $\frac{8}{27}$ |
| Herring ..................... | $\frac{1}{7}$ | $\frac{5}{7}$ | $\frac{1}{7}$ |

The radii of the surfaces of the chrystalline are in

| | |
|---|---|
| Man as ..................... | 12 to 16 |
| Dog ..................... | 12 to 14 |
| Ox ..................... | 6 to 21 |
| Rabbit ..................... | 14 to 14 |
| Owl ..................... | 16 to 14 |

The specific gravities of the different parts are as follows, that of distilled water being 1.

| | *In the Ox.* | *In the Cod Fish.* |
|---|---|---|
| Aqueous humour ..................... | 1 | 1 |
| Vitreous humour ..................... | 1.016 | 1.013 |
| Chrystalline lens (mean) ..................... | 1.114 | 1.165 |
| Outer part of ditto ..................... | 1.070 | 1.140 |
| Inner ..................... | 1.160 | 1.200 |

As to their refractive powers, they must be more considerable than their density indicates, on account of the inflammable particles which enter into their composition.

Dr. Wollaston makes the refracting power of the vitreous humour equal to that of water, and that of the chrystalline lens of the ox greater in the ratio of from 1.38 to 1.447 to 1. Dr. Brewster gives the following Table, deduced from experiments made on a recent human eye :—

| | | |
|---|---|---|
| Refracting power of | Water | 1.3358 |
| | The Aqueous humour | 1.3366 |
| | —— Vitreous humour | 1.3394 |
| | —— outer coat of chrystalline | 1.3767 |
| | —— middle | 1.3786 |
| | —— central parts | 1.3990 |
| | —— whole chrystalline | 1.3839 |

Dr. Brewster also gives the following dimensions :—

| | *Inch.* |
|---|---|
| Diameter of the chrystalline | 0.378 |
| —————— cornea | 0.400 |
| Thickness of the chrystalline | 0.172 |
| —————— cornea | 0.042 |

If the humours of the eye be too convex or too flat, an imperfection in vision is in either case the consequence : a concave lens will remedy the former defect, and a convex one the latter. The following problems embrace nearly every thing connected with the theory of spectacles.

1. Given the distance at which a short-sighted person can see distinctly, to find the focal length of a concave glass which will enable him to see distinctly at any other given distance.

Let $\Delta'' =$ distance at which he can see distinctly, $\Delta$ a greater distance at which he wishes to view objects, $F =$ focal length of the required lens, then *(see Refraction* iii, *Art.* 2.)

$$\frac{1}{\Delta''} = \frac{1}{F} + \frac{1}{\Delta}; \text{ and } F = \frac{\Delta\,\Delta''}{\Delta - \Delta''}.$$

*Cor.* If $\Delta$ be indefinitely great, $F = \Delta''$.

2. Given the distance at which a long-sighted person can see distinctly, to find the focal length of a convex glass which will enable him to see distinctly at any other given distance.

Let $\Delta'' =$ distance at which he can see distinctly, $\Delta$ a shorter distance at which he wishes to view objects, $F =$ focal length of the lens, then

$$\frac{1}{\Delta''} = \frac{1}{\Delta} - \frac{1}{F}; \text{ and } F = \frac{\Delta\,\Delta''}{\Delta'' - \Delta}.$$

*Cor.* If $\Delta''$ be indefinitely great, or the eye require parallel rays, $F = \Delta$.

## TABLE,

*Of the focal length of the convex or magnifying glasses, commonly required at various ages.—(Kitchiner.)*

| *Years of age.* | *Inches. Focus.* | *Remarks.* |
|---|---|---|
| 40 ... | 36 | |
| 45 ... | 30 | Convex Spectacles are seldom wanted except to read by candle light, till 45 or 50. |
| 50 ... | 24 | |
| 55 ... | 20 | |
| 58 ... | 18 | |
| 60 ... | 16 | Concave glasses called No. 1, are equivalent to a convex of 24 inches focus; No. 2 to a 21 inch convex; No. 3 to an 18 inch. |
| 65 ... | 14 | |
| 70 ... | 12 | |
| 75 ... | 10 | |
| 80 ... | 9 | |

The following is an easy method of finding which of two concave or convex glasses magnifies most. Hold one in each hand about one foot from your eye, and about five feet from a window frame, and the lens, through which the panes of glass appear *least*, magnifies *most*. This is the readiest way of ascertaining their comparative power.

# F

FIGURE *of the sines, &c.*

Figure of the sines, cosines, tangents, secants; to find the area of.

1. Figure of the sines.
   Let $\theta =$ arc or abscissa, then area $= r \times$ ver. sin. $\theta$.
   When $\theta$ is a quadrant, area $= r^2$.
2. Figure of the cosines.
   Area $= r \times$ sin. $\theta$.
   When $\theta$ is a quadrant, area $= r^2$.
3. Figure of the tangents.

   $$\text{Area} = r^2 \times \text{h. l. } \frac{\text{sec. } \theta}{r}.$$

   When $\theta$ is a quadrant, area is infinite.

*Cor.* The solid, generated by the revolution of the figure of the tangents about its base, is equal to a cylinder, the base of which is the circle, and height $=$ excess of tang. $\theta$ above $\theta$.

4. Figure of the secants.

$$\text{Area} = r^2 \times \text{h. l.} \frac{\sec.\,\theta + \tan.\,\theta}{r}.$$

When $\theta$ is a quadrant, area is infinite.

*Cor.* The solid, generated by the revolution of the figure of the secants about its base, is equal to a cylinder, the base of which is the circle, and the altitude $= \tan.\,\theta$.

FLEXURE *point of contrary in curves.—See Inflexion.*

FLOATING *bodies.—See Specific Gravity, and Equilibrium.*

FLUENTS.—*See Differentials.*

FLUIDS, *pressure of.—(Vince, Bland.)*

1. The pressure of a fluid against any surface, in a direction perpendicular to it, is as the area of the surface, multiplied into the depth of its centre of gravity below the surface of the fluid, multiplied into the specific gravity of the fluid; and is ∴ equal to the weight of a cylinder of the same fluid, the area of whose bottom = the given surface, and altitude the depth of the centre of gravity.

Hence the pressure is entirely independent of the weight of the fluid.

*Ex.* Compare the pressure on the area of a parabola with that on its circumscribing rectangle, both being immersed perpendicularly to the vertex.

The areas are as 2 : 3, and the depths of their centres of gravity as $\frac{3}{5} : \frac{1}{2}$; ∴ the pressures are as 4 : 5.

2. Hence if a vessel be filled with a fluid, the pressure on any part : the whole weight of the fluid :: the area of that part × the depth of its centre of gravity : the solid content of the fluid.

*Ex.* 1. In a cone, pressure on base = 3 weight of fluid.
2. In a cube, pressure on any side = $\frac{1}{2}$ weight of fluid.
3. In a sphere, pressure on surface = 3 weight.
4. In a paraboloid, pressure on base = 2 weight.
5. In a cylinder, pressure on bottom : pressure on the side :: diamater of base : 2 altitude.

3. If a solid of revolution be filled with fluid, to find the pressure perpendicular to the surface.

Let the height of the solid $= h$, $x$ and $y$ the coordinates, then the pressure on the curve surface

$$= 2\pi\,\text{fl.}\; y\,d\,\Sigma\,(h - x) + \text{C}.$$

*Ex.* Let the surface be a segment of a sphere, with its vertex downwards.

Here $y\,d\,z = r\,d\,x$, ∴ pressure $= 2\,\pi\,r\,\mathrm{fl.}\,(h - x).\ d\,x = 2\,\pi\,r\,(h\,x - \frac{1}{2}\,x^2)$; and for the whole segment $x = h$, ∴ pressure $= \pi\,r\,h^2$.

4. Upon the principle that fluids press equally in all directions, and in proportion to their perpendicular depths, depends the principle of the hydrostatical paradox or hydrostatic bellows.

In the hydrostatic bellows, as the area of the orifice of the pipe : area of the bellows board :: weight of the water in the pipe above the bellows board : the weight sustained on the board.

*Cor.* Supposing a given quantity of fluid to be poured into the tube, to determine how much the weight will rise.

Let $z =$ required height, $x$ and $y =$ the area of the sections of the tube and bellows, and let the quantity poured into the tube $= l\,x$,

$$\text{then } z = \frac{l\,x}{x + y}.$$

5. If two fluids communicate in a bent tube, their perpendicular altitudes, above the plane where they meet, are inversely as their specific gravities.

Hence the *same* fluid will stand at the same altitude on each side. Thus water may be conveyed by pipes from a spring on the side of a hill to a reservoir of equal height on another hill.

*For Centre of Pressure, see Centre.*

*A few practical inferences from the foregoing propositions.*

1. In a vertical gate, dam, or sluice exposed to the pressure of water, the pressure on a square foot at the depth of $d$ feet $= 1000\,d$ in ounces. And if it be rectangular and $b$ its breadth, and D its depth in feet, the pressure by *Art.* 2. *Ex.* 2. $= 1000 \times \frac{1}{2}\,b\,D^2 = 500 \times b\,D^2$ in ounces.

2. If the transverse section of a canal be in the form of a trapezium, widest at the top, then if B and $b$ be the breadth at the top and bottom respectively, and $d$ the depth in feet, and it be required to find the pressure on a gate, which, standing across the canal, would dam the water up, we have area of trapez. $= \frac{1}{2}\,\overline{B + b}.\ d$; and depth of centre of gravity $= \dfrac{\overline{2\,b + B}\,.\ d}{3.\ \overline{B + b}}$; ∴ the whole pressure in ounces $= 500.\ \dfrac{2\,b + B}{3}.\ d^2$.

3. The strongest angle of position for a pair of gates for the lock of a canal or river $= 100°.\ 28'$.

4. The thickness of pipes to convey water is as $\frac{hd}{c}$; where $h$ is the height of the head of water, $d$ the diameter of the pipe, and $c$ the cohesion of a bar of the same material as the pipe, and an inch square. In the same metal, thickness varies as $h\,d$. This result obviously only gives the *proportional* thickness: to determine the actual thickness, we must have a series of experiments on which to found our computation. But these do not appear to have been carried on upon a sufficiently large scale to inspire us with any confidence in the results. In fact, the thickness of pipes is generally determined in practice by experiment, or rather by imitating, as near as circumstances will allow, some other work of a similar kind.

Should we, however, suppose, with Dr Gregory, that a pipe of cast iron 15 inches diameter, and ¾ of an inch thick, will be strong enough for a head of water of 600 feet; and a pipe of oak of the same diameter, and two inches thick, would sustain a head of 180 feet, we should have for any other head $h$ and diameter $d$, thickness of cast iron pipes $= \frac{h\,d}{12000}$, and thickness of oak pipes $= \frac{h\,d}{1350}$.

For the pressure of fluids against dykes—*see Dyke.*

FLUIDS *discharge of, through very small apertures in the bottom or sides of vessels.—(Vince, Bland, Playfair.)*

1. The velocity at the aperture is equal to that acquired in falling freely through ½ the altitude of the fluid above the orifice, and the velocity at the *vena contracta* equal to that acquired in falling through the whole height.

*Cor.* 1. Hence if $h$ = height of the fluid above the orifice, $g = 32\frac{1}{6}$ feet, the velocity at the orifice $= \sqrt{g\,h}$, and velocity at the vena contracta $= \sqrt{2\,g\,h}$.

*Cor.* 2. If any pressure be exerted on the surface of the fluid, the velocity of the issuing fluid will be increased. Thus when water is projected into a vacuum, as the pressure of the atmosphere is equal to that of a column of water of 34 feet, $v = \sqrt{2g.\,(h+34)}$. And in general, if $h'$ be the height of the column of fluid, which would exert the same pressure as is applied at the upper surface,

$$v = \sqrt{2g\,(h+h')}.$$

*Cor.* 3. It is found by experiment that the section of the vena contracta is distant from the orifice a little less than the radius of the orifice, and its magnitude is about $\frac{5}{8}$ of the magnitude of the orifice.

2. If a cylindrical or prismatic vessel, whose altitude is $h$, and the area of whose section is A, empty itself through a very small orifice $a$ at the bottom, the time $t$ of emptying itself

$$= \frac{2}{\sqrt{g}} \times \frac{A}{a} \sqrt{h} = ,3526 \times \frac{A}{a} \sqrt{h}.$$

and the time that the surface takes to sink from the depth $h$ to any other depth $h'$

$$= ,3526 \times \frac{A}{a} (\sqrt{h} - \sqrt{h'}.)$$

*Cor.* The construction of the clepsydra depends upon this Proposition. If the whole depth through which the water sinks in 12 hours be divided into 144 parts, it will sink through 23 of these in the first hour, 21 in the second, 19 in the third, and so on according to the series of the odd numbers.

Any vessel may serve for a clepsydra, but in order that the fluid may descend (which is most commodious) through equal portions of the vertical axis in equal portions of time, the vessel must be a paraboloid of the fourth order.

3. M. Prony deduces from actual experiment, the following formula for computing the discharge due to any altitude, and with any given orifice. Let Q = quantity of water discharged in cubic feet, $d$ = diameter of orifice in inches, H = height of the head of the water in feet, T = time in seconds. then

$$Q = 3.9103\, d^2\, T \sqrt{H}\cdot$$

If instead of the aperture a pipe of one or two inches in length be inserted, the discharge is increased in the ratio of 13 to 10 nearly; in that case

$$Q = 5.1086\, d^2\, T \sqrt{H}\cdot$$

4. Bossut has found that the discharges due to equal intervals of time, through horizontal tubes of the same diameter, and under the same height of water, but of different lengths, not differing greatly from each other, will be very nearly in the inverse ratio of the square roots of these lengths.

5. To find the time of emptying vessels in general; let $g = 32\frac{1}{6}$ feet, $x$ = depth of fluid at any point of time, $z$ = area of surface at the depth $x$, $a$ = area of orifice; then the velocity with which the surface descends

$$= \frac{a \sqrt{g x}}{z}.$$

$$\& \; t = \text{fl.} \frac{z\,dx}{a\sqrt{gx}}.$$

*Cor.* If any pressure be exerted on the surface of the fluid, and $h' =$ the height of a column of the fluid which would exert the same pressure,

$$t = \text{fl.} \frac{z\,dx}{a\sqrt{g \times (x + h')}}.$$

*Ex.* 1. If equal hemispheres are emptied by orifices in the vertex and base, time in the first case : time in last :: 7 : 12; the actual time in the first case being $\frac{14\pi r^{\frac{5}{2}}}{15\,a\sqrt{g}}$, and in the latter $\frac{8\pi r^{\frac{5}{2}}}{5\,a\sqrt{g}}$.

2. In paraboloids, the times are as 1 : 2.
3. In cones, the times are as 3 : 8.
4. In a sphere, time of emptying upper half : time of emptying lower :: $8\sqrt{2} - 7 : 7$.
5. To determine the time in which a cylinder will empty itself into a vacuum, its upper surface being exposed to the pressure of the atmosphere.

Let $h =$ height of the vessel, and $h' =$ the height of a column of fluid, equal to the weight of the atmosphere. Then by Cor. Art. 5.

$$t = \frac{2\pi}{a\sqrt{g}} \times \left\{ (h + h')^{\frac{1}{2}} - h'^{\frac{1}{2}} \right\}.$$

6. If upon the altitude of a fluid in a vessel as diameter we describe a ½ circle, the horizontal space described by the fluid from a perpendicular orifice at any point in the diameter equals twice the ordinate of the ½ circle drawn from that point, and ∴ varies as sin. $\theta$, where $\theta =$ the arc of a circle, whose diameter is the depth of the fluid, and versed sine the depth of the orifice.

7. In jets d'eau, the differences between the heights of the jets and of the reservoirs, are as the squares of the heights of the jets themselves. i.e. if H and H′ be the heights of two reservoirs, $h$ and $h'$ the heights of the actual jets,

$$H - h : H' - h' :: h^2 : h'^2.$$

**FLUIDS,** *resistance of.—( Vince, Bland.)*

The resistance to a body moving in a fluid arises from the inertia, the tenacity, and the friction of the fluid. But the resistance here considered is that arising solely from the inertia of the fluid. The following articles are also deduced upon an hypothesis which cannot obtain in real practice; because it supposes first, that the medium in which the body moves falls in behind the body in motion, as fast as this moves forward, which is not the case, except the velocity is very small; and secondly, that the particles are so constituted, that after the body strikes them their action entirely ceases; whereas the particles, after they are struck. must necessarily diverge, and act upon other particles behind them. Hence will arise some difference between theory and experiment.

1. Required the resistance to a plane, moving in a fluid, in a direction perpendicular to its surface.

Let $a$ = area of the plane; $v$ its velocity, $w$ its weight, $\delta$ the density of the fluid; $g = 32\frac{1}{6}$ feet. R the resistance, R′ the retarding force, then

$$\mathrm{R} = \frac{a\,\delta\,v^2}{2g}.$$

$$\& \ \mathrm{R}' = \frac{\mathrm{R}}{w} = \frac{a\,\delta\,v^2}{2\,g\,w}.$$

*Cor.* If the body be a cylinder (rad. = $r$) moving in the direction of its axis,

$$\mathrm{R} = \frac{\pi\,r^2\,\delta\,v^2}{2\,g}.$$

2. If the direction of motion be not perpendicular to the face of the plane, but inclined to it at any angle $\theta$, the resistance *perpendicular* to the plane, is

$$\frac{a\,\delta\,v^2\,\sin._2\,\theta}{2\,g}.$$

And the resistance *in the direction* of its motion, is

$$\frac{a\,\delta\,v^2\,\sin.^3\,\theta}{2\,g}.$$

And in a direction perpendicular to that of its motion, is

$$\frac{a\,\delta\,v^2\,\sin.^2\,\theta \times \cos.\,\theta}{2\,g}.$$

*Ex.* At what ∠ must the rudder of a vessel be inclined to the stream, that the effect produced may be a maximum?

The effect varies (by the 3d Formula) as $\sin.^2\,\theta \times \cos.\,\theta = \max.$, $\therefore$ $\sin.\,\theta = \sqrt{\frac{2}{3}}$.

3. If a plane figure, or a solid generated by the revolution of a plane figure round its axis, move in a fluid in the direction of its axis; to determine the ratio of the resistances on the curve or surface, and on the base.

In a plane figure,

Res. on base : that on the curve :: $y$ : fl. $\frac{dy}{1+\frac{dx^2}{dy^2}}$

And in a solid,

Res. on base : that on the surface :: $\frac{1}{2}y^2$ : fl. $\frac{y\,dy}{1+\frac{dx^2}{dy^2}}$

*Ex.* 1. Let the curve be a semicircle.

Res. on base : res. on curve :: $y : y - \frac{y^3}{3r^2}$, which, when $y = r$, becomes as 3 : 2.

*Ex.* 2. Let the solid be a sphere,

Res. on base : res. on surface :: $\frac{1}{2}y^2 : \frac{1}{2}y^2 - \frac{y^4}{4r^2}$ :: 2 : 1 when $y = r$.

Hence resistance to a cylinder is double that of the inscribed sphere.

*Cor.* Hence if $n$ = density of a globe, whose radius is $r$, and the specific gravity of the fluid be 1,

$$R = \frac{\pi r^2 v^2}{4g}.$$

and $R' = \frac{R}{w} = \frac{\pi r^2 v^2}{4g} \div \frac{4\pi n r^3}{3} = \frac{3v^2}{16gnr}$; or if $z$ = space fallen through to acquire the velocity $v$

$$R' = \frac{3z}{8nr}.$$

4. Let a sphere of given diameter be projected in a resisting medium, whose specific gravity is to that of the sphere as 1 : $n$. Having given the velocity of projection, to find the velocity of the sphere at any distance $x$, and the time of description.

Let $e$ = No. whose hyp. log. = 1, and suppose when $x = o$, $z = a$, then

$$V = \frac{\sqrt{2ga}}{e^{\frac{3x}{16nr}}}.$$

$$\text{and } T = \frac{16nr}{3\sqrt{2ga}} \times \left(e^{\frac{3x}{16nr}} - 1\right)$$

5. Let a spherical body descend in a fluid from rest by the action of

gravity (the rest as before), to find the velocity at any point of the descent, and the time of description.

$$\text{Here } V = \sqrt{\frac{16gr.\overline{n-1}}{3}} \times \sqrt{1 - e^{\frac{-3x}{8nr}}}.$$

$$\text{and } T = \sqrt{\frac{4n^2 r}{3g.\overline{n-1}}} \times \text{hyp. log.} \frac{1 + \sqrt{1 - e^{\frac{-3x}{8nr}}}}{1 - \sqrt{1 - e^{\frac{-3x}{8nr}}}}.$$

*Cor.* 1. If $x$ be increased sine limite, $e^{\frac{-3x}{8nr}}$ vanishes, and $V = \sqrt{\frac{16gr.\overline{n-1}}{3}}$ = the greatest velocity that can be acquired by a spherical body descending in a fluid.

FLUID *elastic.—See Atmosphere.*

FLUXIONS.—*See Differentials.*

FORCES, *the composition and resolution of.—( Whewell.)*

1. If any two forces act at the same point, the force, which is equivalent to the two, is represented in *direction and magnitude* by the diagonal of the parallelogram, of which the sides represent the magnitude and direction of the component forces.

*Cor.* If $p$ and $q$ be the component forces, which contain an angle $\theta$, the resultant will be $\sqrt{p^2 + 2pq\cos.\theta + q^2}$.

2. Forces may be represented by lines parallel to their direction, and proportional to them in magnitude.

*Cor.* 1. If two sides of a Δ taken in order represent the magnitude and direction of two forces, the third side will represent a force equivalent to them both.

*Cor.* 2. If three forces, represented in magnitude and direction by the three sides of a Δ taken in order, act on a point, they will keep it at rest; and conversely.

*Cor.* 3. If three forces keep a body in equilibrium, and three lines be drawn making with the directions of the forces three equal angles towards the same parts, these three lines will form a Δ, whose sides will represent the three forces respectively.

*Cor.* 4. If three forces keep a point at rest, they are each inversely as the sine of the ∠ contained by the other two.

*Cor.* 5. If the ∠ between two given forces be diminished, the resultant is increased.

*Cor.* 6. If any number of forces be represented by sides of a polygon taken in order, their resultant will be represented by the line which completes the polygon.

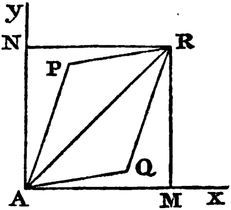
*Cor.* 7. A number of forces which are represented by all the sides of a polygon taken in order, acting upon a point, will keep it at rest.

3. If the edges of a parallelopiped drawn from the same point, represent three component forces, the diagonal will represent the resultant.

*Cor.* 1. If any number of forces be represented by sides, taken in order, of a polygon, which is not in the same plane, their resultant will be represented by the line which completes the polygon.

*Cor.* 2. If any number of forces be represented by all the sides, taken in order, of a polygon, they will keep a point at rest.

4. To find, by means of equations among the symbols, which the forces and their positions introduce, the resultant of two forces acting at a point.



If we suppose a line, as A $x$, to pass through A, we may determine the positions, both of the components and resultant, by the ∠s. which they make with this line.

Let $p$ and $q$ be the forces in A P, AQ; $\alpha$, $\beta$ the ∠s. which they make with A $x$. Resolve $p$ into two forces in the directions A $x$, and A $y$ perpendicular to A $x$, then the resolved parts will be $p$ cos. $\alpha$, $p$ sin. $\alpha$. In like manner $q$ is equivalent to $q$ cos. $\beta$ in the direction A $x$, and $q$ sin. $\beta$ in the direction A$y$. Hence the forces are equivalent to

$$p \cos.\ \alpha,\ q \cos.\ \beta \text{ in A}x.$$

$$p \sin.\ \alpha,\ q \sin.\ \beta \text{ in A}y.$$

And the resultant of $p$ and $q$ will be the resultant of these four forces. If we put

$$p \cos.\ \alpha + q \cos.\ \beta = X.$$

$$p \sin.\ \alpha + q \sin.\ \beta = Y.$$

and take in A $x$, A$y$, A M $=$ X, A N $=$ Y, and complete the rectangle A M R N, A R will be the resultant of $p$ and $q$, and if $r$ be this resultant, and $\theta$ the ∠ which it makes with A $x$, we have

$$r = \sqrt{X^2 + Y^2},\ \tan.\ \theta = \frac{Y}{X}.$$

whence the magnitude and position of the resultant are known.

*Cor.* 1. By putting the values of X and Y in the expression for $r$, we shall get

$$r = \sqrt{\left\{p^2 + 2pq\cos.\ (\alpha - \beta) + q^2\right\}}$$

which agrees with the result obtained in Cor. Art. 1.

*Cor.* 2. If we call $\varphi$ and $\psi$ the ∠s. P A R and Q A R, we shall have

$$\sin.\ \varphi = \frac{q\sin.\ (\alpha - \beta)}{\sqrt{\left\{p^2 + 2pq\cos.\ (\alpha - \beta) + q^2\right\}}}$$

$$\&\ \sin.\ \psi = \frac{p\sin.\ (\alpha - \beta)}{\sqrt{\left\{p^2 + 2pq\cos.\ (\alpha - \beta) + q^2\right\}}}$$

5. To find the resultant of any number of forces, $p_1, p_2, p_3 \ldots\ldots\ldots p_n$, in the same plane; their directions making with the line A $x$ angles $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots\ldots \alpha_n$ respectively.

By proceeding precisely as before, we shall have, by putting

$$p_1 \cos.\ \alpha_1 + p_2 \cos.\ \alpha_2 + p_3 \cos.\ \alpha_3 \ldots\ldots + p_n \cos.\ \alpha_n = X$$

$$p_1 \sin.\ \alpha_1 + p_2 \sin.\ \alpha_2 + p_3 \sin.\ \alpha_3 \ldots\ldots + p_n \sin.\ \alpha_n = Y$$

$$r = \sqrt{(X^2 + Y^2)};\ \tan.\ \theta = \frac{Y}{X}.$$

6. To find the resultant of forces, whose directions are not all in the same plane.

In the preceding case, the forces were resolved in the directions of two lines at right ∠s. to each other. In this case we must resolve them in the directions of three lines each at right ∠s. to the other two, and meeting together in a point. Let us suppose these three lines to be A $x$, A $y$, A $z$, and let $p$ be a force, and $\alpha, \beta, \gamma$ the ∠s. which it makes with A $x$, A $y$, A $z$; the force will then be equivalent to three forces

$p\cos.\ \alpha$ in A $x$, $p\cos.\ \beta$ in A $y$, $p\cos.\ \gamma$ in A $z$.

Hence if we have forces $p_1, p_2, p_3 \ldots\ldots p_n$

making with A $x$ angles $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots \alpha_n$

$$\text{with A}y\text{ angles } \beta_1,\ \beta_2,\ \beta_3 \ \ldots\ldots\ \beta_n$$

$$\text{with A}z\text{ angles } \gamma_1,\ \gamma_2,\ \gamma_3 \ \ldots\ldots\ \gamma_n$$

and make

$$p_1 \cos.\ \alpha_1 + p_2 \cos.\ \alpha_2 + p_3 \cos.\ \alpha_3 \ldots\ldots + p_n \cos.\ \alpha_n = X$$

$$p_1 \cos.\ \beta_1 + p_2 \cos.\ \beta_2 + p_3 \cos.\ \beta_3 \ldots\ldots + p_n \cos.\ \beta_n = Y.$$

$$p_1 \cos.\ \gamma_1 + p_2 \cos.\ \gamma_2 + p_3 \cos.\ \gamma_3 \ldots\ldots + p_n \cos.\ \gamma_n = Z$$

the forces will be equivalent to X in A $x$, Y in A $y$, and Z in A $z$.

If R be the resultant, and $\theta$, $\eta$, $\zeta$ the $\angle$s. which it makes with A $x$, A $y$, A $z$ respectively, we shall have

$$R = \sqrt{(X^2 + Y^2 + Z^2)}$$

$$\cos.\ \theta = \frac{X}{R},\ \cos.\ \eta = \frac{Y}{R},\ \cos.\ \zeta = \frac{Z}{R}$$

One of the three last Equations is superfluous.

7. When a point is acted upon by any forces, to find the conditions of equilibrium.

In order that there may be an equilibrium, the resultant of all the forces must be $o$. And in order that this may be the case, it is evident we must have in Art. 5, $X = o$, $Y = o$; and in Art. 6, $X = o$, $Y = o$, $Z = o$. Hence we have for the conditions of equilibrium in the former case

$$p_1 \cos.\ \alpha_1 + p_2 \cos.\ \alpha_2 + p_3 \cos.\ \alpha_3 + \ldots\ldots = o$$

$$p_1 \sin\ \alpha_1 + p_2 \sin.\ \alpha_2 + p_3 \sin.\ \alpha_3 + \ldots\ldots = o$$

And in the latter case

$$p_1 \cos.\ \alpha_1 + p_2 \cos.\ \alpha_2 + p_3 \cos.\ \alpha_3 + \ldots\ldots = o$$

$$p_1 \cos.\ \beta_1 + p_2 \cos.\ \beta_2 + p_3 \cos.\ \beta_3 + \ldots\ldots = o$$

$$p_1 \cos.\ \gamma_1 + p_2 \cos.\ \gamma_2 + p_3 \cos.\ \gamma_3 + \ldots\ldots = o$$

FORCE.—*See Motion.*

FORCE *moving, or motive.—See Momentum.*

FORCES, *centripetal and centrifugal.—See Central Forces.*

FRACTIONS *continued.*

Continued fractions are very useful when we have a fraction or ratio in very large numbers which are prime to one another, as by their means we may find an approximate value in less terms.

To represent $\frac{a}{b}$ in a continued fraction.

Divide as in the rule for finding the greatest common measure, thus

$b)\,a\,(p$  
$c)\,b\,(q$  
$d)\,c\,(r$  
$e)\,d\,(s$  
&c. &c.

$$\text{Then } \frac{a}{b} = p + \cfrac{1}{q + \cfrac{1}{r + \cfrac{1}{s}\ \&c.}}$$

The first approximation is $p$, which is too small, the next $p + \frac{1}{q}$, which is too large, the next $p + \cfrac{1}{q + \cfrac{1}{r}}$, which is too small; and thus we may form a series of fractions, each succeeding one being nearer the true value of the proposed fraction than the one which preceded it.

This series of fractions requires some trouble in their formation after the first two or three; but the 3d, 4th, &c. may be expeditiously found thus. Arrange the figures of the quotients in a line, as $p, q, r, s, t$, &c. let the successive fractions be $\frac{c}{d}, \frac{e}{f}, \frac{g}{h}, \frac{k}{l}, \frac{m}{n}$, &c. then to find any of them after the 2d, as $\frac{g}{h}$, we have $\frac{g}{h} = \frac{re + c}{rf + d}$; $\frac{k}{l} = \frac{sg + e}{sh + f}$; $\frac{m}{n} = \frac{tk + g}{tl + h}$, &c. where the law of formation is evident.

*Ex.* To approximate to $\frac{277288}{87968}$, proceeding as if finding the greatest common measure we have for the quotients

3, 6, 1, 1, 2, 1, &c.

Now first approximation $= p = 3$; 2d. $= p + \frac{1}{q} = 3 + \frac{1}{6} = \frac{19}{6}$, $\therefore$ we have by the rule the following series of fractions

$3, \frac{19}{6}, \frac{22}{7}, \frac{41}{13}, \frac{104}{33}, \frac{145}{46}$ where 3 is too small, $\frac{19}{6}$ too large, &c.

FRACTIONS *vanishing.*

If $u = \frac{P}{Q}$, where P and Q are functions of $x$, which are both $= o$, when $x = a$, then the value of $u$, in this case, is the same as the values in this case of $\frac{dP}{dQ}, \frac{d^2P}{d^2Q}, \frac{d^3P}{d^3Q}$, &c.

Hence the value of a vanishing fraction may be found by differentiation, as in the following examples :—

*Ex.* 1. Required the value of $\frac{x^2 - a^2}{x - a}$ when $x = a$.

Here $\frac{d\,P}{d\,Q} = \frac{2\,x\,d\,x}{d\,x} = 2\,x = 2\,a$.

*Ex.* 2. Required the value of $\frac{x^{n+1} - x}{x - 1}$, when $x = 1$.

Here $\frac{d\,P}{d\,Q} = (n + 1)\,x^n - 1 = n$.

But if it so happen that on substituting $a$ instead of $x$ in $\frac{d\,P}{d\,Q}$, this fraction also becomes $\frac{o}{o}$, we must treat it in the same manner as the first, and so on, till we arrive at a value of which one term at least is finite.

*Ex.* Let $\frac{P}{Q} = \frac{a\,x^2 + a\,c^2 - 2\,a\,c\,x}{b\,x^2 - 2\,b\,c\,x + b\,c^2}$, which $= \frac{o}{o}$ when $x = c$.

Here $\frac{d\,P}{d\,Q} = \frac{2\,a\,x - 2\,a\,c}{2\,b\,x - 2\,b\,c}$ which also $= \frac{o}{o}$ when $x = c$,

But $\frac{d^2\,P}{d^2\,Q} = \frac{a}{b}$ which is the value of $\frac{P}{Q}$ in this case.

FREEZING.—*See Congelation.*

**FRICTION.**—*(Playfair.)*

The following must only be considered as a short abstract of the most interesting general results on the subject of Friction, as deduced from experiments made by Coulomb and others.

1. The retardation which friction opposes to motion is nearly uniform, or the same for all velocities.

2. The force of friction is the greater, the greater the force with which the surfaces, moving on one another, are pressed together, and is commonly equal to between ⅓ and ¼ of that force ; but it is very little affected by the extent of the surfaces.

3. Friction may be distinguished into two kinds, that of sliding, and that of rolling bodies. The force of the latter is very small compared with that of the former.

4. The distance to which a given body will be moved by percussion in

opposition to friction, is as the square of the velocity communicated to it. Thus a nail is driven by a blow of no great force, into a piece of wood where the mere friction is sufficient to retain it against a great force applied to draw it out.

5. When motion begins, the intensity of friction diminishes; it does not, however, change afterwards as the velocity changes, but continues, as already said, to retard with a uniform force. Coulomb found the friction of wood sliding on wood to become less when the body began to move, than it had been the instant before in the ratio nearly of 2 to 9.

6. Friction may be measured by placing the body on a plane of variable inclination, and increasing that inclination till the body begin to slide. If the weight of the body = W, and the inclination of the plane when the body begins to slide $= \theta$, the friction $= W \times \tan. \theta$.

7. Time is often required for friction to attain its maximum, and in this respect different substances differ much from one another.

8. Friction is diminished by unctuous substances; those that are thinnest and least tenacious are the best; plumbago reduced to powder, and rubbed on the surface of wood, metal, stone, &c. serves greatly to diminish friction.

9. The effect of friction may be diminished by drawing a body in a line inclined at a certain angle to the plane on which it rests. Thus if the weight of a body be to its friction on a horizontal plane as $n$ to 1, it will be drawn with the greatest ease in the direction which makes with that plane an angle, having for its tangent $\frac{1}{n}$.

10. The friction of cylinders rolling upon an horizontal plane is in a direct ratio of their weights, and in the inverse ratio of their diameters.

11. The momentum of friction is diminished by friction wheels in the ratio of the radius of the axis of any one of the wheels (they are supposed equal) to the perpendicular height of the axis that rests upon them, above the line joining their centres.

12. In wheel carriages, the plane on which they move, and the line of draught, being both horizontal, the advantage for surmounting an immoveable obstacle, of a given height, is as the square root of the radius of the wheel.

Let the whole weight to be moved be W, the radius of the wheel $r$, $f$ the force which drawing horizontally will raise the carriage over an immoveable obstacle of the height $h$; then $f = W \times \sqrt{\frac{2h}{r}}$.

13. The stiffness of ropes, or the force requisite to bend them has a great analogy to friction. In different ropes, the forces requisite to bend them are in the direct ratio of their diameters and their tensions jointly, and in the inverse ratio of the radii of the cylinders round which they are bent.

14. The friction of a rope that is wound round a cylinder increases in geometrical progression, while the number of turns increases in arithmetical progression.

If the turns be represented by the numbers 0, 1, 2, 3, 4, &c. the resistance made by the rope may be represented by the numbers 1, 2. 4, 8, 16, &c.

15. Though friction destroys motion, and generates none, it is of essential use in mechanics. It is the cause of stability in the structure of machines; and is necessary to the exertion of the force of animals.

FRIGORIFIC *Mixtures.—(Ure.)*

*Tables of Frigorific Mixtures, sufficient for all useful philosophical purposes.*

FRIGORIFIC MIXTURES WITHOUT ICE.

| *Mixtures.* | | *Thermometer sinks from + 50o.* | *Degree of cold produced.* |
|---|---|---|---|
| Muriate of Ammonia......<br>Nitrate of Potash ...........<br>Water ............................ | 5 parts.<br>5<br>16 | To + 10°. | 40°. |
| Nitrate of Ammonia.......<br>Water ............................ | 1 part.<br>1 | To + 4°. | 46 |
| Nitrate of Ammonia ......<br>Carbonate of Soda .........<br>Water ............................ | 1 part.<br>1<br>1 | To — 7°. | 57 |
| Sulphate of Soda ............<br>Diluted nitric acid ......... | 3 parts.<br>2 | To — 3°. | 53 |
| Sulphate of Soda ............<br>Nitrate of Ammonia ......<br>Diluted nitric acid ......... | 6 parts.<br>5<br>4 | To — 14°. | 64 |
| Phosphate of Soda .........<br>Diluted nitric acid ......... | 9 parts.<br>4 | To — 12°. | 62 |
| Sulphate of Soda ............<br>Muriatic acid .................. | 8 parts.<br>5 | To 0°. | 50 |
| Sulphate of Soda ............<br>Diluted sulphuric acid..... | 5 parts.<br>4 | To + 3°. | 47 |

N.B. If the materials are mixed at a warmer temperature than 50°, the effect will be proportionably greater.

FRIGORIFIC MIXTURES WITH ICE.

| *Mixtures.* | |
|---|---|
| Snow or pounded ice ...... 2 parts.<br>Muriate of Soda ............ 1 | To — 5° from any temperature. |
| Snow or ice .................. 5 parts.<br>Muriate of Soda ........... 2<br>Muriate of Ammonia...... 1 | To — 12° from any temperature. |
| Snow .......................... 3 parts.<br>Diluted sulphuric acid ... 2 | From + 32° to — 23°. |
| Snow .......................... 8 parts.<br>Muriatic acid ............... 5 | From + 32° to — 27°. |
| Snow .......................... 7 parts.<br>Diluted nitric acid ........ 4 | From + 32° to — 30°. |
| Snow .......................... 4 parts.<br>Muriate of lime ............. 5 | From + 32° to — 40°. |
| Snow .......................... 3 parts.<br>Potash .......................... 4 | From + 32 to — 51°. |

Greatest artificial cold yet measured — 91°.

# G

GAUGING.—*(Hutton.)*

Rule for finding the dimensions of a cask, in wine, ale, or imperial gallons.

Let B = bung diameter, H = head diameter, L = length of cask, all in inches ; then

$$(39\ B^2 + 25\ H^2 + 26\ B\ H) \times \frac{L}{114}$$

is the content in inches, which being divided by 231 for wine gallons ; or by 282 for ale gallons ; or by 277.274 for imperial gallons, will be the content required.

**GEOMETRICAL** *Progression.—See Progression.*

**GEORGIUM** *Sidus.*

This planet was discovered by Dr Herschel, March 13, 1781. For its elements, &c.—*see Planets, elements of;* and for its satellites, *see Satellites.*

**GOLDEN** *Number.—See Cycle.*

**GRAVITY,** *Centre of.—See Centre of Gravity.*

**GRAVITY** *specific.*—(*Vince, Bland.*)

1. Of the specific gravities of a body and fluid, having given the one, to find the other.

*Case* 1. When the body is heavier than the fluid.

Let $w$ = weight lost by the body when immersed in the fluid, W its whole weight in vacuo, $s$ = spec. grav. of the fluid, S = that of the body; then

$$w : W :: s : S.$$

whence $s$ or S may be found.

*Cor.* 1. If different bodies be weighed in the same fluid, S is as $\frac{W}{w}$, from whence we may compare the spec. grav. of two bodies.

*Cor.* 2. If the same body is weighed in different fluids, $s$ is as $w$; from whence we can compare the spec. grav. of two fluids.

*Case* 2. When the body Q is lighter than the fluid in which it is weighed.

Connect it with a heavier body P, so that together they may sink. Find the weight lost by P + Q, and the weight lost by P, when immersed; then the difference = the weight lost by Q; and ∴ its specific gravity may be found by the last case.

2. If the specific gravity of air be called $m$, that of water being 1, and W the weight of any body in air, and W′ its weight in water; then its weight *in vacuo* is nearly

$$W + m\,(W - W').$$

3. If $\sigma$ be the specific gravity of a body ascertained by weighing it in air and water, and $m$ the specific gravity of the air at the time when the experiment was made; the correct specific gravity, or that which would have been found, if the body had been weighed in vacuo, instead of air, is

$$\sigma + m\,(1 - \sigma).$$

4. If a body float on a fluid, the part immersed (Q) : the whole body (P + Q) :: sp. grav. ($s$) of the body : sp. grav. (S) of the fluid.

*Cor.* Hence if the same body float on different fluids, Q is as $\frac{1}{S}$; on which principle the *Hydrometer* is constructed. For let the instrument be successively immersed in two fluids, and the magnitudes of the parts immersed be observed. Then the magnitude of the part immersed in the first : that immersed in the second :: spec. grav. of the second fluid : to that of the first.

A considerable improvement has been made in the hydrometer, by placing a small brass cup on the top of the stem, into which small weights may be put, so as to sink it in different fluids to the same point of the stem. Let $W$ = weight necessary to make it sink in one fluid, and $W + w$ the weight necessary to make it sink to the same point in another; then if one of them is water, the spec. grav. of the other $= 1.000 \times (1 \pm \frac{w}{W})$.

5. If a lighter fluid rest upon a heavier, and their spec. grav. be as $a : b$, and a body, whose spec. grav. is $c$, rest with one part $P$ in the upper fluid, and the other part $Q$ in the lower, then

$$P : Q :: b - c : c - a.$$

6. If $a$ and $b$ be the spec. grav. of two fluids or solids to be mixed together, $P$ and $Q$ their magnitudes, and $c$ the spec. grav. of the compound,

$$P : Q :: b - c : c - a, \text{ and}$$
$$\text{weight of } P : \text{weight of } Q :: a\,(b - c) : b\,(c - a).$$

*Cor.* Hence from the first proportion,

$$c = \frac{Pa + Qb}{P + Q}.$$

And from the second, if $W$ and $w$ = weights of $P$ and $Q$,

$$c = \frac{(W + w)\,ab}{Wb + wa}.$$

7. Of the magnitude and weight of a body, having given the one to find the other.

Let $M$ = magnitude in cubic feet, $S$ = its spec. grav. that of water being 1000, $W$ = weight in avoirdupois ounces, then

$$W = M \times S.$$

Or let $W$ = its weight in grains, and $S$ its spec. grav. that of water being 1, $B$ its bulk in cubic inches, then

$$B = \frac{W}{252,576\,S}.$$

*Cor.* If the weight is expressed in pounds Troy, it must be multiplied (to reduce it to grains) by 5760; if in pounds avoirdupois, by 7002.

We may thus find the magnitude of bodies which are too irregular to admit of the application of the common rules of mensuration; or we may, by knowing the spec. grav. and magnitude, find the weight of bodies which are too ponderous to be submitted to the action of the balance or steel yard.

8. To determine the magnitude of an irregular solid, and the capacity of an irregular vessel.

(j) Weigh the solid in air, and water; then since a cubic foot of rain water weighs 1000 ounces,

1000 oz. : weight lost :: 1 cubic foot : magnitude required.

(jj) Weigh the vessel when empty, and full of water, and you have the weight of water it contains, then

1000 oz. : weight of water :: 1 cub. foot : capacity required.

(jjj) To determine the diameter of any small sphere, whose spec. grav. is $s$, its weight in grains ($w$) being known

$$\delta = 1.9612 \sqrt[3]{\frac{w}{s}}.$$

## TABLE OF SPECIFIC GRAVITY.

*Extracted from Davies, Lavoisier, Young, and other authentic sources.*

*Note.*—Water at 60° is assumed 1000 specific gravity.

### MINERAL PRODUCTIONS.

| | |
|---|---|
| Platina, purified | 19500 |
| hammered | 20336 |
| Pure gold, cast | 19258 |
| hammered | 19361 |
| Mercury | 13568 |
| Lead, cast | 11352 |
| Silver, pure, cast | 10474 |
| hammered | 10510 |
| Bismuth, cast | 9822 |
| Copper, cast | 8788 |
| wire | 8878 |
| Brass, cast | 8395 |
| wire | 8544 |
| Cobalt, cast | 7812 |
| Nickel, cast | 7807 |
| Iron, cast | 7207 |
| bar | 7788 |
| Steel, hard, not screwed | 7816 |
| soft, not screwed | 7833 |
| Loadstone | 4800 |
| Tin, cast | 7291 |
| Zinc, cast | 7190 |
| Antimony, cast | 6702 |

| | |
|---|---|
| Tungstein | 6066 |
| Arsenic, cast | 5763 |
| Molybdena | 4738 |
| Spar. ponderous | 4430 |
| Ruby, oriental | 4283 |
| Garnet, Bohemian | 4188 |
| Sapphire of Puy | 4076 |
| Topas, oriental | 4010 |
| Beryl, or oriental aquamarine | 3548 |
| Diamond, rose coloured | 3531 |
| white | 3521 |
| lightest | 3501 |
| Glass, flint | 3329 |
| white | 2892 |
| bottle | 2732 |
| green | 2642 |
| Fluor | 3191 |
| Serpentine, green | 2988 |
| Mica, black | 2900 |
| Basaltes, from the Giants' Causeway | 2864 |
| Marble, white, Parian | 2837 |
| green | 2741 |
| red | 2725 |
| white, of Carrara | 2716 |
| Emerald, Peruvian | 2775 |
| Porphyry, red | 2765 |
| Jasper | 2764 |
| Alabaster, white, antique | 2730 |
| Calcarious spar, rhombic | 2715 |
| pyramidal | 2714 |
| Slate | 2671 |
| Pitch stone | 2669 |
| Onyx, pebble | 2664 |
| Chalcedony, transparent | 2664 |
| Granite, Egyptian, red | 2654 |
| Rock crystal, pure | 2653 |
| Amorphous quartz | 2647 |
| Agate, onyx | 2637 |
| Carnelian | 2613 |
| Sardonyx | 2602 |
| Purbeck stone | 2601 |
| Flint, white | 2594 |

| | |
|---|---|
| Flint, blackish | 2581 |
| Agate, oriental | 2590 |
| Prase | 2580 |
| Portland stone | 2570 |
| Mill-stone | 2483 |
| Paving-stone | 2415 |
| Touchstone | 2415 |
| Porcelain, Chinese | 2384 |
| Lapis obsidianus | 2348 |
| Selenite | 2322 |
| Grindstone | 2142 |
| Salt | 2130 |
| Sulphur, native | 2033 |
| Nitre | 2000 |
| Brick | 2000 |
| Plumbago | 1860 |
| Alum | 1720 |
| Asphaltum | 1400 |
| Coal, Scots | 1300 |
| Newcastle | 1270 |
| Staffordshire | 1240 |
| Jet | 1238 |
| Ice, *probably* | 930 |
| Pumice-stone | 914 |

LIQUIDS.

| | |
|---|---|
| Sulphuric acid | 1840 |
| Ph. London | 1850 |
| Nitrous acid, Ph. London | 1550 |
| Nitric acid | 1217 |
| Water of the Dead Sea | 1240 |
| Sea Water | 1026 |
| Muriatic acid | 1194 |
| Water of the Seine, filtered | 1001 |
| Naphtha | 708 |

ELASTIC FLUIDS.

| | *Kirwan.*<br>Barometer, 30. | *Lavoisier.*<br>Thermometer 52°. |
|---|---|---|
| Sulphureous acid gas | 2·265 | — |
| Carbonic acid gas | 1·500 | ·00176 |
| Nitrous gas | 1·194 | — |

|  | *Kirwan.* Barometer, 30. | *Lavoisier.* Thermometer 52°. |
|---|---|---|
| Hepatic gas | 1·106 | — |
| Oxygen gas | 1·103 | ·00137 |
| Atmospheric air | 1·000 | ·00128 |
| Nitrogen gas | ·985 | ·00120 |
| Ammoniacal gas | ·600 | — |
| Hydrogen gas | ·084 | ·000096 |

### VEGETABLE PRODUCTIONS.

|  |  |
|---|---|
| Sugar, white | 1606 |
| Gum Arabic | 1452 |
| Honey | 1450 |
| Catechu | 1398 |
| Pitch | 1150 |
| Copal, opaque | 1140 |
| Yellow amber | 1078 |
| Malmsey, Madeira | 1038 |
| Cider | 1018 |
| Vinegar, distilled | 1009 |
| Water at 60° | 1000 |
| Bourdeaux wine | 994 |
| Burgundy wine | 991 |
| Turpentine liquid | 991 |
| Camphor | 988 |
| Linseed oil | 940 |
| Elastic gum | 933 |

### ANIMAL SUBSTANCES.

|  |  |
|---|---|
| Pearl | 2750 |
| Coral | 2690 |
| Sheep's bone, recent | 2222 |
| Oyster shell | 2092 |
| Ivory | 1917 |
| Stag's horn | 1875 |
| Ox's horn | 1840 |
| Isinglass | 1111 |
| Egg of a hen | 1090 |
| Human blood | 1053 |
| Milk cow's | 1032 |
| Wax, white | 968 |
| yellow | 965 |

| | |
|---|---|
| Spermaceti | 943 |
| Butter | 942 |
| Tallow | 942 |
| Fat of hogs | 937 |
| veal | 934 |
| mutton | 923 |
| beef | 923 |
| Ambergrease | 926 |
| Lamp oil | 923 |

### WOODS.

| | |
|---|---|
| Pomegranate tree | 1354 |
| Lignum vitæ | 1333 |
| Box, Dutch | 1328 |
| Ebony | 1177 |
| Heart of oak, 60 years felled | 1170 |
| Oak, English, just felled | 1113 |
| the same, seasoned | 743 |
| usually stated at | 925 |
| Bog oak, of Ireland | 1046 |
| Teak, of the East Indies | from 745 to 657 |
| Mahogany | from 1063 to 637 |
| Pear tree trunk | 646 |
| Medlar tree | 944 |
| Olive wood | 927 |
| Logwood | 931 |
| Beech | 852 |
| Ash | from 845 to 600 |
| Yew, Spanish | 807 |
| Dutch | 788 |
| Alder | 800 |
| Elm | from 800 to 600 |
| Apple tree | 793 |
| Plum tree | 755 |
| Maple | 755 |
| Cherry tree | 715 |
| Quince tree | 705 |
| Orange tree | 705 |
| Walnut | 671 |
| Pitch pine | 660 |
| Red pine | 657 |
| Yellow pine | 529 |

| | |
|---|---|
| White pine | 420 |
| Fir, of New England | 553 |
| of Riga | 753 |
| of Mar Forest, Scotland | 696 |
| Cypress | 644 |
| Lime tree | 604 |
| Filbert wood | 600 |
| Willow | 585 |
| Cedar | 560 |
| Juniper | 556 |
| Poplar, white Spanish | 529 |
| common | 383 |
| Sassafras wood | 482 |
| Larch, of Scotland | 530 |
| Cork | 240 |

GREGORIAN *Calendar.—See Calendar.*

GULDINUS' *Property.—See Solids and Surfaces.*

GUNNERY, *leading principles of.—(Hutton.)*

1. To find the initial velocity of a shot.

Let $p$ = weight of powder, B of the ball, $v$ the initial velocity, then

$$v = 2000 \sqrt{\frac{P}{B}}.$$

*Cor.* 1. The initial velocity of a shot varies from 1600 to 2000 feet per second.

*Cor.* 2. $B v^2 = (2000)^2 . P$, i.e. the *effect* of a shot is nearly as the quantity of gunpowder.

2. If $w$ = weight of any ball, $d$ its diameter.

$$w = .5236\, d^3 \text{ in pounds.}$$

3. To find the resistance of the air to any ball or projectile.

Let $d$ = diameter of ball, $v$ its velocity, $r$ = resistance in avoirdupois pounds, then

$$r = \frac{d^2}{1000}\left(\frac{2\,v^2}{3000} - v.\right)$$

*Ex.* Resistance to a ball, whose diameter = 2.78 inches (or weight 3 lbs.), when thrown with a velocity of 1800 feet per second, = 176 lbs., more than 58 times its own weight.

4. Supposing the air to resist according to the law just assigned, required the height to which a ball will ascend perpendicularly.

Let $d$ = diameter of ball, $c$ the velocity of projection, $h$ = height ascended, then

$$h = 760\,d \times \log.\left(\frac{c^2 - 150\,c}{21000\,d} + 1.\right)$$

*Ex.* A ball of 1.05 lbs., discharged with a velocity of 2000 feet, will ascend to the height of 2920 feet; *in vacuo* it would have ascended to the height of 11¾ miles.

5. If a body descending in the atmosphere has acquired such a velocity that the resistance is equal to its weight, the accelerating and retarding forces being equal, its motion will become uniform; to find this terminal velocity,

$$\frac{2\,v^2}{3000} - v = 523.6\,d.$$

a quadratic equation, from whence $v$ may be found.

*Ex.* For an iron ball of 1 lb. the terminal velocity = 244 feet; for one of 42 lbs. it is 456.

6. The best charge of powder is about $\frac{1}{5}$ or $\frac{1}{6}$ of the weight of the ball; for battering $\frac{1}{3}$: a 24-pounder with 16 pounds of gunpowder at an elevation of 45° ranges 20,250 feet, about $\frac{1}{5}$ of the range that would take place in a vacuum. The resistance is at first 400 pounds or more, and reduces the velocity in a second from 2000 to 1200 feet in the first 1500 feet.—(*Young's Nat. Phil.*)

GUNPOWDER.—*See Gunnery and Steam.*

GYRATION, *Centre of.—See Centre of Gyration.*

# H

HARMONICAL *Progression.—See Progression.*

HARVEST *Moon.—(Maddy.)*

To find the retardation of the Moon's rising on successive nights.

Let the moon's daily motion = $m$, the inclination of the moon's orbit to the horizon = $n$, latitude of the place = $l$, moon's declination = $\delta$, then the difference of the times of rising on succeeding days (D) is

$$D = \frac{m.\ \sin.\ n}{\sqrt{\cos.^2 \delta - \sin.^2 l}}.$$

Hence may be explained the phænomenon of the Harvest Moon, premising that when the 1st point of Aries rises, the ecliptic makes the least angle with the horizon. For if the moon's orbit be supposed to coincide with the ecliptic (which it does nearly) sin. $s$ is least when the moon rises in Aries; therefore the numerator of the above expression is then least; and because $\cos.^2 \delta = 1$, the denominator is then greatest; ∴ on both accounts D is least, and if the sun be at the same time in Libra, the moon is then at the full; therefore the full moon, which takes place near the autumnal equinox rises nearly at the same time for several nights, and as this is near the time of harvest in north latitudes, it is called the Harvest Moon.

HEAT, *various Tables relating to.*

### TABLE I.

*Table of the effects of heat on different substances according to Fahrenheit's thermometer and Wedgwood's.—(Wedgwood.)*

| | *Fahr.* | *Wedg.* |
|---|---|---|
| Extremity of the scale of Wedgwood | 32277° | 240° |
| Greatest heat of his small air furnace | 21877 | 160 |
| Chinese porcelain softened | —— | 156 |
| Cast iron melts | 17977 | 130 |
| Greatest heat of a common smith's forge | 17327 | 125 |
| Derby porcelain vitrifies | —— | 112 |
| Welding heat of iron greatest | 13427 | 95 |
| least | 12777 | 90 |
| Fine gold melts | 5237 | 32 |
| Fine silver melts | 4717 | 28 |
| Swedish copper melts | 4587 | 27 |
| Brass melts | 3807 | 21 |
| Enamel colours burnt on | 1857 | 6 |
| Red heat fully visible in day light | 1077 | 0 |
| in the dark | 947 | $-1$ |
| Mercury boils | 600 | $-3\frac{673}{1000}$ |
| Water boils | 212 | $-6\frac{656}{1000}$ |
| Vital heat | 97 | $-7\frac{542}{1000}$ |
| Water freezes | 32 | $-8\frac{42}{1000}$ |
| Proof spirit freezes | 0 | $-8\frac{289}{1000}$ |

| | Fahr. | | Wedg. |
|---|---|---|---|
| Mercury freezes | − 40 | ... | − $8\frac{596}{1000}$ |

### TABLE II.

*Table of the congealing or concreting temperatures of various liquids by Fahrenheit's scale.—(Ure.)*

| | |
|---|---|
| Sulphuric ether | − 46° |
| Liquid ammonia | − 46 |
| Nitric acid sp. gr. 1.424 | − 45.5 |
| Sulphuric acid sp. gr. 1.6415 | − 45 |
| Mercury | − 39 |
| Nitric acid sp. gr. 1.3290 | − 2.4 |
| Brandy | − 7.0 |
| Alochol 1, water 1. | − 7 |
| Alcohol 1, water 3 | + 7 |
| Oil of turpentine | 14 |
| Strong wines | 20 |
| Blood | 25 |
| Vinegar | 28 |
| Sea water | 28 |
| Milk | 30 |
| Water | 32 |
| Olive oil | 36 |
| Sulphuric acid, sp. gr. 1.741 | 42 |
| Tallow | 92 |
| Spermaceti | 112 |
| Yellow wax | 142 |
| White do. | 155 |
| Tin | 442 |
| Lead | 612 |
| Zinc | 680 |

The concreting temperature of the bodies above tallow in this Table, is usually called their freezing or congealing point, and of tallow and the bodies below it the fusing or melting point.

### TABLE III.

*Table of the boiling points by Fahrenheit's scale of a few of the most important liquids, under a mean barometrical pressure of 30 inches.—(Ure.)*

| | | |
|---|---|---|
| Ether sp. gr. 0.7365 at 48° | Gay Lussac | 100° |
| Alcohol sp. gr. 0.813 | Ure | 173.5 |

| | | | |
|---|---|---|---|
| Nitric acid sp. gr. 1.500 | | Dalton | 210 |
| Water | | | 212 |
| Muriatic acid sp. gr. 1.094 | | Dalton | 232 |
| Do. | 1.047 | Do. | 222 |
| Nitric acid | 1.16 | Do. | 220 |
| Oil of turpentine | | Ure | 316 |
| Sulphuric acid, sp. gr. 1.30 | | Dalton | 240 |
| Do. | 1.848 | Ure | 600 |
| Linseed oil | | | 640 |
| Mercury | | | 656 |

### Table IV.

Boiling temperature of water.

*Height of the boiling point in Fahrenheit's Thermometer at different heights of the Barometer.*

| Barom. | Ht. of boiling point. |
|---|---|
| 31°. 0 | 213°. 57 |
| 30. 5 | 212. 79 |
| 30. 0 | 212. 00 |
| 29. 5 | 211. 20 |
| 29. 0 | 210. 38 |
| 28. 5 | 209. 55 |
| 28. 0 | 208. 69 |
| 27. 5 | 207. 84 |
| 27. 0 | 206. 96 |

And in general Dr Horsley's rule deduced from De Luc is, height $= \frac{99}{8990000} \times \log. n. - 92.804$, where $n =$ height of Barometer in 10ths of an inch.

In an exhausted receiver water boils at 98°. or 100°. in Papin's digester at 412°.

From this variation in the height of the boiling point, arising from the variation of the pressure of the atmosphere, an ingenious instrument called the Thermometrical Barometer has been invented by Mr Wollaston, for ascertaining the heights of mountains; it appearing from General Roy's experiments, that a difference of 1°. in the boiling point corresponds to 535 feet in height. Let ∴ $n =$ difference of boiling points at the bottom and top of a mountain, then $1° : n° :: 535$ feet $: n \times 535$ $=$ approximate height. To correct it for the temperature of the air, let $m =$ mean temperature of the top and bottom, ascertained by a common thermometer, then *(see Barometer)* $n.\ 535 \times (1 + \overline{m - 32°} \times .00244)$ $=$ correct height.—*(Phil. Trans.)*

### Table V.

Linear expansion of solids by heat.

*Dimensions which a bar takes at* 212° *whose length at* 32° *is* 1.000000.—*(Ure.)*

| | | |
|---|---|---|
| Glass tube | Smeaton | 1.00083333 |
| Do. | Roy | 1.00077615 |
| Deal | Roy, as glass | — |

| | | |
|---|---|---|
| Platina | Troughton | 1.00099180 |
| Cast iron prism | Roy | 1.00110940 |
| Steel rod | Roy | 1.00114470 |
| Iron | Smeaton | 1.00125800 |
| Iron wire | Troughton | 1.00144010 |
| Gold | Ellicot | 1.00150000 |
| Copper | Troughton | 1.00191880 |
| Brass | Laplace | 1.00186671 |
| Brass wire | Smeaton | 1.00193000 |
| Silver | Troughton | 1.0020826 |
| Tin | Laplace | 1.00217298 |
| Lead | Smeaton | 1.00286700 |

### TABLE VI.

### Expansion of liquids.

*Dilatation of the volume of liquids by being heated from 32° to 212°.—( Ure.)*

| | | | |
|---|---|---|---|
| Mercury | Lord C. Cavendish | 0.018870 | $\frac{1}{53}$ |
| Do. | Roy | 0.017000 | $\frac{1}{59}$ |
| Do. | Shuckburgh | 0.018510 | $\frac{1}{54}$ |
| Do. | Du Long and Petit | 0.0180180 | $\frac{1}{55.5}$ |
| Do. | Do. from 212° to 392° | 0.1301843 | $\frac{1}{54.25}$ |
| Do. | Do. from 392° to 572° | 0.0188700 | $\frac{1}{53}$ |
| Water | Kirwan from 39°. its max. dens. | 0.04332 | $\frac{1}{23.08}$ |
| Muriatic acid sp. gr. 1.137 | Dalton | 0.0600 | $\frac{1}{17}$ |
| Nitric acid, sp. gr. 1.40 | Do. | 0.1100 | $\frac{1}{9}$ |
| Sulphuric acid sp. gr. 1.85 | Do. | 0.0600 | $\frac{1}{17}$ |
| Alcohol | Do. | 0.1100 | $\frac{1}{9}$ |
| Water saturated with salt | Do. | 0.0500 | $\frac{1}{20}$ |

| | | | |
|---|---|---|---|
| Sulphuric ether | Do. | 0.0700 | $\frac{1}{14}$ |
| Fixed oils | Do: | 0.0800 | $\frac{1}{12.5}$ |

### TABLE VII.

*Expansion of water.—(Ure.)*

The maximum density of water is at 39°., and it is a singular fact that the expansion of water is the same for any number of degrees above or below the maximum of density; thus the density of water at 32° and at 46° is precisely the same. The following Table, the result of experiments by Sir Charles Blagden and Mr Gilpin, shews this in a clear light.

| *Sp. Gr.* | *Bulk of water.* | *Temperat.* | | *Bulk of water.* | *Sp. Gr.* |
|---|---|---|---|---|---|
| | 1.00000 | 39° | | 1.00000 | |
| 1.00000 | 1.00000 | 38 | 40 | 1.00000 | 1.00000 |
| 0.99999 | 1.00001 | 37 | 41 | 1.00001 | 0.99999 |
| 0.99998 | 1.00002 | 36 | 42 | 1.00002 | 0.99998 |
| 0.99996 | 1.00004 | 35 | 43 | 1.00004 | 0.99996 |
| 0.99994 | 1.00006 | 34 | 44 | 1.00006 | 0.99994 |
| 0.99991 | 1.00008 | 33 | 45 | 1.00008 | 0.99991 |
| 0.99988 | 1.00012 | 32 | 46 | 1.00012 | 0.99988 |

This law of maximum density does not prevail in the case of sea water; on the contrary, Dr Marcet fonnd that sea water gradually increases in weight down to the freezing point.

HORIZON, *Dip or depression of.*

In observing an altitude at sea with the sextant or reflecting circle, the image of the object is made to coincide with the visible horizon, but as the eye is elevated above the surface of the sea by the height of the ship's deck, the visible horizon will be below the true horizontal plane.

The following Table gives the dip or apparent depression of the horizon for different elevations of the eye, allowing $\frac{1}{10}$ for terrestrial refraction. The dip must be always subtracted from the observed altitude when taken by the fore observation, but added to it in the back observation.

TABLE I.—*Of the dip of the horizon.*

| H. of Eye. | Dip of Horiz. | H. of Eye. | Dip. of Horiz. | H. of Eye. | Dip of Horiz. | H. of Eye. | Dip of Horiz. |
|---|---|---|---|---|---|---|---|
| Feet. | ′ ″ | Feet. | ′ ″ | Feet. | ′ ″ | Feet. | ′ ″ |
| 1 | 1 0 | 6 | 2 26 | 16 | 3 58 | 32 | 5 37 |
| 1¼ | 1 7 | 6½ | 2 32 | 16½ | 4 2 | 33 | 5 42 |
| 1½ | 1 13 | 7 | 2 38 | 17 | 4 5 | 34 | 5 47 |
| 1¾ | 1 19 | 7½ | 2 43 | 17½ | 4 9 | 35 | 5 53 |
| 2 | 1 24 | 8 | 2 48 | 18 | 4 12 | 36 | 5 58 |
| 2¼ | 1 29 | 8½ | 2 53 | 18½ | 4 16 | 37 | 6 2 |
| 2½ | 1 34 | 9 | 2 58 | 19 | 4 19 | 38 | 6 7 |
| 2¾ | 1 39 | 9½ | 3 3 | 19½ | 4 23 | 39 | 6 12 |
| 3 | 1 43 | 10 | 3 8 | 20 | 4 26 | 40 | 6 17 |
| 3¼ | 1 47 | 10½ | 3 12 | 21 | 4 33 | 42 | 6 26 |
| 3½ | 1 51 | 11 | 3 17 | 22 | 4 39 | 44 | 6 35 |
| 3¾ | 1 55 | 11½ | 3 21 | 23 | 4 46 | 46 | 6 44 |
| 4 | 1 59 | 12 | 3 26 | 24 | 4 52 | 48 | 6 52 |
| 4¼ | 2 3 | 12½ | 3 31 | 25 | 4 58 | 50 | 7 1 |
| 4½ | 2 6 | 13 | 3 35 | 26 | 5 4 | 55 | 7 21 |
| 4¾ | 2 10 | 13½ | 3 39 | 27 | 5 10 | 60 | 7 41 |
| 5 | 2 13 | 14 | 3 43 | 28 | 5 16 | 70 | 8 18 |
| 5¼ | 2 17 | 14½ | 3 47 | 29 | 5 22 | 80 | 8 53 |
| 5½ | 2 20 | 15 | 3 51 | 30 | 5 27 | 90 | 9 25 |
| 5¾ | 2 23 | 15½ | 3 55 | 31 | 5 32 | 100 | 9 56 |

If the land is not sufficiently distant to afford a free horizon, it may be sometimes necessary to obtain an altitude referred to the surface of the sea at some known or estimated distance. Under such circumstances, the dip may be taken from the following Table.

TABLE II.—*The dip at different distances from the Observer.*

| Miles. | Height of the eye in feet. | | | | | |
|---|---|---|---|---|---|---|
| | 5 | 10 | 15 | 20 | 25 | 30 |
| ¼ | 11′ | 23′ | 34′ | 45′ | 57′ | 68′ |
| ½ | 6 | 12 | 17 | 23 | 28 | 34 |
| ¾ | 4 | 8 | 12 | 15 | 19 | 23 |
| 1 | 3 | 6 | 9 | 12 | 15 | 17 |
| 1¼ | 3 | 5 | 7 | 10 | 12 | 14 |
| 1½ | 3 | 4 | 6 | 8 | 10 | 12 |
| 2 | 2 | 4 | 5 | 7 | 8 | 9 |
| 2½ | 2 | 3 | 4 | 6 | 7 | 8 |
| 3 | 2 | 3 | 4 | 5 | 6 | 7 |
| 3½ | 2 | 3 | 4 | 5 | 6 | 6 |
| 4 | 2 | 3 | 4 | 5 | 5 | 6 |
| 5 | 2 | 3 | 4 | 4 | 5 | 6 |
| 6 | 2 | 3 | 4 | 4 | 5 | 5 |

**HOUR,** *decimal parts of.—See Time.*

**HYDROMETER.**—*See Gravity specific.*

**HYDROSTATICAL** *paradox, or bellows.—See Fluids, pressure of.*

**HYPERBOLA,** *principal properties of.—See Conic Sections.*

# I

ICEBERG.

According to the experiments of Boyle and Mairan, the volume of solid compact ice is to that of sea water as 10 to 9; therefore the volume of ice which rises above the surface of the water is to that which sinks below it as 1 to 9. Supposing ∴ a cylinder of ice to rise above the surface of the sea 200 feet, which does not exceed the height of some ice islands described by navigators, its depth under water would be 1800 feet, and its whole height 2000 feet. But it is probable that this considerably exceeds the actual height of the Polar Icebergs. For first, the shape of these floating bodies is probably somewhat pyramidal, the part immersed being the broader end. And in the next place, as Mr Wales observes, the ice, which composes these masses, is comparatively light and porous, being chiefly snow and salt water frozen together, and bearing not perhaps a greater proportion to the weight of salt water than that of 5 to 6, or 6 to 7 at the utmost.

Icebergs in both ½ spheres are sometimes carried by currents as low as $40^0$ latitude.

JETS *d'eau.—See Fluids, discharge of.*

IMPACT *of hard and elastic bodies.—See Collision.*

IMPERIAL *weights and measures.—See Weights.*

INCLINED *Plane.*

1. *Equilibrium of bodies upon inclined planes.*

Let $P =$ power, $W =$ weight, $p =$ pressure, $H =$ height of the plane, $B =$ base, and $L =$ length, $\alpha = \angle$ of inclination of the plane, $\beta = \angle$ which the direction of the power makes with a perpendicular to the plane, $\gamma = \angle$ which the direction of the power makes with a perpendicular to the horizon; then when a body is sustained upon the plane, we have the following proportions:—

$$P : W :: \sin. \alpha : \sin. \beta.$$

$$P : p :: \sin. \alpha : \sin. \gamma.$$

$$p : W :: \sin. \gamma : \sin. \beta.$$

*Cor.* 1. When the power acts parallel to the plane,

$$P : W :: H : L.$$
$$P : p :: H : B.$$
$$p : W :: B : L.$$

*Cor.* 2. When the power acts parallel to the base,

$$P : W :: H : B.$$
$$P : p :: H : L$$
$$p : W :: L : B.$$

*Cor.* 3. If W and $\alpha$ be invariable, P varies as $\frac{1}{\sin. \beta}$, $\therefore$ P is least, when it acts in the direction of the plane; and is indefinitely great, when it acts perpendicular to the plane.

*Cor.* 4. If P and $\alpha$ be invariable, W varies as sin. $\beta$; $\therefore$ W is the greatest, when P acts in the direction of the plane, and the least when P acts perpendicular to the plane.

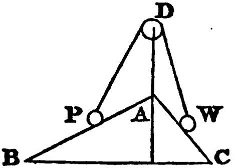
*Cor.* 5. If P and $\alpha$ be given, $p$ varies as sin. $\gamma$, and $\therefore$ is the greatest when P acts parallel to the base.

*Cor.* 6. If two weights P and W sustain each other on two planes, whose lengths are L and l, and which have a common altitude by means of a string passing over a pulley fixed at the intersection of the planes,

$$P : W :: L : l.$$

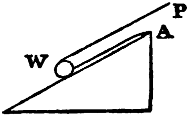
*Cor.* 7. If the pulley be above the intersection of the planes, as in the annexed figure,

$P : W ::$ sin. B C A $\times$ cos. D P A : sin. A B C $\times$ cos. D W A.

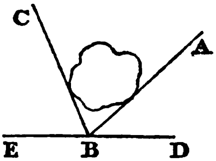


*Cor.* 8. If a string fixed at A pass round the weight W, and then be parallel to the plane,

$$P : W :: \tfrac{1}{2} H : L.$$



*Cor.* 9. If a body be supported on two inclined planes, the pressures on A B, B C and the weight of the body are represented by sin. C B E, sin. A B D, and sin. C B A respectively.



II. *Inclined planes, motion of bodies down.*

1. The force which accelerates or retards a body's motion upon an inclined plane, is to the force of gravity, as the height of the plane to its length.

Hence if $g = 32\frac{1}{6}$ feet, accelerating force $= \frac{g\,H}{L}$; or if $\alpha =$ plane's inclination, accelerating force $= g \times \sin.\ \alpha$.

*Cor.* 1. Hence if in the formulæ for the rectilinear descent of bodies *(see Motion)* we substitute $g \times \frac{H}{L}$, or $g \times \sin.\ \alpha$ for F, we shall have, if the body descends from rest,

$$v = \frac{H}{L} \times g\,t = \sin.\ \alpha \times g\,t.$$

$$s = \frac{H}{L} \times \frac{g\,t^2}{2} = \sin.\ \alpha \times \frac{g\,t^2}{2}.$$

$$s = \frac{L}{H} \times \frac{v^2}{2g} = \frac{v^2}{2\,g \times \sin.\ \alpha}.$$

*Cor.* 2. The velocity acquired in falling down the whole length of an inclined plane varies as $\sqrt{H}$.

*Cor.* 3. The time of descent down the whole length of an inclined plane varies as $\frac{L}{\sqrt{H}}$. Or if the inclination be given, i.e. if H varies as L, T varies as $\sqrt{L}$.

2. If chords be drawn in a circle from the extremity of that diameter which is perpendicular to the horizon, the velocities which bodies acquire by falling down them are proportional to their lengths; and the times of descent are equal.

*Cor.* The times of descent down chords in different circles are as the square roots of the diameters.

3. If a body descend down a system of inclined planes, the velocity acquired, on the supposition that no motion is lost in passing from one plane to another, is equal to that which would be acquired in falling through the perpendicular height of the system.

4. If a body fall from a state of rest down a curve surface which is perfectly smooth, the velocity acquired is equal to that which would be acquired in falling through the same perpendicular height.

5. The times of descent down similar systems of inclined planes, similarly situated, are as the square roots of their lengths, on the supposition that no velocity is lost in passing from one plane to another.

INFLEXION, *point of in curves.*

To ascertain the point of contrary flexure in any curve, find the 2d differential of the equation of the curve, supposing $dx$ constant, and we shall have a finite value of $\frac{-d^2y}{dx^2}$, which must be put equal to either zero or infinity. By means of this equation, and that of the curve, we can determine those values of $x$ and $y$, which belong to the point or points of contrary flexure.

*Ex.* 1. Let the equation be $y = 3x + 18x^2 - 2x^3$.

$$\text{Here } -\frac{d^2y}{dx^2} = 12x - 36 = o, \therefore x = 3.$$

2. Let the curve be the cubical parabola, whose equation is $y^3 = a^2x$.

Here $\frac{-d^2y}{dx^2} = \frac{2}{9}x^{-\frac{5}{3}}a^{\frac{2}{3}} = o, \therefore x = o$, or the point of inflexion is at the vertex.

For the point of inflexion in spirals—*see Spirals.*

In general there cannot be a point of contrary flexure, unless the first differential coefficient, which does not vanish, for a particular value of the abscissa, be of an odd order.—*See Maxima and Minima.*

INTEGRAL.—*See Differential.*

INTEREST.

*Interest simple.*

Let P = principal, $r$ = interest of £1. for one year, I the interest of P, and M its amount in the time $n$; then we have the following equations, from which any of the quantities may be found, the rest being given.

$$I = n r P.$$

$$M = P + n r P = (1 + n r.) P.$$

$$\text{Discount of M £} = M - \frac{M}{1 + nr}.$$

**The following Tables will much facilitate the computation of simple Interest :—**

## TABLE I.

*Of the Interest of £1. for any number of days at different rates of Interest.*

| No. of Days. | 3 per Cent. | 3½ per Cent. | 4 per Cent. | 4½ per Cent. | 5 per Cent. |
|---|---|---|---|---|---|
| 1 | ·0000821 | ·0000958 | ·0001095 | ·0001232 | ·0001369 |
| 2 | ·0001641 | ·0001916 | ·0002191 | ·0002465 | ·0002739 |
| 3 | ·0002465 | ·0002876 | ·0003287 | ·0003698 | ·0004109 |
| 4 | ·0003287 | ·0003835 | ·0004383 | ·0004931 | ·0005479 |
| 5 | ·0004109 | ·0004794 | ·0005479 | ·0006164 | ·0006849 |
| 6 | ·0004931 | ·0005753 | ·0006575 | ·0007397 | ·0008219 |
| 7 | ·0005753 | ·0006712 | ·0007671 | ·0008630 | ·0009589 |
| 8 | ·0006575 | ·0007671 | ·0008767 | ·0009863 | ·0010958 |
| 9 | ·0007397 | ·0008630 | ·0009863 | ·0011095 | ·0012328 |
| 10 | ·0008219 | ·0009589 | ·0010958 | ·0012328 | ·0013698 |
| 20 | ·0016438 | ·0019178 | ·0021917 | ·0024657 | ·0027397 |
| 30 | ·0024657 | ·0028767 | ·0032876 | ·0036986 | ·0041095 |
| 40 | ·0032876 | ·0038356 | ·0043835 | ·0049315 | ·0054794 |
| 50 | ·0041095 | ·0047945 | ·0054794 | ·0061643 | ·0068493 |
| 60 | ·0049315 | ·0057534 | ·0065753 | ·0073972 | ·0082191 |
| 70 | ·0057534 | ·0067123 | ·0076712 | ·0086301 | ·0095890 |
| 80 | ·0065753 | ·0076712 | ·0087671 | ·0098630 | ·0109589 |
| 90 | ·0073972 | ·0086301 | ·0098630 | ·0110958 | ·0123287 |
| 100 | ·0082191 | ·0095890 | ·0109589 | ·0123287 | ·0136986 |
| 200 | ·0164382 | ·0191780 | ·0219178 | ·0246574 | ·0273972 |
| 300 | ·0246573 | ·0287670 | ·0328767 | ·0369861 | ·0410958 |

**This Table it is obvious will furnish, by the addition of two or three of its numbers, the interest for any number of days, and the following will in the same way find it for any number of years.**

## TABLE II.

*Of the Interest of £1. for any number of years not exceeding 25, at different rates of Interest.*

| No. of Years. | 3 per Cent. | 3½ per Cent. | 4 per Cent. | 4½ per Cent. | 5 per Cent. |
|---|---|---|---|---|---|
| 1 | ·0300000 | ·0350000 | ·0400000 | ·0450000 | ·0500000 |
| 2 | ·0600000 | ·0700000 | ·0800000 | ·0900000 | ·1000000 |
| 3 | ·0900000 | ·1050000 | ·1200000 | ·1350000 | ·1500000 |
| 4 | ·1200000 | ·1400000 | ·1600000 | ·1800000 | ·2000000 |
| 5 | ·1500000 | ·1750000 | ·2000000 | ·2250000 | ·2500000 |
| 6 | ·1800000 | ·2100000 | ·2400000 | ·2700000 | ·3000000 |
| 7 | ·2100000 | ·2450000 | ·2800000 | ·3150000 | ·3500000 |
| 8 | ·2400000 | ·2800000 | ·3200000 | ·3600000 | ·4000000 |
| 9 | ·2700000 | ·3150000 | ·3600000 | ·4050000 | ·4500000 |
| 10 | ·3000000 | ·3500000 | ·4000000 | ·4500000 | ·5000000 |
| 11 | ·3300000 | ·3850000 | ·4400000 | ·4950000 | ·5500000 |
| 12 | ·3600000 | ·4200000 | ·4800000 | ·5400000 | ·6000000 |
| 13 | ·3900000 | ·4550000 | ·5200000 | ·5850000 | ·6500000 |
| 14 | ·4200000 | ·4900000 | ·5600000 | ·6300000 | ·7000000 |
| 15 | ·4500000 | ·5250000 | ·6000000 | ·6750000 | ·7500000 |
| 16 | ·4800000 | ·5600000 | ·6400000 | ·7200000 | ·8000000 |
| 17 | ·5100000 | ·5950000 | ·6800000 | ·7650000 | ·8500000 |
| 18 | ·5400000 | ·6300000 | ·7200000 | ·8100000 | ·9000000 |
| 19 | ·5700000 | ·6650000 | ·7600000 | ·8550000 | ·9500000 |
| 20 | ·6000000 | ·7000000 | ·8000000 | ·9000000 | 1·0000000 |
| 21 | ·6300000 | ·7350000 | ·8400000 | ·9450000 | 1·0500000 |
| 22 | ·6600000 | ·7700000 | ·8800000 | ·9900000 | 1·1000000 |
| 23 | ·6900000 | ·8050000 | ·9200000 | 1,0350000 | 1·1500000 |
| 24 | ·7200000 | ·8400000 | ·9600000 | 1·0800000 | 1·2000000 |
| 25 | ·7500000 | ·8750000 | 1·0000000 | 1·1250000 | 1,2500000 |

*To find the Interest of any sum, for a given time, by the preceding Tables.*

Add together the interests for the several periods corresponding with the proposed rate of per cent. and that sum multiplied by the principal will be the interest required.

*Interest compound.*

Let R = £1. and its interest for one year $= 1 + r$, M the amount of P £ in $n$ years, then

$$M = P R^n$$

Discount of M £ $= M - \frac{M}{R^n}$ where $n$ must be greater than one year, otherwise only simple interest can be allowed.

If besides the interest being converted into principal at the end of every year, the sum P is at the same time annually invested in capital then at the end of $n$ years.

$$M = \frac{P R (R^n - 1)}{R - 1}.$$

## TABLE I.

*Showing the sum to which one pound will increase when improved at Compound Interest during any number of years not exceeding* 50.

| Years. | 2½ per Cent. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|---|
| 1 | 1·02500 | 1·030000 | 1·040000 | 1·050000 | 1·060000 |
| 2 | 1·05063 | 1·060900 | 1·081600 | 1·102500 | 1·123600 |
| 3 | 1·07689 | 1·092727 | 1·124864 | 1·157625 | 1·191016 |
| 4 | 1·10381 | 1·125509 | 1·169859 | 1·215506 | 1·262477 |
| 5 | 1·13141 | 1·159274 | 1·216653 | 1·276282 | 1·338226 |
| 6 | 1·15969 | 1·194052 | 1·265319 | 1·340096 | 1·418519 |
| 7. | 1·18869 | 1·229874 | 1·315932 | 1·407100 | 1·503630 |
| 8 | 1·21840 | 1·266770 | 1·368569 | 1·477455 | 1·593848 |
| 9 | 1·24886 | 1·304773 | 1·423312 | 1·551328 | 1·689479 |
| 10 | 1·28008 | 1·343916 | 1·480244 | 1·628895 | 1·790848 |
| 11 | 1·31209 | 1·384234 | 1·539454 | 1·710339 | 1·898299 |
| 12 | 1·34489 | 1·425761 | 1·601032 | 1·795856 | 2·012196 |
| 13 | 1·37851 | 1·468534 | 1·665074 | 1·885649 | 2·132928 |
| 14 | 1·41297 | 1·512590 | 1·731676 | 1·979932 | 2·260904 |
| 15 | 1·44830 | 1·557967 | 1·800944 | 2·078928 | 2·396558 |
| 16 | 1·48151 | 1·604706 | 1·872981 | 2·182875 | 2·540352 |
| 17 | 1·52162 | 1·652848 | 1·947901 | 2·292018 | 2·692773 |
| 18 | 1·55966 | 1·702433 | 2·025817 | 2·406619 | 2·854339 |
| 19 | 1·59865 | 1·753506 | 2·106849 | 2·526950 | 3·025600 |
| 20 | 1·63862 | 1·806111 | 2·191123 | 2·653298 | 3·207135 |
| 21 | 1·67958 | 1·860295 | 2·278768 | 2·785963 | 3·399564 |
| 22 | 1·72157 | 1·916103 | 2·369919 | 2·925261 | 3·603537 |
| 23 | 1·76461 | 1·973587 | 2·461716 | 3·071524 | 3·819750 |
| 24 | 1·80873 | 2·032794 | 2·563304 | 3·225100 | 4·048935 |
| 25 | 1·85394 | 2·093778 | 2·665836 | 3·386355 | 4·291871 |
| 26 | 1·90029 | 2·156591 | 2·772470 | 3.555673 | 4·549383 |
| 27 | 1·94780 | 2·221289 | 2·883369 | 3·733456 | 4·822346 |
| 28 | 1·99650 | 2·287928 | 2·998703 | 3·920129 | 5·111687 |
| 29 | 2·01641 | 2·356566 | 3·118651 | 4·116136 | 5·418388 |
| 30 | 2·09757 | 2·427262 | 3·243398 | 4·321942 | 5·743491 |
| 31 | 2·15001 | 2·500080 | 3·373133 | 4·538039 | 6·088101 |
| 32 | 2·20376 | 2·575083 | 3·508059 | 4·764941 | 6·453387 |
| 33 | 2·25885 | 2·652335 | 3·648381 | 5·003189 | 6·840590 |
| 34 | 2·31532 | 2·731905 | 3·794316 | 5·253348 | 7·251025 |
| 25 | 2·37321 | 2·813862 | 3·946089 | 5·516015 | 7·686087 |
| 36 | 2·43254 | 2·898278 | 4·103933 | 5·791816 | 8·147252 |
| 37 | 2·49335 | 2·985227 | 4·268090 | 6·081407 | 8·636087 |
| 38 | 2·55568 | 3·074783 | 4·438813 | 6·385457 | 9·154252 |
| 39 | 2·61957 | 3·167027 | 4·616366 | 6·704751 | 9·703507 |
| 40 | 2·68506 | 3·262038 | 4·801021 | 7·039989 | 10·285718 |
| 41 | 2·75219 | 3·359899 | 4·993061 | 7·391988 | 10·902861 |
| 42 | 2·82100 | 3·460696 | 5·192784 | 7·761588 | 11·557033 |
| 43 | 2·89152 | 3·564517 | 5·400495 | 8·149667 | 12·250155 |
| 44 | 2·96381 | 3·671452 | 5·616515 | 8·557150 | 12·985482 |
| 45 | 3·03790 | 3·781596 | 5·841176 | 8·985008 | 13·764611 |
| 46 | 3·11385 | 3·895044 | 6·074823 | 9·434258 | 14·590487 |
| 47 | 3·19170 | 4·011895 | 6·317816 | 9·905971 | 15·465917 |
| 48 | 3·27149 | 4·132252 | 6·570528 | 10·401270 | 16·393872 |
| 49 | 3·35328 | 4·256219 | 6·833349 | 10·921333 | 17·377504 |
| 50 | 3·43711 | 4·383906 | 7·106683 | 11·467400 | 18·420154 |

## TABLE II.

*Shewing the present value of one pound to be received at the end of any number of years not exceeding 50.*

| Years. | 2½ per Cent. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|---|
| 1 | ·975610 | ·970874 | ·961538 | ·952381 | ·943396 |
| 2 | ·951814 | ·942596 | ·924556 | ·907029 | ·889996 |
| 3 | ·928599 | ·915142 | ·888996 | ·863838 | ·839619 |
| 4 | ·905951 | ·888487 | ·854804 | ·822702 | ·792094 |
| 5 | ·883854 | ·862609 | ·821927 | ·783526 | ·747258 |
| 6 | ·862297 | ·837484 | ·790315 | ·746215 | ·704961 |
| 7 | ·841265 | ·813092 | ·759918 | ·710681 | ·665057 |
| 8 | ·820747 | ·789409 | ·730690 | ·676839 | ·627412 |
| 9 | ·800728 | ·766417 | ·702587 | ·644609 | ·591898 |
| 10 | ·781198 | ·744094 | ·675564 | ·613913 | ·558395 |
| 11 | ·762145 | ·722421 | ·619581 | ·584679 | ·526788 |
| 12 | ·743556 | ·701380 | ·624597 | ·556837 | ·496969 |
| 13 | ·725420 | ·680951 | ·600574 | ·530321 | ·468839 |
| 14 | ·707727 | ·661118 | ·577475 | ·505068 | ·442301 |
| 15 | ·690466 | ·641862 | ·555265 | ·481017 | ·417265 |
| 16 | ·673625 | ·623167 | ·533908 | ·458112 | ·393646 |
| 17 | ·657195 | ·605016 | ·513373 | ·436297 | ·371364 |
| 18 | ·641166 | ·587395 | ·493628 | ·415521 | ·350344 |
| 19 | ·625528 | ·570286 | ·474642 | ·395734 | ·330513 |
| 20 | ·610271 | ·553676 | ·456387 | ·376889 | ·311805 |
| 21 | ·595386 | ·537549 | ·438834 | ·358942 | ·294155 |
| 22 | ·580865 | ·521893 | ·421955 | ·341850 | ·277505 |
| 23 | ·566697 | ·506692 | ·405726 | ·325571 | ·261797 |
| 24 | ·552875 | ·491934 | ·390121 | ·310068 | ·246979 |
| 25 | ·539391 | ·477606 | ·375117 | ·295303 | ·232999 |
| 26 | ·526235 | ·463695 | ·360689 | ·281241 | ·219810 |
| 27 | ·513400 | ·450189 | ·346817 | ·267818 | ·207368 |
| 28 | ·500878 | ·437077 | ·333477 | ·255094 | ·195630 |
| 29 | ·488661 | ·424346 | ·320651 | ·242946 | ·184557 |
| 30 | ·476743 | ·411987 | ·308319 | ·231377 | ·174110 |
| 31 | ·465115 | ·399987 | ·296460 | ·220359 | ·164255 |
| 32 | ·453771 | ·388337 | ·285058 | ·209866 | ·154957 |
| 33 | ·442703 | ·377026 | ·274094 | ·199873 | ·146186 |
| 34 | ·431905 | ·366045 | ·263552 | ·190355 | ·137912 |
| 35 | ·421371 | ·355383 | ·253415 | ·181290 | ·130105 |
| 36 | ·411094 | ·345032 | ·243669 | ·172657 | ·122741 |
| 37 | ·401067 | ·334983 | ·234297 | ·164436 | ·115793 |
| 38 | ·391285 | ·325226 | ·225285 | ·156605 | ·109239 |
| 39 | ·381741 | ·315754 | ·216621 | ·149148 | ·103056 |
| 40 | ·372431 | ·306557 | ·208289 | ·142046 | ·097222 |
| 41 | ·363347 | ·297628 | ·200278 | ·135282 | ·091719 |
| 42 | ·354485 | ·288959 | ·192575 | ·128840 | ·086527 |
| 43 | ·345839 | ·280543 | ·185168 | ·122704 | ·081630 |
| 44 | ·337404 | ·272372 | ·178046 | ·116861 | ·077009 |
| 45 | ·329174 | ·264439 | ·171198 | ·111297 | ·072650 |
| 46 | ·321146 | ·256737 | ·164614 | ·105997 | ·068538 |
| 47 | ·313313 | ·249259 | ·156283 | ·100949 | ·061658 |
| 48 | ·305671 | ·241999 | ·152195 | ·096142 | ·060998 |
| 49 | ·298216 | ·234950 | ·146341 | ·091564 | ·057546 |
| 50 | ·290942 | ·228107 | ·140713 | ·087204 | ·054288 |

**INTERPOLATIONS.**—(*Woodhouse, Vince.*)

If $a, a', a''$, &c. are successive values of a quantity $a$, differing by a constant interval 1, and if the 1st, 2d, 3d, &c. differences be $d', d'', d'''$, &c.; then any intermediate value ($y$), distant from $a$ by the interval $x$, is equal to

$$a + x\,d' + x.\,\frac{x-1}{2}\,d'' + x.\,\frac{x-1}{2}.\,\frac{x-2}{3}\,d'''\ \text{\&c.}$$

*Note.*—In taking the differences, the *preceding* quantity must always be subtracted from the succeeding; they will ∴ be positive or negative according as the series of quantities is increasing or decreasing.

If the law of the quantities be such that their last differences always become $= o$, we shall get at any intermediate time the *accurate* value of that quantity; but if the differences do not at last become accurately $= o$, we shall then get only an approximate value.

In general the quantities $d'$, $d''$, &c. diminish very fast, and it will not often be necessary to proceed farther than $d'''$.

*Ex.* 1. Given the squares of 2, 3, 4, and 5, to find the square of $2\frac{1}{2}$.

4, 9, 16, 25 ...... quantities
5, 7, 9 ......... 1st order of differences.
2, 2 .......... 2d do.
0 ............. 3d do.

Here $a = 4$, $d' = 5$, $d'' = 2$, $d''' = 0$, $x$ the required interval $= \frac{1}{2}$; ∴

$$y = 4 + \tfrac{1}{2} \times 5 - \tfrac{1}{8} \times 2 = 6,25.$$

*Ex.* 2. Given the log. of 110 = 2.04139, of 111 = 2.04532, of 112 = 2.04922, and of 113 = 2.05308; required the log. of 110.5.

2.04139, 2.04532, 2,04922, 2,05308
.00393, .00390, .00386
—. 00003, —. 00004

Here $a = 2.04139$. $d' = .00393$, $d'' = -.00003$, and $x = \frac{1}{2}$, ∴

$$y = 2.04139 + \tfrac{1}{2} \times .00393 - \tfrac{1}{8} \times -.00003 = 2.043359.$$

*Ex.* 3. Given five places of a comet as follows; on Nov. 5th at 8*h*. 17*m*. in Cancer 2°. 30′ = 150′; on the 6th at 8*h*. 17*m*. in 4°. 7′ = 247′; on the 7th at 8*h*. 17*m*. in 6°. 20′ = 380′; on the 8th at 8*h*. 17*m*. in 9°. 10′ = 550′; on the 9th at 8*h*. 17*m*. in 12°. 40′ = 760′. To find its place on the 7th at 14*h*. 17*m*.

First subtract 5*d*. 8*h*. 17*m*. from 7*d*. 14*h*. 17*m*., and there remains 2*d*. 6*h*. = 2,25 for the interval of time between the first observation and the given time at which the place is required; this ∴ is the value of $x$, to which we want to find the corresponding value of $y$; hence

150, 247, 380, 550, 760
97, 133, 170, 210
36, 37, 40
1, 3
2

Here $a = 150$, $d' = 97$, $d'' = 36$, $d''' = 1$, $d'''' = 2$; hence $y = 150 + 97 \times 2{,}25 + \frac{36}{2} \times 2{,}25 \times 1{,}25 + \frac{1}{2.3} \times 2{,}25 \times 1{,}25 \times {,}25 + \frac{2}{2.3.4} + 2{,}25 \times 1{,}25 \times {,}25 \times -{,}75 = 418', 96 = 6^{0}. 58'. 57''$, the place required.

But besides the use of the above equation, to find the value of any term of a series from its position being given, the converse is often required, *i. e.* having given any term, to find its position or distance from the first term.

*Ex.* On March, 1783, the sun's declination at noon at Greenwich was as follows:—On the 19th, N. $28'. 41'' = 1721''$; on the 20th, N. $5' = 300''$; on the 21st, S. $-18'. 41'' = -1121''$; to find the time of the equinox.

$$\begin{array}{c} 1721,\ 300,\ -1121 \\ -1421,\ -1421 \\ 0 \end{array}$$

Here $a = 1721$, $d' = -1421$, hence $y = 1721 - 1421 \times x$; now when the sun comes to the Equator, $y$ the declination becomes $= o$; $\therefore 1721 - 1421\,x = o$, and $x = \frac{1721}{1421} = 1d.\ 5h.\ 3m.\ 53s.$, the time from the 19th; hence $20d.\ 5h.\ 3m.\ 53s.$ is the time required.

We have here supposed that the quantities to be interpolated were taken at equal intervals of time; for a formula when the intervals are unequal, *see Vince's Astronomy*, vol. 2.

## INVOLUTION *and Evolution.*

TABLE *of the first nine powers of numbers.*

| 1st | 2d | 3d | 4th | 5th | 6th | 7th | 8th | 9th |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 |
| 3 | 9 | 27 | 81 | 243 | 729 | 2187 | 6561 | 19683 |
| 4 | 16 | 64 | 256 | 1024 | 4096 | 16384 | 65536 | 262144 |
| 5 | 25 | 125 | 625 | 3125 | 15625 | 78125 | 390625 | 1953125 |
| 6 | 36 | 216 | 1296 | 7776 | 46656 | 279936 | 1679616 | 10077696 |
| 7 | 49 | 343 | 2401 | 16807 | 117649 | 823543 | 5764801 | 40353607 |
| 8 | 64 | 512 | 4096 | 32768 | 262144 | 2097152 | 16777216 | 134217728 |
| 9 | 81 | 729 | 6561 | 59049 | 531441 | 4782969 | 43046721 | 387420489 |

**TABLE** *of squares, cubes, square roots, cube roots, and reciprocals, of of all numbers from* 1 *to* 100.—*(Barlow.)*

| Num. | Squares. | Cubes. | Sq. Roots. | Cu. Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 1·4142136 | 1·2599210 | ·500000000 |
| 3 | 9 | 27 | 1·7320508 | 1·4422496 | ·333333333 |
| 4 | 16 | 64 | 2·0000000 | 1·5874011 | ·250000000 |
| 5 | 25 | 125 | 2·2360680 | 1·7099759 | ·200000000 |
| 6 | 36 | 216 | 2·4494897 | 1·8171206 | ·166666667 |
| 7 | 49 | 343 | 2·6457513 | 1·9129312 | ·142857143 |
| 8 | 64 | 512 | 2·8284271 | 2·0000000 | ·125000000 |
| 9 | 81 | 729 | 3·0000000 | 2·0800837 | ·111111111 |
| 10 | 100 | 1000 | 3·1622777 | 2·1544347 | ·100000000 |
| 11 | 121 | 1331 | 3·3166248 | 2·2239801 | ·090909091 |
| 12 | 144 | 1728 | 3·4641016 | 2·2894286 | ·083333333 |
| 13 | 169 | 2197 | 3·6055513 | 2·3513347 | ·076923077 |
| 14 | 196 | 2744 | 3·7416574 | 2·4101422 | ·071428571 |
| 15 | 225 | 3375 | 3·8729833 | 2·4662121 | ·066666667 |
| 16 | 256 | 4096 | 4·0000000 | 2·5198421 | ·062500000 |
| 17 | 289 | 4913 | 4·1231056 | 2·5712816 | ·058823529 |
| 18 | 324 | 5832 | 4·2426407 | 2·6207414 | ·055555556 |
| 19 | 361 | 6859 | 4·3588989 | 2·6684016 | ·052631579 |
| 20 | 400 | 8000 | 4·4721360 | 2·7144177 | ·050000000 |
| 21 | 441 | 9261 | 4·5825757 | 2·7589243 | ·047619048 |
| 22 | 484 | 10648 | 4·6904158 | 2·8020393 | ·045454545 |
| 23 | 529 | 12167 | 4·7958315 | 2·8438670 | ·043478261 |
| 24 | 576 | 13824 | 4·8989795 | 2·8844991 | ·041666667 |
| 25 | 625 | 15625 | 5·0000000 | 2·9240177 | ·040000000 |
| 26 | 676 | 17576 | 5·0990195 | 2·9624960 | ·038461538 |
| 27 | 729 | 19683 | 5·1961524 | 3·0000000 | ·037037037 |
| 38 | 784 | 21952 | 5·2915026 | 3·0365889 | ·035714286 |
| 29 | 841 | 24389 | 5·3851648 | 3·0723168 | ·034482759 |
| 30 | 900 | 27000 | 5·4772256 | 3·1072325 | ·033333333 |
| 31 | 961 | 29791 | 5·5677644 | 3·1413806 | ·032258065 |
| 32 | 1024 | 32768 | 5·6568542 | 3·1748021 | ·031250000 |
| 33. | 1089 | 35937 | 5·7445626 | 3·2075343 | ·030303030 |
| 34 | 1156 | 39304 | 5·8309519 | 3·2396118 | ·029411765 |
| 35 | 1225 | 42875 | 5·9160798 | 3·2710663 | ·028571429 |
| 36 | 1296 | 46656 | 6·0000000 | 3·3019272 | ·027777778 |
| 37 | 1369 | 50653 | 6·0827625 | 3·3322218 | ·027027027 |
| 38 | 1444 | 54872 | 6·1644140 | 3·3619754 | ·026315789 |
| 39 | 1521 | 59319 | 6·2449980 | 3·3912114 | ·025641026 |
| 40 | 1600 | 64000 | 6·3245553 | 3·4199519 | ·025000000 |
| 41 | 1681 | 68921 | 6·4031242 | 3·4482172 | ·024390244 |
| 42 | 1764 | 74088 | 6·4807407 | 3·4760266 | ·023809524 |
| 43 | 1849 | 79507 | 6·5574385 | 3·5033981 | ·023255814 |
| 44 | 1936 | 85184 | 6·6332496 | 3·5303483 | ·022727273 |
| 45 | 2025 | 91125 | 6·7082039 | 3·5568933 | ·022222222 |
| 46 | 2116 | 97336 | 6·7823300 | 3·5830479 | ·021739130 |
| 47 | 2209 | 103823 | 6·8556546 | 3·6088261 | ·021276600 |
| 48 | 2304 | 110592 | 6·9282032 | 3·6342411 | ·020833333 |
| 49 | 2401 | 117649 | 7·0000000 | 3·6593057 | ·020408163 |
| 50 | 2500 | 125000 | 7·0710678 | 3·6840314 | ·020000000 |

| Num. | Squares. | Cubes. | Sq. Roots. | Cu. Roots. | Reciprocals. |
|---|---|---|---|---|---|
| 51 | 2601 | 132651 | 7·1414284 | 3·7084298 | ·019607843 |
| 52 | 2704 | 140608 | 7·2111026 | 3·7325111 | ·019230769 |
| 53 | 2809 | 148877 | 7·2801099 | 3·7562858 | ·018867925 |
| 54 | 2916 | 157464 | 7·3484692 | 3·7797631 | ·018518519 |
| 55 | 3025 | 166375 | 7·4161985 | 3·8029525 | ·018181818 |
| 56 | 3136 | 175616 | 7·4833148 | 3·8258624 | ·017857143 |
| 57 | 3249 | 185193 | 7·5498344 | 3·8485011 | ·017543860 |
| 58 | 3364 | 195112 | 7·6157731 | 3·8708766 | ·017241379 |
| 59 | 3481 | 205379 | 7·6811457 | 3·8929965 | ·016949153 |
| 60 | 3600 | 216000 | 7·7459667 | 3·9148676 | ·016666667 |
| 61 | 3721 | 226981 | 7·8102497 | 3·9364972 | ·016393443 |
| 62 | 3844 | 238328 | 7·8740079 | 3·9578915 | ·016129032 |
| 63 | 3969 | 250047 | 7·9372539 | 3·9790571 | ·015873016 |
| 64 | 4096 | 262144 | 8·0000000 | 4·0000000 | ·015625000 |
| 65 | 4225 | 274625 | 8·0622577 | 4·0207256 | ·015384615 |
| 66 | 4356 | 287496 | 8·1240384 | 4·0412401 | ·015151515 |
| 67 | 4489 | 300763 | 8·1853528 | 4·0615480 | ·014925373 |
| 68 | 4624 | 314432 | 8·2462113 | 4·0816551 | ·014705882 |
| 69 | 4761 | 328509 | 8·3066239 | 4·1015661 | ·014492754 |
| 70 | 4900 | 343000 | 8·3666003 | 4·1212853 | ·014285714 |
| 71 | 5041 | 357911 | 8·4261498 | 4·1408178 | ·014084507 |
| 72 | 5184 | 373248 | 8·4852814 | 4·1601676 | ·013888889 |
| 73 | 5329 | 389017 | 8·5440037 | 4·1793390 | ·013698630 |
| 74 | 5476 | 405224 | 8·6023253 | 4·1983364 | ·013513514 |
| 75 | 5625 | 421875 | 8.6602540 | 4·2171633 | ·013333333 |
| 76 | 5776 | 438976 | 8·7177979 | 4·2358236 | ·013157895 |
| 77 | 5929 | 456533 | 8·7749644 | 4·2543210 | ·012987013 |
| 78 | 6084 | 474552 | 8·8317609 | 4·2726586 | ·012820513 |
| 79 | 6241 | 493039 | 8·8881944 | 4·2908404 | ·012658228 |
| 80 | 6400 | 512000 | 8 9442719 | 4·3088695 | ·012500000 |
| 81 | 6561 | 531441 | 9·0000000 | 4·3267487 | ·012345679 |
| 82 | 6724 | 551368 | 9·0553851 | 4·3444815 | ·012195122 |
| 83 | 6889 | 571787 | 9·1104336 | 4·3620707 | ·012048193 |
| 84 | 7056 | 592704 | 9·1651514 | 4·3795191 | ·011904762 |
| 85 | 7225 | 614125 | 9·2195445 | 4·3968296 | ·011764706 |
| 86 | 7396 | 636056 | 9·2736185 | 4·4140049 | ·011627907 |
| 87 | 7569 | 658503 | 9·3273791 | 4·4310476 | ·011494253 |
| 88 | 7744 | 681472 | 9·3808315 | 4·4479602 | ·011363636 |
| 89 | 7921 | 704969 | 9·4339811 | 4·4647451 | ·011235955 |
| 90 | 8100 | 729000 | 9·4868330 | 4·4814047 | ·011111111 |
| 91 | 8281 | 753571 | 9·5393920 | 4·4979414 | ·010989011 |
| 92 | 8464 | 778688 | 9·5916630 | 4·5143574 | ·010869565 |
| 93 | 8649 | 804357 | 9·6436508 | 4·5306549 | ·010752688 |
| 94 | 8836 | 830584 | 9·6953597 | 4·5468359 | ·010638298 |
| 95 | 9025 | 857375 | 9·7467943 | 4·5629026 | ·010526316 |
| 96 | 9216 | 884736 | 9·7979590 | 4·5788570 | ·010416667 |
| 97 | 9409 | 912673 | 9·8488578 | 4·5947009 | ·010309278 |
| 98 | 9604 | 941192 | 9·8994949 | 4·6104363 | ·010204082 |
| 99 | 9801 | 970299 | 9·9498744 | 4·6260650 | ·010101010 |

The use of the first five columns is obvious: the column of reciprocals is useful for converting a vulgar into a decimal fraction, as in the following example.

$$\text{Express } \frac{3}{28} \text{ as a decimal.}$$

$$\text{By Table } \frac{1}{28} \text{ — is — } .035714286$$

$$\therefore \frac{3}{28} \text{ — is — } .107142858$$

JULIAN *Period.—See Cycle.*

JUNO.

This planet was discovered by Mr Harding, at Lilienthal, September 1st, 1804. For its elements, &c.—*see Planets, elements of.*

JUPITER.—*See Planets, elements of.*

JUPITER'S *Satellites.—See Satellites.*

## L.

LAND *Surveying.—See Surveying.*

LATITUDE *Geographical.—(Woodhouse.)*

*1st Method, by the Altitudes of circumpolar stars.*

Co-latitude = half the sum of the greatest and least zenith distances corrected for refraction.

Or the latitude may be found by Captain Kater's method, from an observed altitude of the pole star when out of the meridian thus.—*(Galbraith.)*

To the constant log. 5.314425, add the log. tangent of the star's polar distance $p$, and the log. cos. of the meridian distance $t$ in degrees, the sum of these will be log. of an arc $u$ in seconds. Now to the log. secant $p$ add the log. cosine $u$, and cosine of the zenith distance $z$; the sum will be the cosine of $(\psi \pm u)$ an arc which being increased or diminished by the arc $u$, will be the co-latitude $\psi$.

To find $t$, calculate the time of the star's meridian passage *(see Time)*, the difference between which and the time of observation gives $t$.

In the application of $u$ attention must be paid to the sign of the arc $t$, according to its situation in the circle which the star describes round the pole in its diurnal revolution. If $t$ is in the 1st or 4th quadrant it is additive; but if in the 2d or 3d, it is subtractive.

*Ex.* On the 22d of February, 1826, at 7*h.* 42*m.* 49*s.* mean time, the altitude of the Pole star was observed to be 51°. 58′. 11″; required the latitude.

First to find the mean solar time when the star was upon the meridian.

| | *h.* | *m.* | *s.* | | | | |
|---|---|---|---|---|---|---|---|
| Star's app. R. A. ......... | 0 | 58 | 15,2 | App. alt. | 51°. | 58′ | 11″ |
| Sun's R. A. at noon ...... | 22 | 21 | 18,3 | Refract. | | | — 45,4 |
| | 2 | 36 | 56,9 | True alt. | 51 | 57 | 25,6 |
| Diff. *(see Time Table 6)* | | — | 25,7 | $z$ = | 38 | 2 | 34,4 |
| | 2 | 36 | 31,2 | | | | |
| Equat. of time for noon | + | 13 | 50,7 | | | | |
| Star upon meridian ...... | 2 | 50 | 21,9 | | | | |
| Time of observation ...... | 7 | 42 | 49,0 | | | | |
| Distance of star from meridian in mean time | 4 | 52 | 27,1 | = 73°. 18′. 47″. *(see Time Table.)* | | | |

| | | | |
|---|---|---|---|
| Constant log. ............... | 5.314425 | | |
| $p = 1^\circ.\ 36'.\ 48''$ tang. ... | 8.449716 | secant | 0.000172 |
| $t = 73.\ 18.\ 47$ cosine ... | 9.458097 | | |
| $u = 0.\ 27.\ 48{,}2 = 1668'',2$ | 3.222238 | cosine | 9.999986 |
| | $z = 38^\circ.\ 2'.\ 34{,}4''$ | cosine | 9.896278 |
| $(\psi - u) = 38^\circ.\ 0'.\ 58{,}2''$ | | cosine | 9.896436 |
| $\psi$ = | 38. 28. 46.4 | | |
| $\lambda$ = | 51. 31. 13,6 | | |

*2d Method, by the zenith distances of stars near the zenith.*

This method determines merely the difference of latitude by means of the zenith sector, measuring small zenith distances with great exactness.

*Ex.* By observation at the College of Mazarin.

| | |
|---|---|
| Z. D. of $\gamma$ *Draconis* reduced to January, 1750 ......... | 2°. 40.′ 15″ |
| At Greenwich Z. D. reduced to same epoch ............ | 0. 3. 4,5 |
| Difference of latitude .................................. | 2. 37. 10,5 |
| Hence if latitude of Greenwich be ....................... | 51. 28. 39,5 |
| Latitude of Observatory of College of Mazarin ...... | 48. 51. 29 |

This method, which is capable of great accuracy, was employed in the Trigonometrical Survey of England.

*3d Method by observations of Altitudes made near the Meridian, and reduced to the Meridian.*

Let $z'$, $z''$, &c. $z_{\prime}$, $z_{\prime\prime}$, &c. be $n$ zenith distances to the east and west of the meridian.

$h,'$ $h,''$ &c. $h_{\prime}$, $h_{\prime\prime}$. &c. the corresponding times determined by means of a time-keeper.

$\delta'$, $\delta''$, &c. $\delta_{\prime}$, $\delta_{\prime\prime}$, &c., the calculated corrections, or the reductions to the meridian: then the true or corrected meridional zenith distances will be (if the passage of the star be above the pole.)

$$z' - \delta', \; z'' - \delta'' \text{ \&c. } z_{\prime} - \delta_{\prime}, \; z_{\prime\prime} - \delta_{\prime\prime} \text{ \&c.}$$

and the true mean meridional zenith distance will be

$$= \frac{z' - \delta' + z'' - \delta'' + \text{\&c.} + z_{\prime} - \delta_{\prime} + z_{\prime\prime} - \delta_{\prime\prime} + \text{\&c.}}{n} =$$

$$\frac{z' + z'' + \text{\&c.} + z_{\prime} + z_{\prime\prime} + \text{\&c.}}{n} - \frac{\delta' + \delta'' + \text{\&c.} + \delta_{\prime} + \delta_{\prime\prime} + \text{\&c.}}{n}$$

In the above formula $\delta = \frac{2}{\sin. 1''} \times \sin.^2 \frac{h'}{2} . \frac{\cos. d. \cos. L}{\sin. z}$, where $d$ is known from the Tables; and for L the *approximate* value of the latitude may be taken, and for $z$ the observed meridional zenith distance.

Having thus found the meridional Z. D. and knowing the N. P. D. the latitude may be ascertained.

This method of determining the latitude was used by the French Astronomers in measuring an arc of the meridian, and is capable of determining the latitude within the fraction of a second. It is peculiarly adapted to Borda's *Circle of Repetition.*

*Latitude of a vessel at sea by the sextant.*

*Method by the Meridional Altitude of the Sun.*

If the latitude and the declination be of the same denomination, then the latitude =

Z. D. of sun + declination of sun;

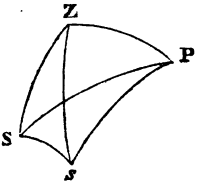
or if declination be greater than latitude, = declination sun — Z. D. of sun.

If the latitude and declination be of different denominations, then latitude =

Z. D. of sun — declination of sun.

This method is commonly used at sea, but as the sun must be on the meridian, clouds may prevent its being used. A subsidiary method therefore is provided, in which the latitude may be computed from two observed altitudes of the sun, and the interval of time between the observations.

Let Z be the zenith, P the pole, S, $s$ two positions of the sun; then the following are the steps in this process.



(1.) Find S $s$; let $t$ = interval of time, $p$ = P S then

$$2 \sin.^2 \frac{\mathrm{S}\,s}{2} = \sin._2 p.\ 2.\ \sin._2 \frac{t}{2};$$

$$\therefore \log. \sin. \frac{\mathrm{S}\,s}{2} = \log. \sin. p + \log. \sin. \frac{t}{2} - 10.$$

(2.) Find $\angle$ S $s$ P;

$$\tan.\ \mathrm{S}\,s\,\mathrm{P} = \frac{\cot. \frac{t}{2}}{\cos. p};$$

$$\therefore \log. \tan.\ \mathrm{S}\,s\,\mathrm{P} = 10 + \log. \cot. \frac{t}{2} - \log. \cos. p.$$

(3.) Find $\angle$ Z $s$ S;

Let $a$ and $a'$ be the observed altitudes, then $\sin._2 \frac{1}{2}\,\mathrm{Z}\,s\,\mathrm{S}$

$$= \frac{\cos. \frac{1}{2} (\mathrm{S}\,s + a' + a).\ \sin. \frac{1}{2} (\mathrm{S}\,s + a' - a)}{\sin.\ \mathrm{S}\,s \times \cos. a};\ \therefore 2 \log. \sin. \tfrac{1}{2}\,\mathrm{Z}\,s\,\mathrm{S} =$$

$$20 + \log. \cos. \tfrac{1}{2} (\mathrm{S}\,s + a' + a) + \log. \sin. \tfrac{1}{2} (\mathrm{S}\,s + a' - a) - \log. \sin.\ \mathrm{S}\,s - \log. \cos. a.$$

Hence Z $s$ P = S $s$ P — Z $s$ S is known.

(4.) Find Z P;

Assume $\theta$ such that

$$\tan.^2 \theta = \frac{\cos. a'.\ \sin. p.\ \text{ver. sin. } \mathrm{Z}\,s\,\mathrm{P}}{\text{ver. sin. } (90^\circ - a' - p)};$$

$$\text{then } \sin. \frac{\mathrm{Z\,P}}{2} = \sin. \frac{90^\circ - a' - p}{2} \times \sec. \theta;$$

$$\therefore \log. \sin. \frac{\mathrm{Z\,P}}{2} = 10 + \log. \sin. \tfrac{1}{2} (90^\circ - a' - p) - \log. \cos. \theta.$$

**LEAP** *Year.—See Calendar.*

**LEMNISCATA**, *equation to.*

$$a\,y = x\,\sqrt{a^2 - x^2}$$

**Lemniscata of James Bernouilli.**

$$(x^2 + y^2)^2 = a^2\,(x^2 - y^2)$$

or considered as a spiral

$$\varrho = a\,\sqrt{\cos.\,2\,\theta}.$$

LENGTHS *of curves.—See Rectification.*

LENS.—*See Refraction.*

LEVELLING.

Two or more places are on a true level, when they are equally distant from the centre of the earth; and a line equally distant from that centre in all its points, is called the line of true level. This line is nearly an arc of a circle, and will evidently pass below the line of apparent level, which, as determined by the instrument, will be a tangent or a parallel to a tangent at the earth's surface at the point of observation. Hence the depression of the true below the apparent level is always equal to the excess of the secant of the arc of distance above the radius of the earth. To find this depression, let L be the arc of distance in English miles, D the depression in feet; then

$$D = \frac{2\,L^2}{3}.$$

TABLE *shewing the height of the apparent above the true level for every* 100 *yards of distance on the one hand, and for every mile on the other.*

| *Distance of base.* | *Diff. of level.* | *Distance of base.* | *Difference of level.* | |
|---|---|---|---|---|
| Yards. | Inches. | Miles. | Feet. | In. |
| 100 | 0.026 | ¼ | 0. | 0½ |
| 200 | 0.103 | ½ | 0. | 2 |
| 300 | 0.231 | ¾ | 0. | 4½ |
| 400 | 0.411 | 1 | 0. | 8 |
| 500 | 0.643 | 2 | 2. | 8 |
| 600 | 0.925 | 3 | 6. | 0 |
| 700 | 1.260 | 4 | 10. | 7 |
| 800 | 1.645 | 5 | 16. | 7 |
| 900 | 2.081 | 6 | 23. | 11 |
| 1000 | 2.570 | 7 | 32. | 6 |
| 1100 | 3.110 | 8 | 42. | 6 |
| 1200 | 3.701 | 9 | 53. | 9 |
| 1300 | 4.344 | 10 | 66. | 4 |
| 1400 | 5.098 | 11 | 80. | 3 |
| 1500 | 5.784 | 12 | 95. | 7 |
| 1600 | 6.580 | 13 | 112. | 2 |
| 1700 | 7.425 | 14 | 130. | 1 |

*Example.* Suppose a spring to be on one side of a hill, and a house on an opposite hill, with a valley between them; and that the spring seen from the house appears by a levelling instrument to be on a level with the foundation of the house, which suppose is at a mile distance from it; then *(by Table)* the spring is eight inches above the true level of the house; and this difference would be barely sufficient for the water to be brought in pipes from the spring to the house, the pipes being laid all the way in the ground.

In the above Table, the effects of refraction have not been considered, which, however, should not be neglected, if the distances are considerable. In that case, the correct formula is

$$D = \frac{4 L^2}{7};$$

which expression includes the effects both of curvature and refraction. *See Refraction terrestrial.*

LEVER.

Levers may be divided into three kinds. In levers of the first kind, the fulcrum is between the power and the weight, as in the balance, steelyard, scissors, poker, &c. In levers of the second kind, the weight is between the fulcrum and the power, as in oars, doors, cutting knives fixed at one end, &c. In levers of the third kind, the power acts between the fulcrum and the weight, as in tongs, sheers for sheep, muscles of animals, &c.

1. Two weights or forces, acting perpendicularly upon a straight lever, will balance each other, when they are reciprocally proportional to their distances from the fulcrum.

*Cor.* 1. When the power and weight act on the same side of the fulcrum, and keep each other in equilibrio, the weight sustained by the fulcrum is equal to the difference between the power and the weight.

*Cor.* 2. If the same body be weighed at the two ends of a false balance (one arm of which is longer than the other), its true weight is a mean proportional between the apparent weights,

*Cor.* 3. If a weight be placed upon a lever supported upon two props, the pressures upon the props are inversely proportional to their distances from the weight.

2. If two forces, acting upon the arms of *any* lever, keep it at rest, they are to each other inversely as the perpendiculars drawn from the centre of motion to the directions in which the forces act; or inversely as the arms, multiplied into the sines of the angles, which the direction of the forces make with them.

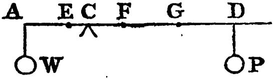
*Cor.* If a man, balanced in a common pair of scales, press upwards, by means of a rod, against any point of the beam, except that from which the scale is suspended, he will preponderate.

3. In a compound lever, where one is made to turn another, there is an equilibrium, when $W : P ::$ the product of all the arms taken alternately, beginning with that to which the power is applied : the product of all the other arms.

4. Any weights will keep each other in equilibrio on the arms of a straight lever, when the products, which arise from multiplying each weight by its distance from the fulcrum, are equal on each side of the fulcrum.

*Cor.* 1. If in the above Propositions we would allow for the weight of the lever itself, we must suppose its weight to be united in the centre of gravity, and to act there as a third force added to the power or the weight, according to the side of the fulcrum on which it is placed.

*Cor.* 2. If the weights do not act perpendicularly to the arms of the lever, we must for the distances substitute the perpendiculars, (see Art. 2.)



*Cor.* 3. Let $AD$ be the common steelyard, whose fulcrum is $C$, and let the moveable weight $P$, when placed at $E$, keep the lever at rest; then when $W$ and $P$ are suspended upon the lever, and the whole remains at rest, $W \times AC = P \times DC + P \times EC = P \times DE$; $\therefore$ $W$ varies as $ED$; the graduation must $\therefore$ begin from $E$, and if $P$ when placed at $F$ support a weight of one pound at $A$, take $FG$, $GD$, &c. equal to one another and to $EF$; and when $P$ is placed at $G$ it will support two pounds; and when at $D$ it will support three pounds, &c.

LIFE *Annuities.—See Annuities Life.*

LIFE *Assurances.—See Annuities.*

LIGHT, *Phænomena of.*

*Light, propagation of.*

1. In a free medium the force and intensity of light, which propagates itself in rays emanating from the same point, are inversely as the squares of the distances from that point.

*Prob.* Having given the position of two lights of known intensities, to determine the nature and equation of the surface, of which every point shall be equally illuminated by the two lights.

Let $A$ and $B$ be the two points at which the lights are placed, $m$ and $n$

their intensities at any assumed unit of distance, and let $a = AB$; then it may be shewn that the required surface is a sphere of which the radius $= \frac{a}{n-m} \sqrt{mn}$, and whose centre has for an abscissa $\frac{ma}{m-n}$.

*Cor.* If $m = n$ the radius is infinite, as also the abscissa from the centre; in this case the surface is a plane perpendicular to the middle of the line A B.

*Light, velocity of.*

2. Light takes up about 16¼ minutes in passing over a space = the diameter of the earth's orbit, which is nearly 190 millions of miles; ∴ it travels at the rate of almost 200,000 miles per secnod.

*Light, diminution of, under various circumstances.*

3. If the spaces through which light passes through a uniformly dense diaphonous medium increase in arithmetical progression, the quantity will decrease in geometrical progression.

Let the space be divided into equal portions or laminæ, and suppose $\frac{1}{n}$th part of the whole light to be lost or absorbed in its passage thro' the 1st lamina; then $\frac{n-1}{n}$ = quantity of light entering the 2d lamina; $\frac{(n-1)^2}{n^2}$ = do. entering the 3d; $\frac{(n-)^3}{n^3}$ = do. entering the 4th, &c.

TABLE *from Bouguer, shewing the intensity of the sun's light at different altitudes, and the thickness of air it has to penetrate at each angle.*

| *Sun's altitude.* | *Thickness of air in toises.* | *Intensity of light the whole being* 10,000. |
|---|---|---|
| 90° …… | 3911 …… | 8123 |
| 80 …… | 3971 …… | 8098 |
| 70 …… | 4162 …… | 8016 |
| 60 …… | 4516 …… | 7866 |
| 50 …… | 5104 …… | 7624 |
| 40 …… | 6086 …… | 7237 |
| 30 …… | 7784 …… | 6613 |
| 20 …… | 11341 …… | 5474 |
| 15 …… | 14880 …… | 4535 |
| 10 …… | 21745 …… | 3149 |
| 5 …… | 39893 …… | 1201 |
| 3 …… | 58182 …… | 454 |
| 1 …… | 100930 …… | 47 |
| 0 …… | 138823 …… | 6 |

4. According to Leslie, in passing through sea water, light is diminished four times for every five fathoms of vertical descent; and Bouguer asserts, that the whole effect of the sun's light would be lost by passing through 679 feet of sea water, and that the same effect would take place by its passing through 3,110,310 feet of air.

5. Bouguer computes that of 300,000 rays which the moon receives; 172,000, or perhaps 204,100 are absorbed; and that the light of the sun : ditto of the full moon :: 300,000 : 1.

6. Euler makes the light of the sun equal to that of 6560 candles at one foot distance; that of the moon to a candle at 7½ feet; of Venus to a candle at 421 feet; and of Jupiter to a candle at 1620 feet, partly from Bouguer's experiments. Hence the sun would appear like Jupiter, if removed to 131,000 times his present distance.—(*Young's Nat. Phil.*)

*Light, refrangibility of.*

7. The sun's light consists of rays which differ in refrangibility and colour.

The 7 *primary* colours are red, orange, yellow, green, blue, indigo, and violet, of which the red rays are the least refrangible, and the violet ones the most; while green and blue are the colours which have a mean degree of refrangibility. Sir Isaac Newton found their degrees of refrangibility in passing out of glass into air to be as the numbers 77, $77\frac{1}{8}$, $77\frac{1}{5}$, $77\frac{1}{3}$, $77\frac{1}{2}$, $77\frac{2}{3}$, $77\frac{7}{9}$, and 78, those being the values of the sines of refraction to the common sine of incidence 50. Some substances, however, separate the different coloured rays more widely than others, and the *dispersive* power of media does not appear to depend at all upon their mean refracting power.

To find a measure of the *dispersing* power, take a constant small ∠ $\theta$ for the ∠ of refraction, the ∠ of incidence will then be $m\,\theta$ and will differ according to the value of $m$. The difference between these two or $(m-1)\,\theta$ is the refraction; and if $m_r$ and $m_v$ be values of $m$ for red and violet rays, the difference of refraction will be $(m_v - 1)\,\theta - (m_r - 1)\,\theta$ or $(m_v - m_r)\,\theta$. Its ratio to the refraction will consequently be $\frac{m_v - m_r}{m}$, taking the mean value of $m$ : this is the usual measure of the dispersing power.

In flint glass its value is about 0.05; in crown glass 0,033.

8. Having given the refracting powers of two mediums, to find the

ratio of the focal lengths of two lenses formed of these substances, which, when united, will produce images nearly free from colour.

Let $\varrho$ and $\varrho'$ be the focal lengths of the lenses, $1 + r$ and $1 + v$ the *ratio of refraction* belonging to the red and violet rays respectively in the 1st lens, and $1 + r'$ and $1 + v' =$ ditto of the other; then

$$\frac{\varrho'}{\varrho} = -\frac{v' - r'}{v - r}.$$

Hence it appears that $\varrho'$ and $\varrho$ must be of different signs, or one lens concave and the other convex; and that they are as the respective dispersive powers of the substances of which the lenses are made.

The common practice of opticians, is to use flint glass and crown glass, the dispersive powers of which are in the ratio of 50 to 33; and ∴ a compound lens, in which the separate focal lengths for the same kind of homogeneous light, are as 50 : 33 will make the red and violet rays, converge accurately to one point.

9. Having given the aperture of any lens, and the foci to which rays of different colours, belonging to the same pencil, converge; to find the least circle of aberration through which these rays pass.

Let D = diameter of the least circle of aberration, $\alpha$ = aperture of the lens, the rest as before; then

$$D = \frac{v - r}{v + r} . \alpha.$$

Suppose, for instance, the lens be of crown glass, $v = .56$, $r = .54$; $\therefore \frac{v - r}{v + r} = \frac{1}{55}$; D ∴ is $\frac{1}{55}$ of the aperture.

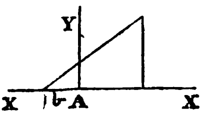
*Light, aberration of—see Aberration.*

For a concise account of other physical properties of light, such as the phænomena of coloured rings, double refraction, polarization of light, &c. *see Coddington's Optics*; these subjects, as requiring diffuse explanations, cannot here be entered upon.

LINE *right.*

Equations and Problems relating to, the co-ordinates being supposed *rectangular.*—(*Hamilton.*)

1. The equation to a straight line is $y = a x + b$, where $a$ is the tangent of the angle which the line makes with the axis X A $x$, and $b$ is the distance from A at which it intersects the axis A $y$.



2. Required the equation to a straight line passing through a given point, whose co-ordinates are $x'$, $y'$.

Any point of which the co-ordinates are $x$, $y$ being assumed in the line, we have $y = a\,x + b$; also $y' = a\,x' + b$; $\therefore$ equation required is

$$y - y' = a\ (x - x').$$

For the sake of brevity it is usual to designate the point, whose co-ordinates are $x'$, $y'$, as the point $(x', y')$; and the straight line, whose equation is $y = a\,x + b$, as the straight line $y = a\,x + b$.

2. Required the equation to the line which passes through *two* given points $(x', y')$ and $(x'', y'')$.

$$y - y' = \frac{y'' - y'}{x'' - x'}\ (x - x').$$

3. Required the angle formed by the intersection of two given lines.

Let $y = a\,x + b$ and $y = a'x + b'$ be the given lines, and $\theta$ the given angle; then

$$\tan.\ \theta = \frac{a - a'}{1 + a\,a'}$$

$$\sin.\ \theta = \frac{a - a'}{\surd(1 + a^2)\ (1 + a'^2)}$$

$$\cos.\ \theta = \pm \frac{1 + a\,a'}{\surd(1 + a^2)\ (1 + a'^2)}$$

the positive sign being used when the $\angle$ is acute, the negative when it is obtuse.

4. Required the equation to a straight line drawn through a given point $(x', y')$, and making an angle $\tan.^{-1} m$ with the line $y = a\,x + b$.

Here $y - y' = \dfrac{a - m}{1 + a\,m}\ (x - x')$.

Hence (1) when the lines are perpendicular,

$$y - y' = -\frac{1}{a}\ (x - x').$$

(2) When they are parallel,

$$y - y' = a\ (x - x').$$

5. Required the distance $(r)$ between two points $(x, y)$ and $(x', y')$.

$$r = \surd \left\{ (x' - x)^2 + (y' - y)^2 \right\}$$

When $x'$ and $y' = o$, $r = \surd\ (x^2 + y^2)$; which therefore expresses the distance of a point from the origin.

6. If $(p)$ be the perpendicular dropped from a given point $(x', y',)$ on the straight line $y = a\,x + b$; then

$$p = \pm \frac{y' - a\,x' - b}{\sqrt{(1 + a^2)}}.$$

LITUUS.—*See Spiral.*

## LOGARITHMS.

### 1. Properties of Logarithms.

$$\text{Log. } a \times b = \text{Log. } a + \text{Log. } b.$$

$$\text{Log. } \frac{a}{b} = \text{Log. } a - \text{Log. } b.$$

$$\text{Log. } a^{m} = m \text{ Log. } a.$$

$$\text{Log. } a^{\frac{1}{m}} = \frac{1}{m} \text{ Log. } a.$$

$$- \text{Log. } a = \text{Log. } \frac{1}{a}.$$

*Ex.* 1. $\text{Log. } \dfrac{2^{20} \times 3^{7} \times 2.031}{17 \times 9350} = 20 \text{ log. } 2 + 7 \text{ log. } 3 + \text{log. } 2.013 -$ $(\text{log. } 17 + \text{log. } 9350)$.

*Ex.* 2. $\text{Log. } \sqrt[5]{\dfrac{317^2 \times \sqrt{3}\ \sqrt[3]{5}}{251}} = \dfrac{1}{5}\,(2 \text{ log. } 317 + \tfrac{1}{2} \text{ log. } 3 + \tfrac{1}{3}$ $\text{log. } 5 - \text{log. } 251)$.

### 2. Given a number, to find its Logarithm.

Let $1 + x$ be the number, $m$ the modulus,

$$\text{then log. } 1 + x = m\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \&c.\right)$$

$$\text{and log. } 1 - x = m \times \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \&c.\right)$$

$$\text{Hence log. } \frac{1 + x}{1 - x} = 2\,m \times \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \&c.\right)$$

Or since $\text{N} = \dfrac{1 + \dfrac{\text{N} - 1}{\text{N} + 1}}{1 - \dfrac{\text{N} - 1}{\text{N} + 1}}$, we may for $x$ substitute $\dfrac{\text{N} - 1}{\text{N} + 1}$, and we shall have

$$\text{Log. } N = 2m \times \left\{\frac{N-1}{N+1} + \frac{1}{3}\left(\frac{N-1}{N+1}\right)^3 + \frac{1}{5}\left(\frac{N-1}{N+1}\right)^5 + \&c.\right\}$$

both of which last series converge very fast.

*Ex.* If $N = 2, \frac{N-1}{N+1} = \frac{1}{3}$; $\therefore$ log. 2 = .3010300.

In hyp. logarithms $m = 1$, in the common system $m = \frac{1}{2.30258509} =$ .43424948. And since different systems of logs. are as their moduli, if any common log. be divided by this modulus, it gives the corresponding hyp. log.; or if any hyp. log. be multiplied by it, it gives the corresponding common logarithm.

3. Given a logarithm, to find its number.

Let $1 + x =$ No., $y$ its log. $m$ the modulus.

$$\text{then } 1 + x = 1 + \frac{y}{m} + \frac{y^2}{2m^2} + \frac{y^3}{2.3.m^3} + \&c.$$

If $m = 1$, $1 + x = 1 + y + \frac{y^2}{2} + \frac{y^3}{2.3} + \&c. =$ No. whose hyp. log. is $y$.

4. *Modular ratio* is the ratio of which the modulus is the measure, or the number of which the modulus is the logarithm, and $= 1 + 1 + \frac{1}{2} + \frac{1}{2.3} + \&c. : 1$; or 2.7182818 : 1; which is therefore the same for every system, being independent of $m$ and $y$.

Hence in Napier's or hyp. logs., where the modulus is 1, the log. of 2.7182818 is 1; in Brigg's or the common system, log. 2.7182818 is .43424948.

Hence also since in every system the log. of the base is 1; 2.7182818 is the base of Napier's logs.; in Brigg's the base is 10.

In general if $a =$ base of any system, whose modulus is $m$, $m = \frac{1}{\text{h. l. } a}$.

*The following Table of Logarithmic series will be found useful on various occasions.*

1. $\text{Log. } a = \frac{1}{M} \times \left\{(a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \&c.\right\}$

2. $\text{Log. } a = \frac{1}{M} \times \left\{\left(\frac{a-1}{a}\right) + \frac{1}{2}\left(\frac{a-1}{a}\right)^2 + \frac{1}{3}\left(\frac{a-1}{a}\right)^3 + \&c.\right\}$

$$3.\ \text{Log.}\ a = \frac{2}{\text{M}} \times \left\{ \left(\frac{a-1}{a+1}\right) + \tfrac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \tfrac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \&\text{c.} \right\}$$

$$4.\ \text{Log.}\ \frac{a}{b} = \frac{1}{\text{M}} \times \left\{ \left(\frac{a \backsim b}{b}\right) - \tfrac{1}{2}\left(\frac{a \backsim b}{b}\right)^2 + \tfrac{1}{3}\left(\frac{a \backsim b}{b}\right)^3 - \&\text{c.} \right\}$$

$$5.\ \text{Log.}\ \frac{a}{b} = \frac{1}{\text{M}} \times \left\{ \left(\frac{a \backsim b}{a}\right) + \tfrac{1}{2}\left(\frac{a \backsim b}{a}\right)^2 + \tfrac{1}{3}\left(\frac{a \backsim b}{a}\right)^3 + \&\text{c.} \right\}$$

$$6.\ \text{Log.}\ \frac{a}{b} = \frac{2}{\text{M}} \times \left\{ \left(\frac{a \backsim b}{a+b}\right) + \tfrac{1}{3}\left(\frac{a \backsim b}{a+b}\right)^3 + \tfrac{1}{5}\left(\frac{a \backsim b}{a+b}\right)^5 + \&\text{c.} \right\}$$

$$7.\ \text{Log.}\ a = \log.\ (a-1) + \frac{1}{\text{M}} \times \left\{ \frac{1}{a} + \frac{1}{2\,a^2} + \frac{1}{3\,a^3} + \frac{1}{4\,a^4} + \&\text{c.} \right\}$$

$$8.\ \text{Log.}\ a = \log.\ (a-1) + \frac{1}{\text{M}} \times \left\{ \frac{1}{a-1} - \frac{1}{2\,(a-1)^2} + \frac{1}{3\,(a-1)^3} - \&\text{c.} \right\}$$

$$9.\ \text{Log.}\ a = \log.\ (a-1) + \frac{2}{\text{M}} \times \left\{ \frac{1}{a-1} + \frac{1}{3\,(a-1)^3} + \frac{1}{5\,(a-1)^5} + \&\text{c.} \right\}$$

$$10.\ \text{Log.}\ a = \frac{1}{\text{M}} \times \left\{ (a - a^{-1}) - \tfrac{1}{2}\,(a^2 - a^{-2}) + \tfrac{1}{3}\,(a^3 - a^{-3}) - \&\text{c.} \right\}$$

$$11.\ \text{Log.}\ (a+z) = \log.\ a + \frac{1}{\text{M}} \times \left\{ \frac{z}{a} - \tfrac{1}{2}\frac{z^2}{a^2} + \tfrac{1}{3}\frac{z^3}{a^3} - \tfrac{1}{4}\frac{z^4}{a^4} + \&\text{c.} \right\}$$

$$12.\ \text{Log.}\ (a-z) = \log.\ a - \frac{1}{\text{M}} \times \left\{ \frac{z}{a} + \tfrac{1}{2}\frac{z^2}{a^2} + \tfrac{1}{3}\frac{z^3}{a^3} + \tfrac{1}{4}\frac{z^4}{a^4} + \&\text{c.} \right\}$$

$$13.\ \text{Log.}\ (a \pm z) = \log.\ a \pm \frac{2}{\text{M}} \times \left\{ \left(\frac{z}{a+z}\right) + \tfrac{1}{3}\left(\frac{z}{a+z}\right)^3 + \tfrac{1}{5} \left(\frac{z}{a+z}\right)^5 + \&\text{c.} \right\}$$

$$14.\ \text{Log.}\ a = \frac{m}{\text{M}} \times \left\{ (m\sqrt{a}-1) - \tfrac{1}{2}\,(m\sqrt{a}-1)^2 + \tfrac{1}{3}\,(m\sqrt{a}-1)^3 - \&\text{c.} \right\}$$

LOGARITHMIC *Curve, Equation to, &c.—(Higman.)*

$$y = a^x.$$

The curve consists of one branch infinite on each side of the origin, to which the axis of abscissas is an asymptote.

If $x = o$, $y = 1$; and if $x = 1$, $y = a$.

If the abscissas increase in arithmetic progression, the ordinates increase in geometric.

The subtangent is a constant quantity, and = modulus of the system of logarithms, whose base $= a$.

Area between any two ordinates $y$ and $b = m\,(y - b)$, where $m$ is the modulus or subtangent.

$$\text{Content} = \frac{\pi m}{2}\,(y^2 - b^2).$$

$$\text{Arc} = \sqrt{(m^2 + y^2)} - \sqrt{(m^2 + b^2)}$$

$$+ m \log. \frac{b\left(\sqrt{(m^2 + y^2)} - m\right)}{y\left(\sqrt{(m^2 + b^2)} - m\right)}.$$

$$\text{Surface} = \pi\left(y\sqrt{(m^2 + y^2)} - b\sqrt{(m^2 + b^2)}\right.$$

$$\left. + m^2 \log. \frac{y + \sqrt{(m^2 + y^2)}}{b + \sqrt{(m^2 + b^2)}}\right).$$

**LOGARITHMIC** *Spiral.—See Spiral.*

**LONGITUDE** *Geographical.—(Woodhouse, Vince.)*

*1st Method, by a chronometer.*

Suppose a chronometer to be adjusted to mean solar time at Greenwich, then if its motion were equable, and of the proper rate, we should always know, whatever the place, the time at Greenwich. Compute ∴ the apparent, and by means of the equation of time, the mean time, at the place of observation. The difference between this latter time, and that shewn by the chronometer, would be the longitude, east or west of Greenwich.

*2d Method, by an eclipse of the moon or of Jupiter's satellites.*

Having the times calculated when the eclipse begins and ends at Greenwich, observe the times when it begins and ends at any other place; the difference of these times, converted into degrees, gives the difference of longitudes.

*3d. Method, by the moon's distance from the sun or a fixed star.*

The steps by which we find the longitude by this method are these:

From the observed altitudes of the moon and the sun or a fixed star, and their observed distance, compute the moon's true distance from the sun or star.

From the Nautical Almanack find the time at Greenwich when the moon was at that distance.

From the altitude of the sun or star, find the time at the place of observation.

The difference of the times thus found, gives the difference of the longitudes.

*Formula for deducing the true from the observed distance.*

Conceive S, M to be the true places of the star and moon in two vertical circles S Z, M Z forming at the zenith Z the $\angle$ M Z S; then since both parallax and refraction take place entirely in the direction of vertical circles, some point *s* above S, in the circle Z S, will be the apparent place of the star, and *m* below M (since in the case of the moon the depression by parallax is greater than the elevation by refraction) will be the apparent place of the moon: let

D (S M) be the true, *d* (*s m*) the apparent distance,

A, *a* ($90^0$ — Z M, $90^0$ — Z S) the true altitudes,

H, *h* ($90^0$ — Z *m*, $90^0$ — Z *s*) the apparent altitudes,

$$\text{then if } F = \frac{\cos. A. \cos. a}{\cos. H. \cos. h},$$

$$\sin.^2 \frac{D}{2} = \cos.^2 \tfrac{1}{2}(A + a)\left[1 - \frac{\cos. \frac{1}{2}(H + h + d) . \cos. \frac{1}{2}(H + h - d) . F}{\cos.^2 \frac{1}{2}(A + a)}\right]$$

or if we make the fraction, on the right hand side of the equation = $\sin.^2 \theta$, we shall have

$$\sin.^2 \frac{D}{2} = \cos.^2 \tfrac{1}{2}(A + a) . \cos.^2 \theta.$$

$$\text{and } \sin. \frac{D}{2} = \cos. \tfrac{1}{2}(A + a) . \cos. \theta.$$

The true distance of the moon from the sun or star being thus found, we are next to find the time at Greenwich corresponding to this true distance. To do this, we must observe that the true distance is computed in the Naut. Almanack for every three hours for the meridian of Greenwich. Hence considering that distance as varying uniformly, the time corresponding to any other distance may be thus computed. Look into the Naut. Almanack, and take out two distances, one next greater, and the other next less, than the true distance deduced from observation, and the difference D of these distances gives the access of the moon to, or recess from, the sun or star in three hours; then take the difference *d* between the moon's distance at the beginning of that interval, and the true distance deduced from observation, and then say, D : *d* :: 3 hours : the time the moon is acceding to or receding from the sun or star by the quantity $d$, which added to the time at the beginning of the interval, gives the apparent time at Greenwich corresponding to the given true distance of the moon from the sun or star.

Having thus found the time at Greenwich, compute the time at the place of observation from the corrected altitude of the sun or star, the sun's or star's north polar distance (furnished by Tables) and the latitude.

The difference between this latter time and the time at Greenwich, is the longitude.

The other methods of finding the longitude are, by an occultation of a fixed star by the moon; by a Solar eclipse; and by the passage of the moon over the meridian.

LOOKING *Glass, method of judging of.—(Coddington.)*

To find the thickness of a looking glass, bring a pin or other slender object into contact with the fore surface of the glass, and observe its image, as shown by reflection; then the thickness of the glass will be equal to $\frac{3}{4}$ths of the apparent distance between the objects and its image.

In a looking glass it is not only necessary that each plane should be perfect, but they must be also parallel to each other. If the images of a candle seen very obliquely, and under different degrees of obliquity, and from all parts of the glass, do not always keep pretty nearly at equal distances from one another, it is a proof that the sides of the glass are neither plane nor parallel.

Another method of trying the goodness of a glass is as follows:—Stick a pin or slender wire in the bar of a window sash, so that the pin may be nearly horizontal, and in the plane of the window. Then hold the looking-glass, and turn it about so as to see the image of the pin very obliquely and from all parts of the glass. In this case two images will be visible; and if these images keep always straight, parallel, and at regular distances one from another, the glass may be considered as being well figured. These phænomena will be more conspicuous if two pins be stuck parallel to one another, and at a small distance asunder.

With respect to the polish of a glass; we may observe, cæteris paribus, that the darker the colour of the glass of the speculum is, the better generally is the polish.

For the theory of plane mirrors—*see Reflection.*

LUNAR *inequalities.—See Moon.*

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# M

MACLAURIN'S *Theorem.—See Taylor's Theorem.*

MAGNETIC *Needle, variation and dip of.—See Variation.*

MARS.—*See Planets, elements of.*

MARS *phases of.—See Venus.*

MAXIMA *and Minima of quantities.*

1. To determine in what cases any quantity $y$, depending upon $x$, may become a maximum or minimum, we must find the differential of the equation which expresses the relation that they bear to each other, and make the quantity $\frac{dy}{dx} = o$. The resulting equation, combined with the original one, will give the values of $x$ and $y$ in which $y$ is a maximum or minimum.

2. To determine when $y$ is a maximum and when a minimum; find the value of $\frac{d^2y}{dx^2}$, and if it be negative, $y$ is a maximum; if it be positive, a minimum.

3. If $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both vanish, but $\frac{d^3y}{dx^3}$ remain, then $y$ will be neither a maximum or minimum at that place, but will pass through a point of contrary flexure parallel to the abscissa. In like manner, if $dy$, $d^2y$, $d^3y$ vanish, but $d^4y$ remain, the ordinate $y$ will be a maximum or mininum; and if $dy$, $d^2y$, $d^3y$, and $d^4y$ vanish, but $d^5y$ remain, it will pass through a point of contrary flexure, and so on alternately. This follows immediately from Taylor's theorem.

4. If a quantity be a maximum or minimum, any power or root, multiple, or part, of the original quantity, will be a max. or min.

*Ex.* 1. To divide a right line $a$ into two parts, such that their rectangle may be either a max. or min.

Here $ax - x^2 =$ maximum or minimum. Suppose $y = ax - x^2$, then $\frac{dy}{dx} = a - 2x = o$; $\therefore x = \frac{a}{2}$. To find whether this solution gives a max. or min., take the differential of the equation $\frac{dy}{dx} = a - 2x$; $\therefore$

$\frac{d^2 y}{d x^2} = -2$ a negative quantity; $\therefore$ the value $x = \frac{a}{2}$ gives a max.: also $y = \frac{a^2}{4}$.

*Ex.* 2. To divide a given line into two parts $x$ and $y$, so that $\frac{x}{y} + \frac{y}{x}$ = min. Here $x = y$.

*Ex.* 3. To inscribe the greatest rectangle in a given triangle and parabola.

Let $x$ = that part of the perpendicular measured from the vertex, which determines the required rectangle, $a$ = perpendicular, then

$$(1)\ x = \frac{a}{2}. \qquad (2)\ x = \frac{a}{3}.$$

*Ex.* 4. To inscribe the greatest cylinder in a given cone.

$$\text{Using the same notation } x = \frac{2a}{3}.$$

*Ex.* 5. To inscribe the greatest rectangle in a given ellipse.

Let $x$ = the part of the ½ ax. maj. measured from the centre, which determines the rectangle; then $x = \frac{a}{\sqrt{2}}$.

*Ex.* 6. To find $y$ a max. in the equation $(x^2 + y^2)^2 = a^4 x^2$.

$$\text{Here } x = \frac{a}{\sqrt{3}}, \text{ and } y = \sqrt[3]{\frac{2}{3\sqrt{3}}}.$$

When a quantity is a max. or min. it frequently shortens the operation to assume its logarithm a max. or min. Thus to find when $\sqrt{x^2 - ax + b} \times \sqrt[3]{m - x^3}$ is a max. or min. assume log. $\sqrt{x^2 - ax + b} \times \sqrt[3]{m - x^3}$ a max. or min. or log. $\sqrt{x^2 - ax + b}$ + log. $\sqrt[3]{m - x^3}$ max. or mix.; hence $\frac{1}{2} \times \frac{2x\,dx - a\,dx}{x^2 - ax + b} - \frac{1}{3}\,\frac{3x^2\,dx}{m - x^3} = o$.

6. If $a, b, c, d$, &c. be the real roots of the equation $\frac{dy}{dx} = o$, taken in the order of their magnitude, they will render $y$ a minimum and maximum alternately.

*Cor.* If there be $m$ roots equal to $a$, and $n$ roots equal to $b$, then there will be one minimum value of $y$ for the root $a$, and one maximum for $b$, if $m$ and $n$ be odd; and neither max. nor min. values when they are even.

**MEASURES.**—*See Weights and Measures.*

MECHANICAL *Powers.*

The simple mechanical powers into which more complex machines are resolved, are these :—1. The Lever; 2. The Wheel and Axle; 3. The Pulley; 4. The Inclined Plane; 5. The Wedge; 6. The Screw; for which see the respective heads.

The following property is common to all the mechanical powers, and indeed to all machines whatsoever; it is known by the name of the principle of *Virtual velocities.*

If a power and weight sustain each other on any machine, and the whole be put in motion, the velocity of the power : the velocity of the weight :: the weight : the power. In other words, if the equilibrium of a machine be disturbed by a quantity indefinitely small, and if the velocity of each force be multiplied into its quantity, the sum of these products, reckoning the forces which are in opposite directions positive and negative with respect to one another, will be equal to nothing.

MERCURY.—*See Planets, elements of.*

MERCURY, *Phases of.—See Venus.*

MERCURY, *Transit of.—See Transit.*

MERIDIAN, *Transit of a star or planet over.—See Time.*

MERIDIAN, *to place a Telescope in.—See Telescope.*

MERIDIONAL *parts.—See Projection.*

METEOROLOGY.—*See Atmosphere; Rain.*

MICROSCOPE.—*(Wood, Coddington.)*

1. The visual angle of an object, when seen through a single microscope, is to its visual angle, when seen with the naked eye at the least distance of distinct vision, as that least distance to the focal length of the glass.

Let $c$ = least distance of distinct vision, which in common eyes is about 7 or 8 inches, F = the focal length of the lens; then

$$\text{Magn. power} = \frac{c}{F}.$$

*Ex.* Let $F = .02$ inch, and $c = 7$ inches, then the number of times that the length of the object is magnified $= \frac{7}{.02} = 350$; and the number of times the surface is magnified $= 122500$.

2. Required the same in the double microscope.

Let $\Delta$ = distance of the object from the object glass, F and $f$ = focal lengths of the object and eye glasses, $c$ as before, then*

$$\text{Magn. power} = \frac{c\,F}{\Delta\,f}.$$

**MILL,** *Water.—See Wheel.*

**MILL,** *Wind.—See Windmill.*

**MODULUS.**—*See Logarithms.*

**MODULUS** *of Elasticity.—See Elastic bodies, equilibrium of.*

**MOMENTUM,** *and the moving or motive force which produces it.*

Let M be the momentum or the moving force which produces it; Q the quantity of matter moved; A the accelerating force; then

$$\text{M varies as } Q \times A, \text{ or A varies as } \frac{M}{Q}.$$

*Cor.* Since A varies as $\frac{V}{T}$, or varies as $\frac{V^2}{S}$ we have

$$\text{M varies as } \frac{Q \times V}{T} \text{ or varies as } \frac{Q \times V^2}{S}.$$

∴ when T is given, M varies as $Q \times V$, and when S is given, M varies as $Q \times V^2$ i.e. the moving force, estimated by its effect produced in a *given time*, is as $Q \times V$; but when estimated by its effect produced through a *given space*, is as $Q \times V^2$.

**MONEY.**—*(Enc. Brit. Supp.)*

*Principal gold coins of different countries, with their value in sterling nearly.*

| | | | STERLING. s. | d. |
|---|---|---|---|---|
| AUSTRIA ......... | Souverain . . . . . . . . . . | | 13 | 11 |
| | Ducat, coins of this name are current in Bavaria, Bern, Brunswick, Cologne, Denmark, Francfort, Hamburgh, Hanover, Holland, Poland, Prussia, Russia, Saxony, Sweden, Treves, Wirtemberg, and Zurich; and vary in sterling value . . | from<br>to | 9<br>9 | 2<br>6 |
| BAVARIA ......... | Carolin . . . . . . . . . . . | | 20 | 4 |
| | Max. d'or, or Maximilian . . . . . | | 13 | 7 |
| BERN ............. | Pistole, also used in Brunswick . . . . | | 18 | 8 |
| DENMARK ...... | Christian d'or . . . . . . . . . | | 16 | 6 |
| ENGLAND ....... | Guinea . . . . . . . . . . | | 21 | 0 |
| | Sovereign . . . . . . . . . . | | 20 | 0 |

| | | s. | d. |
|---|---|---|---|
| **FRANCE** ……… | Louis, since 1786 | 18 | 10 |
| | Napoleon, or piece of 20 francs | 15 | 10 |
| | New Louis | ditto. | |
| **GENEVA** ……… | Pistole, old | 16 | 4 |
| | Pistole, new | 14 | 2 |
| **GENOA** ……… | Sequin | 9 | 5 |
| **HANOVER** ……… | George d'or | 16 | 5 |
| | Gold florin | 6 | 11 |
| **HOLLAND** ……… | Ryder | 24 | 10 |
| **MALTA** ……… | Louis | 19 | 1 |
| **MILAN** ……… | Sequin | 9 | 5 |
| | Doppia or pistole | 15 | 8 |
| | 40 lire piece of 1808 | 31 | 10 |
| **NAPLES** ……… | Two ducat piece or sequin | 6 | 7 |
| **NETHERLANDS** | Gold lion, or 14 florin piece | 20 | 9 |
| **PARMA** ……… | Pistole, or doppia of 1796 | 17 | 0 |
| | Maria Theresa of 1818 | 15 | 10 |
| **PERSIA** ……… | Tomaun | 10 | 0 |
| **PIEDMONT** ……… | Pistole since 1785 | 22 | 3 |
| | Sequin | 9 | 4 |
| | Marengo (20 francs) | 14 | 8 |
| **PORTUGAL** ……… | Dobra of 12,800 rees | 71 | 1 |
| | Moidore | 26 | 11 |
| | Piece of 16 testoons, or 1600 rees | 8 | 9 |
| | New crusado of 480 rees | 2 | 7 |
| **PRUSSIA** ……… | Frederick of 1800 | 16 | 3 |
| **ROME** ……… | Sequin since 1760 | 9 | 3 |
| | Scudo of the Republic | 64 | 11 |
| **RUSSIA** ……… | Ruble of 1799 | 3 | 0 |
| | Gold poltin of 1777 | 1 | 5 |
| | Imperial of 1801 | 32 | 2 |
| **SARDINIA** ……… | Carlino | 30 | 8 |
| **SAXONY** ……… | Augustus of 1784 | 16 | 4 |
| **SICILY** ……… | Ounce of 1751 | 10 | 4 |
| **SPAIN** ……… | Doubloon of 1772 | 65 | 10 |
| | Pistole of 1801 | 15 | 11 |
| | Coronilla, gold dollar or vintem of 1801 | 4 | 0 |
| **SWITZERLAND** | Pistole of 1800 | 18 | 9 |
| **TURKEY** ……… | Sequin fonducli of 1789 | 7 | 7 |
| | Half misseir of 1818 | 2 | 2 |
| | Yermeebeshlek | 12 | 5 |

| | | s. | d. |
|---|---|---|---|
| **Tuscany** | Zecchino or sequin | 9 | 6 |
| | Ruspone | 28 | 6 |
| **United States** | Eagle | 43 | 7 |
| **Venice** | Zecchino or sequin | 9 | 6 |
| **Wirtemberg** | Carolin | 20 | 1 |
| **East Indies** | Rupee of Bombay and Madras | 29 | 2 |
| | Pagoda, Star | 7 | 5 |

*Principal silver coins of different countries, with their value in sterling nearly.*

| | | | | |
|---|---|---|---|---|
| **Austria** | Rix-dollar, coins of the same name are current in Baden, Bavaria, Brunswick, Denmark, Hamburgh, Hanover, Hesse Cassel, Holland, Lubec, Poland, Prussia, Saxony, Spain, Sweden, Switzerland, United States, Wirtemberg; and vary in sterling value | from<br>to | 4<br>4 | 2<br>8 |
| | Copftsuck, or 20 creutzer piece | | 0 | 3 |
| | 17 creutzer piece | | 0 | 7 |
| **Bern** | Patagon or crown | | 4 | 9 |
| | Piece of 10 batzen | | 1 | 2 |
| **Bremen** | Piece of 48 grotes | | 2 | 4 |
| **Brunswick** | Guilder of 1795 | | 2 | 4 |
| **Denmark** | Rykadaler of 1798 | | 4 | 6 |
| | Mark, or ⅙ rykadaler | | 0 | 7 |
| | Piece of 24 skillings | | 0 | 10 |
| **England** | Crown (old) | | 5 | 0 |
| | Shilling | | 1 | 0 |
| | Crown (new) | | 4 | 8 |
| | Shilling | | 0 | 11 |
| **France** | Ecu of 6 livres | | 4 | 8 |
| | Piece of 24 sous | | 1 | 0 |
| | Piece of 30 sous | | 1 | 2 |
| | Franc and Franc (Louis) | | 0 | 10 |
| **Geneva** | Patagon | | 4 | 1 |
| | Piece of 15 sous of 1794 | | 0 | 5 |
| **Genoa** | Scudo | | 5 | 4 |
| **Hamburgh** | Piece of 8 schillings | | 0 | 7 |
| **Hanover** | Florin | | 2 | 4 |
| **Hesse Cassel** | Florin | | 2 | 1 |

| | | *s.* | *d.* |
|---|---|---|---|
| HESSE CASSEL | Ecu (1815) | 4 | 1 |
| | Bon gros | 0 | 1 |
| HOLLAND | Ducatoon | 5 | 6 |
| | Florin or guilder | 1 | 8 |
| | 12 stiver piece | 1 | 1 |
| LUBECK | Mark | 1 | 3 |
| LUCCA | Scudo | 4 | 4 |
| | Barbone | 0 | 4 |
| MALTA | Ounce of 30 tari | 3 | 11 |
| MILAN | Scudo | 3 | 9 |
| | Lira | 0 | 7 |
| | Piece of 30 soldi | 0 | 11 |
| MODENA | Scudo of 1796 | 3 | 4 |
| NAPLES | Ducat | 3 | 5 |
| | Piece of 12 Carlini | 4 | 2 |
| NETHERLANDS | Ducatoon of Maria Theresa | 5 | 2 |
| | Crown | 4 | 7 |
| | 5 stiver piece | 0 | 4 |
| | Florin of 1816 | 1 | 9 |
| PARMA | Ducat of 1796 | 4 | 2 |
| | Piece of 3 lire | 1 | 1 |
| PERSIA | Real | 1 | 3 |
| PIEDMONT | Scudo (1770) | 5 | 8 |
| | 5 franc piece | 4 | 0 |
| POLAND | Florin or gulder | 1 | 0 |
| PORTUGAL | New crusado (1809) | 2 | 5 |
| | Seis vintems, or 120 rees (1802) | 0 | 6 |
| | Testoon (1802) | 0 | 6 |
| PRUSSIA | Florin | 2 | 4 |
| | Florin of Silesia | 2 | 0 |
| | Drittel, or 8 good groschen | 1 | 0 |
| ROME | Scudo (1799) | 4 | 3 |
| | Testone (1785) | 1 | 3 |
| | Paolo (1785) | 0 | 5 |
| RUSSIA | Ruble (1805) | 3 | 3 |
| | 10 copeck piece (1802) | 0 | 4 |
| SARDINIA | Scudo or crown | 3 | 9 |
| SAXONY | Piece of 16 groschen | 2 | 0 |
| | ⅙ thaler (1809) | 0 | 6 |
| SICILY | Scudo | 4 | 1 |
| | Piece of 40 grains | 1 | 4 |

| | | s. | d. |
|---|---|---|---|
| **SPAIN** ............ | Peceta of 2 reals of new plate (1775) | 0 | 10 |
| | Real of new plate | 0 | 5 |
| **SWITZERLAND** | Ecu of 40 batzen (1796) | 4 | 9 |
| | Florin of 40 schillings (1793) | 1 | 1 |
| **TURKEY** ........ | Piastre of Selim (1801) | 1 | 1 |
| | Piastre (1818) | 0 | 9 |
| **TUSCANY** ....... | Piece of 10 Paoli (1801) | 4 | 5 |
| | Scudo Pisa (1803) | 4 | 6 |
| | Lira (1803) | 0 | 7 |
| **UNITED STATES** | Dime, or one-tenth of a dollar (1796) | 0 | 6 |
| **VENICE** .......... | Piece of 2 lire | 0 | 4 |
| **WIRTEMBERG** | Copftsuck | 0 | 8 |
| **EAST INDIA** ... | Rupee of Sicca and Calcutta | 2 | 0 |
| | ——— Bombay, new, or Surat | 1 | 11 |
| | Fanam, Cananore, Bombay | 0 | 5 |
| | ——— Pondicherry | 0 | 3 |
| | Gulden of the Dutch East India Company | 1 | 9 |

*Weight of English gold and silver coin.*

| | DWTS. | GR. |
|---|---|---|
| Guinea | 5 | $9\frac{39}{89}$. |
| Sovereign | 5 | $3\frac{171}{623}$. |
| Half guinea | 2 | $16\frac{64}{89}$. |
| Angel or ½ Sovereign | 2 | $13\frac{397}{623}$. |
| Crown | 18 | $4\frac{4}{11}$. |
| Half-Crown | 9 | $2\frac{2}{11}$. |
| Shilling | 3 | $15\frac{3}{11}$. |
| Sixpence | 1 | $19\frac{7}{11}$. |

MONSOON.—*See Wind.*

MOON, *elements and principal phænomena of.—(Vince, Playfair.)*

Secular motion for 100 years, of which 25 are bissextiles $10^{s}\,7^{\circ}\,53'\,12''$

| | |
|---|---|
| Secular motion of the apogee | $3^s\ 19^\circ\ 11'\ 15''$ |
| Secular motion of the node | 4 14 11 15 |
| Epoch of the mean longitude for 1750 | 6 8 22 20 |
| Epoch of the longitude of the apogee for 1750 | 5 20 54 56 |
| Epoch of the longitude of the node for 1750 | 9 10 19 59 |
| Mean equation of the orbit | 6 18 32 |

| | *y.* | *d.* | *h.* | *m.* | *s.* |
|---|---|---|---|---|---|
| Tropical revolution | | 27 | 7 | 43 | 4,7 |
| Sidereal revolution | | 27 | 7 | 43 | 11,5 |
| Synodic revolution | | 29 | 12 | 44 | 2,8 |
| Anomalistic revolution | | 27 | 13 | 18 | 33,9 |
| Revolution in respect to the node | | 27 | 5 | 5 | 35,6 |
| Tropical revolution of the apogee | 8 | 311 | 8 | 34 | 57,6 |
| Sidereal revolution of do. | 8 | 312 | 11 | 11 | 39.4 |
| Tropical revolution of the node | 18 | 228 | 4 | 52 | 52,0 |
| Sidereal revolution of do. | 18 | 223 | 7 | 13 | 17,7 |

| | |
|---|---|
| Diurnal motion of the moon in respect to the Equinox | $13^\circ\ 10'\ 35{,}0''$ |
| Diurnal motion of the apogee | 0 6 41,1 |
| Diurnal motion of the node | 0 3 10,6 |
| Inclination of orbit to Ecliptic | 5 9 0 |
| Inclination of axis to orbit | 88 17 0 |
| Mean apparent diameter as seen from earth | 31 8 |
| ——————————— sun | 4,6 |
| Greatest parallax | 1 1 22,99 |
| Least ——— | 53 50,99 |
| Greatest distance from earth in ½ diam. of earth | 63,8419 |
| Least ——————————— | 55,9164 |
| Mean distance ——————————— | 59,8791 |
| Eccentricity, mean distance being 1 | ,05518 |
| Mean diameter in miles | 2180 |
| Density (earth's density being 1) | 0,6149 |
| Quantity of matter (earth's being 1) | 0,01245 |
| Gravity at surface (earth's being 1) | 0,1677 |

The least difference between the times of the moon's rising on two successive nights in the latitude of London is 17*m*; and the greatest difference 1*h*. 17*m*.

MOON, *inequalities affecting the orbit of, usually called the Lunar inequalities.*

Of these the following are the most important:—

1. Evection, or a correction applied to the equation of the centre,

arising from an increase of the eccentricity of the moon's orbit at the quadratures, and a diminution of it at the syzygies. Let M and S be the mean longitudes of the moon and sun, $x$ the mean anomaly of the moon, then the evection is

$$1^{\circ}.\ 21'.\ 5'',5 \times \sin.\ \left(2\,(M - S) - x\right).$$

The evection runs through all its changes in 31*d*. 19*h*. 28*m*. nearly. It is called the second lunar inequality, the equation of the centre being the first.

2. Variation.—i.e. a variation in the moon's velocity, which is nothing in syzygy and quadratures, and greatest at the octants. It =

$$35'.\ 42'' \times \sin.\ 2\,(M - S).$$

Its period is half a lunar month. This is the third lunar inequality.

3. The annual equation or fourth lunar inequality, is an irregularity in the moon's motion, arising from the variation of the sun's distance from the earth. It is

$$11'.\ 11'',9 \times \sin.\ \text{mean anomaly of sun.}$$

Its period is an anomalistic year.

These three inequalities were known before Newton's time: they are applied as corrections to the equation of the centre in determining the moon's longitude.

Other inequalities there are which have a much longer period; one for instance, discovered by Laplace, depending upon the position of three lines, the axis of the moon's orbit, the axis of the earth's orbit, and the line of the moon's nodes which takes up a period of 85 years, and amounts to

$$14'' \times \sin.\ (2\ \text{longitude node} \times \text{longitude perigee of moon} - 3\ \text{longitude perigee of sun}).$$

Others again there are which do not run through the circle of their changes but in the course of several thousand years, and are usually expounded by their aggregate in 100 years. The moon's nodes, the apogee, the eccentricity, the inclination of the orbit, the moon's mean motion, are all subject to secular inequalities. Of these, the most remarkable is the acceleration of the moon's mean motion (depending on a change in the eccentricity of the earth's orbit), by which her velocity continually encreases, and periodic time decreases from age to age. For many ages to come, it may be nearly expressed by this formula, where $n$ denotes the number of centuries from 1700.

$$10''.\ 181621268\, n^2 + 0''.\ 0185384408\, n^3.$$

The tables of the moon's motion contain at present 28 equations for the moon's longitude, 12 for her latitude, and 13 for the horizontal parallax; their greatest probable error does not exceed twelve seconds.

MOON, *libration of.*

The libration in longitude, or the alternate appearance and disappearance of small segments of the moon on the east and west limbs, arises from the uniform angular velocity round her axis, and the variable angular velocity in her orbit. It nearly = the equation of the orbit, or about $7\frac{1}{2}^0$ at its maximum.

The libration in latitude, or the alternate appearance and disappearance of the north and south poles, arises from the moon's axis not being perpendicular to the plane of her orbit.

Diurnal libration, or the appearance of a small segment of the western limb at the rising, and of the eastern limb at the setting of the moon, arises from the spectator being situated on the surface, instead of at the centre of the earth.

MOON, *augmentation of diameter of.*

When the moon is at different altitudes above the horizon, it is at different distances from the spectator, and therefore there is a change of the apparent diameter. Let the altitude of the moon reckoned from the earth's centre or true altitude $= a$; apparent altitude, or altitude reckoned from the earth's surface $=$ A; then increase of ½ diameter =

$$\text{Hor. } \tfrac{1}{2} \text{ diam.} \times \frac{\sin.\left(\frac{1}{2}a + \frac{1}{2}A\right) \times \sin.\left(\frac{1}{2}a - \frac{1}{2}A\right)}{\cos.\, a}.$$

Hence the following Table :—

*Augmentation of moon's semi-diameter,*

| *Altitude* | *Augmentation.* | *Altitude* | *Augmentation.* |
|---|---|---|---|
| 0° — | 0″ | 40° — | 10″ |
| 5 — | 1 | 45 — | 11 |
| 10 — | 3 | 50 — | 12 |
| 15 — | 4 | 55 — | 13 |
| 20 — | 6 | 60 — | 14 |
| 25 — | 7 | 70 — | 15 |
| 30 — | 8 | 80 — | 15 |
| 35 — | 9 | 90 — | 16 |

**MOON,** *Harvest.—See Harvest Moon.*

**MOON'S** *mean age to find.*

The moon's mean age at any time may be found for the next 50 years, by the following Tables :—

### TABLE I.

*Epacts of Years.*

| *Years.* | *Epacts.* | *Years.* | *Epacts.* | *Years.* | *Epacts.* |
|---|---|---|---|---|---|
| 1827 | 2*d* 8*h* 24′ | B 1844 | 10*d* 22*h* 14′ | 1861 | 18*d* 12*h* 4′ |
| B 1828 | 13 23 35 | 1845 | 21 13 26 | 1862 | 29 3 16 |
| 1829 | 24 14 47 | 1846 | 2 15 53 | 1863 | 10 5 43 |
| 1830 | 5 17 14 | 1847 | 13 7 4 | B 1864 | 21 20 55 |
| 1831 | 16 8 26 | B 1848 | 24 22 16 | 1865 | 2 23 22 |
| B 1832 | 27 23 37 | 1849 | 6 0 43 | 1866 | 13 14 33 |
| 1833 | 9 2 4 | 1850 | 16 15 55 | 1867 | 24 5 45 |
| 1834 | 19 17 16 | 1851 | 27 7 6 | B 1868 | 6 8 12 |
| 1835 | 0 19 43 | B 1852 | 9 9 34 | 1869 | 16 23 24 |
| B 1836 | 12 10 55 | 1853 | 20 0 45 | 1870 | 27 14 36 |
| 1837 | 23 2 6 | 1854 | 1 3 12 | 1871 | 8 17 3 |
| 1838 | 4 4 34 | 1855 | 11 18 24 | B 1872 | 20 8 14 |
| 1839 | 14 19 45 | B 1856 | 23 9 35 | 1873 | 1 10 42 |
| B 1840 | 26 10 56 | 1857 | 4 12 2 | 1874 | 12 1 53 |
| 1841 | 7 13 24 | 1858 | 15 3 14 | 1875 | 22 17 4 |
| 1842 | 18 4 35 | 1859 | 25 17 26 | B 1876 | 4 20 32 |
| 1843 | 28 19 47 | B 1860 | 7 20 53 | | |

### TABLE II.

*Epacts of Months.*

| *Months.* | *Epacts.* | *Months.* | *Epacts.* |
|---|---|---|---|
| January | 0*d* 0*h* 0′ | July | 3*d* 19*h* 36′ |
| February | 1 11 16 | August | 5 6 52 |
| March | 29 11 16 | September | 6 18 8 |
| April | 1 9 48 | October | 7 5 24 |
| May | 1 21 4 | November | 8 16 40 |
| June | 3 8 20 | December | 9 3 55 |

**In Leap Years, a day is to be subtracted from the sum of the Epacts, in the months of January and February.**

**Rule.—Add the Epacts of the given year and month, and the proposed time reduced to the meridian of Greenwich. If this sum exceeds a mean lunation or 29*d.* 12*h.* 44*m.*, deduct it therefrom, and the remainder is the moon's mean age.**

MOON's *time of passing Meridian.—See Time.*

**MOON** *Eclipses of.—See Eclipses.*

**MOTION** *accelerated.—( Whewell.)*

Formulæ for accelerated motion, whether the body is acted upon by constant or variable forces, gravity being represented by $g = 32\frac{1}{6}$ feet, its effect produced in 1 second.

*Force constant.*

$$v = ft \;\ldots\ldots\ldots\ldots\ldots\ldots\; s = \tfrac{1}{2} f t^2.$$

From these two equations we obtain, by simple eliminations, the following results:

$$s = \tfrac{1}{2} f t^2 = \tfrac{1}{2} t v = \frac{v^2}{2f};$$

$$v = f t = \frac{2s}{t} = \sqrt{(2fs)};$$

$$t = \frac{v}{f} = \frac{2s}{v} = \sqrt{\frac{2s}{f}};$$

$$f = \frac{v}{t} = \frac{v^2}{2s} = \frac{2s}{t^2}.$$

If gravity be the constant force, substitute $g$ for $f$ in the above formulæ.

Let a body be projected with a given velocity $u$, and acted on in the same direction by a constant force $f$; it is required to determine the relation of the space, time, and velocity.

Let $s$ be the space described in the time $t$, then

$$s = tu + \tfrac{1}{2} f t^2;$$
$$v = u + ft.$$

If the body is projected in a direction opposite to that in which the force acts;

$$s = tu - \tfrac{1}{2} f t^2;$$
$$v = u - ft.$$

*Ex*. 1. Space described by gravity in 10″ = 1608⅓ feet; and velocity acquired = 321⅔ feet.

*Ex*. 2. A body is projected upwards with a velocity of 100 feet; to find how high it will ascend in 2″.

$$s = tu - \tfrac{1}{2} g t^2 = 2 \times 100 - \tfrac{1}{2} \times 32.2 \times 4 = 135.6 \text{ feet.}$$

*Ex*. 3. Spaces described by gravity in the 1st, 2d, 3d, &c. seconds are $\frac{g}{2}, \frac{3g}{2}, \frac{5g}{2}, \frac{7g}{2}$ &c.

*Ex.* 4. If a body has fallen $t''$, the space described in the last $n'' = \frac{g}{2}(2nt - n^2)$.

*Ex.* 5. Space described in the second immediately previous to the last $n'' = \frac{g}{2}(2t - 2n - 1)$.

### *Force variable.*

$$v = \frac{ds}{dt} \ \ldots\ldots\ldots\ldots\ldots\ldots \ f = \frac{dv}{dt}.$$

When the force is a function of the space, we may hence find the velocity and time. For multiplying the above equations crossways, to eliminate $dt$,

$$v\,dv = f\,ds.$$

If we put for $f$ its value in terms of $s$, we can integrate the last equation, and thus obtain $\frac{1}{2}v^2 = \text{fl.}\, f\,ds$, whence $v$ is known.

After this $t$ is determined by the equation

$$dt = \frac{ds}{v}. \quad \therefore t = \text{fl.}\,\frac{ds}{v}.$$

*Ex.* 1. Let a body fall from rest from the distance $a$ towards a centre of force varying directly as the distance; to determine the motion.

Let $m$ be the absolute force tending to the centre, then

$$f = mx, \therefore v\,dv = mx\,ds = -mx\,dx \therefore v^2 = C - mx^2 = m(a^2 - x^2).$$

$$\text{And } v = m^{\frac{1}{2}}(a^2 - x^2)^{\frac{1}{2}}.$$

$$dt = \frac{ds}{v} = -\frac{dx}{m^{\frac{1}{2}}(a^2 - x^2)^{\frac{1}{2}}}$$

$$\therefore t = \frac{1}{m^{\frac{1}{2}}}.\ \text{arc}\left(\cos. = \frac{x}{a}\right)$$

*Ex.* 2. Required the same when $f$ varies as $\frac{1}{D^2}$.

$$\text{Here } v\,dv = -\frac{m\,dx}{x^2}$$

$$\therefore v = \sqrt{\frac{2m}{a}} \times \sqrt{\frac{a-x}{x}}.$$

Also $dt = -\frac{dx}{v} = \sqrt{\frac{a}{2m}} \times \frac{-x^{\frac{1}{2}}\,dx}{\sqrt{(a-x)}} \quad \therefore t = \sqrt{\frac{a}{2m}} \times$ (arc $+$ sin.) whose ver. sin. is $a - x$.

*Ex*. 3. Required the same when the force is as $\frac{1}{D^n}$.

$$v\,dv = f\,ds = -\frac{m\,dx}{x^n}.$$

$$v = \sqrt{\frac{2m}{n-1}} \cdot \sqrt{\frac{a^{n-1} - x^{n-1}}{a^{n-1}\,x^{n-1}}}$$

$$dt = \sqrt{\frac{(n-1)\,.\,a^{n-1}}{2m}} \times \frac{-x^{\frac{n-1}{2}}\,dx}{\sqrt{\left(a^{n-1} - x^{n-1}\right)}},$$ which can be integrated only in particular cases.

MOVING *Force.—See Momentum.*

MOUNTAINS, *height of the principal, from the best authorities; together with the rocks of which they are composed.*

| | | FEET. |
|---|---|---|
| Dhawalageri (Napaul) slaty primitive rocks, as gneiss, mica slate, scherl rock; details unknown | | 27,677 |
| 36 Peaks of the Himalaya mountains, observed and calculated by Captain Hodgson | Do. from<br>to | 25,749<br>17,017 |
| Jamaturi (Napaul) do. | do. | 25,500 |
| Chimborazo (highest of the Andes) | All these high summits of the Andes are of volcanic matter resting on primitive rocks, such as gneiss, mica slate, &c. | 21,470 |
| Cajambe (Quito Andes) | | 19,480 |
| Antisana (highest volcano, Andes) | | 19,150 |
| Cotopaxi (volcano, Andes) | | 18,875 |
| Mount St Elie | | 18,090 |
| Popocatepetl (volcano of Puebla Mexico) | | 17,720 |
| Cotocatche (Andes) | | 16,450 |
| Tonguragua (volcano, Quito) | | 16,270 |
| Mouna Roa (Owhyhee), volcanic from top to bottom; all the rocks of the island are igneous | | 15,871 |
| Mont Blanc (Alps, highest in Europe) granite, syenite, hornblende slate, in vertical layers | | 15,665 |

| | |
|---|---|
| Mont Rosa (Alps), talc slate and serpentine | 15,527 |
| Ortler Spiltze (Tyrol) alpine or Jura limestone, with organic remains | 15,430 |
| Mount Cervin (Switzerland) primitive slaty rocks | 14,780 |
| Mount Ophir (Sumatra) | 13,842 |
| Peak of Jungfraa (Switzerland) alpine limestone | 13,735 |
| Pambamarca (Andes) | 13,500 |
| Bræt-horn (Switzerland) granite and gneiss | 12,815 |
| Sochonda (China) primitive, probably granite | 12,800 |
| Finisteraaharn (Alps) granite and gneiss | 12,210 |
| Lake of Toluca (Mexico) | 12,195 |
| Peak of Teneriffe, volcanic from top to bottom | 12,176 |
| Town of Micupampa (Peru) | 11,670 |
| Mulahacen (Spain) | 11,670 |
| Peak of Venlatta (Spain) | 11,390 |
| Mont Perdu (Pyrenees) calcareous, with organic remains | 11,265 |
| Le Viguemal (do.) summit granite, flanks calcareous | 11,010 |
| Mount Ætna (Sicily) volcanic ashes, scoria, lava, &c. | 10,955 |
| Italitzkoi (Altaic chain, Asia) | 10,735 |
| Pic Blanc (Alps) | 10,205 |
| Quito | 9,630 |
| Awatsha (volcano, Kamtchatka) | 9,600 |
| Mount Libanus (Turkey) | 9,585 |
| Real del Monte (mine, N. Spain) poryhyry slate | 9,125 |
| Imbabura (Quito) a great volcanic dome resting on primitive rocks | 8,960 |
| Mont St Gothard (Alps) granite and gneiss | 8,930 |
| Peak of Lomnitz (Carpathian Mountains) primitive rocks, details unknown | 8,640 |
| Mount Valina (highest of the Apennines) | 8,300 |
| Sneebutten (Norway) gneiss, mica slate, and other primitive slates | 8,295 |
| Blue Mountains (Jamaica) | 8,180 |
| Volcano, Isle of Bourbon | 7,680 |

| | |
|---|---|
| **Mexico** | 7,525 |
| **Mount Cenis (Alps) transition slate, &c.** | 6,780 |
| **Mount Olympus (Turkey) primitive limestone, with serpentine, syenite, porphyry** | 6,500 |
| Rocky = **Stony Mountains (N. America)** | 6,250 |
| **Mont d'Or (France)** | 6,130 |
| **Roettruck (Sweden) gneiss and mica slate** | 6,000 |
| **Mount Reculet (Switzerland)** | 5,500 |
| **Puy de Dome (France) ancient volcanic rocks (trachyte)** | 5,225 |
| **La Souffriere (Guadaloupe) volcanic** | 5,110 |
| **Hecla (Iceland) volcanic from top to bottom, scoria, lava, tuff, porphyry, slate, &c.** | 5,010 |
| **Mount Ida (Turkey)** | 4,960 |
| **Ben Nevis (highest in Britain) feldspathic slate, greenstone slate, &c.** | 4,370 |
| **Jorulla (volcano, Mexico) volcanic scoria** | 4,265 |
| **Ben Lawers (Scotland)** | 4,015 |
| **Mount Vesuvius (Italy) volcanic ashes, scoria, lava** | 3,935 |
| **Ben Wyvis (Scotland)** | 3,720 |
| **Snowden (highest in Wales) transition slate, with organic remains, greenstone slate, &c.** | 3,571 |
| **Town of Caracas** | 3,490 |
| **Ben Lomond (Scotland) feldspathic slate, greenstone slate, &c.** | 3,250 |
| **Sca Fell (Cumberland) chloritic slate, greenstone slate, compact felspar, &c.** | 3,166 |
| **Helvellyn do.** | 3,055 |
| **Skiddaw, clay slate, with crystals of chiastolite** | 3,022 |
| **Cadir Idris, compact felspar, greenstone slate, &c.** | 2,914 |
| **Cross Fell, mountain limestone, and millstone grit.** | 2,901 |
| **Cheviot, porphyry** | 2,658 |
| **Plynlimmon** | 2,463 |
| **Whernside (Ingleton Fells) mountain limestone, gritstone, &c.** | 2,384 |

| | | |
|---|---|---|
| Ingleborough, mountain limestone, gritstone, &c. | | 2,361 |
| Madrid | | 2,276 |
| Pennigent | do. | 2,270 |
| Whernside (Kettlewell) | do. | 2,263 |
| Fountains Fell | do. | 2,180 |
| Snea Fell (Isle of Man) clay slate | | 2,004 |
| Pendle Hill, mountain limestone, millstone, grit. | | 1,803 |
| Rye Loaf Hill | do. | 1,753 |
| Malvern Hills, Syenitic granite | | 1,444 |
| Cataract of Tequendama (S. America) | | 600 |
| St Peter's, summit of the Cross | | 535 |
| Pyramids (Egypt) 457 | | 500 |
| Natural Bridge of Iconozo (S. America) | | 300 |
| Caspian sea below the ocean 83·7 | | 306 |
| Gay Lussac—highest altitude ever attained by a balloon, Sept. 16, 1804 | | 22,900 |
| Highest flight of the Condor | | 21,000 |
| Height attained by Humboldt up the Andes, June 23, 1802 | | 19,400 |
| Highest limit of lichen plants | | 18,225 |
| Lower limit of perpetual snow on the Equator | | 15,730 |
| Farm House of Antisana | | 13,435 |
| Highest limit of trees | | 11,125 |
| Superior limit of oaks in the torrid zone | | 10,500 |
| Convent on Mont St Bernard (Switzerland) | | 8,040 |
| Do. of St Gothard (Alps) | | 6,810 |

**MOUNTAINS,** *visibility of.—See Refraction.*

**MOUNTAINS,** *attraction of.*

Dr Maskelyne was the first who satisfactorily proved the attraction of mountains by their effect in drawing the plumb line from its vertical direction. The mountain selected was Schehallien in Scotland, the mean height of which above the surrounding valley is 2000 feet, and above the level of the sea 3550. The attraction of this mountain was found = 5″,8: from which Dr Hutton calculated the mean density of the earth to be near 5 times that of water, or as 99 to 20, and almost double the density of rocks near the earth's surface. Mr Cavendish, upon totally different principles, found the density of the earth to be to that of water as 5.48 : 1. The internal parts of the earth are ∴ much denser than those at the surface; though in what manner the dense parts are disposed of must be uncertain.

MOUNTAIN, *correction for height of.—See Refraction.*

MOUNTAINS, *visibility of.—See Refraction.*

# N

NEBULÆ, *and Clusters of Stars.—(Herschel.)*

*On Herschel's Catalogue of new Nebulæ, and clusters of stars.*

The telescope used was a Newtonian reflector of 20 feet focal length, and $18\frac{7}{10}$ inches aperture. The sweeping power was 157. The field of view 15′. 4″.

The Nebulæ are divided into classes like the double stars. *(See Stars double.)* Thus in the 1st class, the degree of brightness of the Nebulæ has been the leading feature, as most likely to point out those which ordinary instruments may be expected to reach. The 1st class ∴ contains the brightest of them; the 2d those which shine but with a feeble light; and in the 3d are placed all the very faint ones. It should be observed, that what Herschel calls bright, or very bright among those of the first class, are commonly less distinguishable than what Messier, in his *Catalogue des Nebules* (given in Wollaston) calls faint; on account of the superiority in the instruments of the former observer.

Besides this general division, there are added a 4th and 5th class, which contain Nebulæ deserving a more particular description. The 4th class contains Planetary Nebulæ, i.e. stars with burs, with milky chevelure, with short rays, remarkable shapes, &c. The 5th class very large Nebulæ.

The 6th, 7th, and 8th classes contain clusters of stars sorted according to their apparent compression, like the Catalogue of double stars, so that the closest and richest clusters take up the first or 6th class; the brightest, largest, and pretty much compressed ones, the second or 7th class; and those which consist only of scattered and less collected large stars, are put into the last.

*Note.*—When a superior power and telescope increase the brightness of a nebula, but at the same time only make the tinge of it more uniformly united, and of a milky appearance, it may be concluded to be purely nebulous; but when by using a superior instrument, its appearance is a mixture of nebulosity and extremely fine points, so that we can almost see stars, the nebula is said to be *easily resolvable*, and may be concluded to be a cluster of stars.

*Conjecture on the nature of nebulæ, not resolvable.*

In the Philosophical Transactions for 1811, Herschel has started a new conjecture respecting the nature of nebulæ. He no longer considers them as clusters of stars, which assume a nebulous appearance by reason of their immense distance, but that they consist of a luminous and extremely rare substance. That this substance, at its first formation, is pretty equally diffused through the nebula; but that in the course of ages, this matter, by the preponderance of some part of it, forms one or more centres, to which all the other matter gravitates; that in consequence of this, the nebula gradually decreases in size, and increases in density, till at last a nucleus is formed; and the nebula becomes planetary surrounded by nebulous matter; which last again is finally absorbed by the central body; and the whole then is, or has all the appearance of, a fixed star. This connexion between nebulous matter and a fixed star, and the conversion of the one into the other, he endeavours to establish, by arranging the nebulæ into classes, according to their supposed age and degree of condensation, beginning with extensive and uniformly diffused nebulosity, and establishing the connexion between this and a fixed star by such nearly allied intermediate steps, as makes it not improbable that every succeeding state of the nebulous matter is the result of the action of gravitation upon it while in a foregoing one, and by such steps the successive condensation of it has been brought up to the planetary condition. From this the transit to the stellar form requires but a very small additional compression of the nebulous matter; and in Herschel's observations of many of these it became doubtful whether they were not stars already.

The steps by which he arrives at this conclusion are nearly as follows:—

1. Extensively diffused nebulosity.
2. Nebulosities joined to nebulæ.
3. Nebulæ of various shapes, but nearly uniform brightness.
4. Nebulæ that are gradually a *little* brighter in the middle.
5. Nebulæ which are gradually brighter in the middle.
6. Nebulæ which are gradually *much* brighter in the middle.

7. Nebulæ that have a cometic appearance.

8. Nebulæ that are suddenly much brighter in the middle.

9. Round nebulæ increasing gradually in brightness up to a nucleus in the middle.

10. Nebulæ that have a nucleus.

11. Round nebulæ that are of an almost uniform light.

12. Nebulæ that draw progressively towards a period of final condensation.

13. Planetary nebulæ.

14. Stellar nebulæ.

15. Stellar nebulæ nearly approaching to the appearance of stars.

### *Clusters of stars.*

We have seen according to Herschel's doctrine, that extensive nebulosities are in process of time broken up into separate and distinct nebulæ; and that these last again, after becoming gradually more and more condensed, form stars. Upon the same principle he accounts for the formation of clusters of stars. He conceives that in rich portions of the heavens, as for instance the milky way, various centres of attraction are formed, to which the neighbouring stars gravitate; that thus the whole is broken up into separate systems or clusters of stars. That these clusters at first are of various irregular figures, and consist of stars coarsely and unequally scattered over the mass; that by the progress of condensation they become more insulated and detached from the neighbouring stars, their figures are more regular and spherical, and the stars more rich and closely connected; till they at length form those minute and beautiful phænomena which are undoubtedly the most interesting objects for our finest telescopes. He arranges them as follows, according to their degree of condensation.

1. Aggregation of stars, or patches of stars, which seem beginning to form clusters.

2. Irregular clusters of various unascertained sizes.

3. Clusters variously extended and compressed.

4. Considerably compressed clusters of stars.

5. Gradual concentration and insulation of clusters of stars.

6. Globular clusters of stars requiring a very fine telescope.

7. More distant globular clusters of stars.

8. Still more distant globular clusters.

NEUTRAL *Point.—See Elastic bodies, equilibrium of.*

NIGHT-GLASS, *or Sweeper.*

These are Telescopes of two, or two and a half feet in length, with large apertures, the object glass either a single lens of 3 or 3½ inches diameter, or an achromatic of 2½; their magnifying power, 6, 8, or 10 times; field of view 5 or 6 degrees: they are occasionally furnished with a system of cross wires, and a diagonal eye piece. Their use is for a rapid survey of any part of the heavens, and for fixing upon such objects as may be proper for examination with finer telescopes. They are also useful, provided the observations are recorded, in detecting minute changes in the heavens upon a subsequent review; or in searching for any object supposed to be moveable, as an asteroid. For this purpose delineations of the telescopic constellations, near the place where it is suspected to be, should be drawn upon paper; and after some days interval, the moving star will be discovered. This can only be done with a night glass of very low magnifying power. Herschel's small sweeper was a Newtonian reflector of 2 feet focal length, aperture 4,2 inches, magnifying power 24, and field of view 2° 12′.—*(Phil. Trans.)*

NONIUS.—*See Vernier.*

NORMAL.—*See Subnormal.*

NORMAL, *equation to.—See Tangent.*

NUTATION *of the Earth's axis.—See Precession.*

# O

OBSERVATORY.

TABLE,

*Of the Latitudes and Longitudes of the principal Observatories of Europe, from the most recent and accurate determinations.—(Lax.)*

| | LATITUDE N. | LONGITUDE. |
|---|---|---|
| Amsterdam | 52° 22′ 17″ | 0*h* 19*m* 33*s* E |
| Armagh | 54 21 15 | 0 26 30 W |
| Berlin | 52 31 45 | 0 53 29 E |
| Berne | 46 56 55 | 0 29 45 |
| Bologna | 44 30 12 | 0 45 26 |
| Bremen | 53 4 38 | 0 35 12 |

| | Latitude N. | | | Longitude. | | | |
|---|---|---|---|---|---|---|---|
| Brunswick | 52° | 16′ | 29″ | 0h | 42m | 8s | |
| Buda | 47 | 29 | 44 | 1 | 16 | 10 | |
| Cadiz | 36 | 32 | 0 | 0 | 25 | 9 | W |
| Cambridge | 52 | 12 | 36 | 0 | 0 | 17 | E |
| Cassel | 51 | 19 | 20 | 0 | 38 | 21 | |
| Coimbra | 40 | 12 | 30 | 0 | 33 | 39 | W |
| Copenhagen | 55 | 41 | 4 | 0 | 31 | 38 | E |
| Cracow | 50 | 3 | 38 | 1 | 19 | 49 | |
| Dantzic | 54 | 20 | 48 | 1 | 14 | 32 | |
| Dorpat | 58 | 22 | 47 | 1 | 46 | 48 | |
| Dresden | 51 | 2 | 50 | 0 | 54 | 52 | |
| Dublin | 53 | 23 | 13 | 0 | 25 | 22 | W |
| Edinburgh | 55 | 57 | 21 | 0 | 12 | 41 | |
| Florence | 43 | 46 | 41 | 0 | 45 | 3 | E |
| Geneva | 46 | 12 | 0 | 0 | 24 | 38 | |
| Genoa | 44 | 25 | 0 | 0 | 35 | 52 | |
| Glasgow | 55 | 51 | 32 | 0 | 17 | 4 | W |
| Gotha | 50 | 56 | 8 | 0 | 42 | 56 | E |
| Gottingen | 51 | 31 | 50 | 0 | 39 | 46 | |
| Greenwich | 51 | 28 | 39 | 0 | 0 | 0 | |
| Koenigsburg | 54 | 42 | 12 | 1 | 21 | 57 | |
| Leipsic | 51 | 20 | 16 | 0 | 49 | 27 | |
| Lilienthal | 53 | 8 | 30 | 0 | 35 | 37 | |
| Lisbon | 38 | 42 | 24 | 0 | 36 | 34 | W |
| London | 51 | 30 | 49 | 0 | 0 | 23 | |
| Madrid | 40 | 24 | 57 | 0 | 14 | 49 | |
| Marseilles | 43 | 17 | 49 | 0 | 21 | 29 | E |
| Milan | 45 | 28 | 2 | 0 | 36 | 46 | |
| Moscow | 55 | 45 | 45 | 2 | 30 | 12 | |
| Munich | 48 | 8 | 20 | 0 | 46 | 18 | |
| Naples | 40 | 50 | 15 | 0 | 57 | 3 | |
| Nuremberg | 49 | 26 | 55 | 0 | 44 | 17 | |

| | Latitude N. | | | Longitude. | | | |
|---|---|---|---|---|---|---|---|
| Oxford | 51 | 45 | 39 | 0*h* | 5*m* | 1*s* | W |
| Padua | 45 | 24 | 2 | 0 | 47 | 26 | E |
| Palermo | 38 | 6 | 44 | 0 | 53 | 28 | |
| Paris | 48 | 50 | 14 | 0 | 9 | 21 | |
| Pavia | 45 | 10 | 47 | 0 | 36 | 39 | |
| Petersburg | 59 | 56 | 23 | 2 | 1 | 15 | |
| Pisa | 43 | 43 | 11 | 0 | 41 | 36 | |
| Portsmouth | 50 | 48 | 3 | 0 | 4 | 24 | W |
| Rome | 41 | 53 | 54 | 0 | 49 | 59 | E |
| Slough | 51 | 30 | 20 | 0 | 2 | 24 | W |
| Stockholm | 59 | 20 | 31 | 1 | 12 | 14 | E |
| Strasburgh | 48 | 34 | 56 | 0 | 30 | 59 | |
| Toulouse | 43 | 35 | 46 | 0 | 5 | 46 | |
| Turin | 45 | 4 | 0 | 0 | 30 | 41 | |
| Upsal | 59 | 51 | 50 | 1 | 10 | 36 | |
| Utrecht | 52 | 5 | 31 | 0 | 20 | 29 | |
| Venice | 45 | 25 | 32 | 0 | 49 | 24 | |
| Verona | 45 | 26 | 7 | 0 | 44 | 5 | |
| Vienna | 48 | 12 | 40 | 1 | 5 | 31 | |
| Wilna | 54 | 41 | 2 | 1 | 41 | 12 | |

OPERA-GLASS.—*Kitchiner.*

An Opera-glass should not magnify more than three, or at the most four times; this also makes a pleasant prospect glass. If it have besides a power magnifying twice, it will be an excellent assistant in giving a general view of the constellations, and will be a good finder for sweeping the sky for a comet. The best Opera-glasses at present made by opticians, have an achromatic object glass of one and a half inch in diameter, magnifying four times; the price in a plain mounting, about two guineas and a half.

OPTICS, *laws of.*

The theory of Optics reposes on three *laws*, which depend for their proof upon observation and induction.

1. The rays of light are straight lines.

2. The angles of incidence and reflexion are in the same plane and equal.

3. The angles of incidence and refraction are in the same plane, and their sines bear an invariable ratio to one another for the same medium.

For the various subjects connected with this branch of science, see the respective heads.

**OSCILLATION**, *Centre of.—See Centre.*

# P

**PALLAS.**

This planet was discovered by Dr Olbers, of Bremen, March 28, 1802. For its elements—*see Planets, elements of.*

**PARABOLA**, *principal properties of.—See Conic Sections.*

**PARACENTRIC** *velocity.—See Central Forces.*

**PARALLAX.**—*(Woodhouse, Playfair.)*

1. If P be the horizontal parallax of a heavenly body, $p$ the parallax at a zenith distance $z$.

$$p = \mathrm{P} \times \sin. z.$$

*Cor.* If R be the radius of the earth; $r$ the tabular radius, $d$ the distance of the body, then

$$d = \frac{r}{\mathrm{P}} \times \mathrm{R}.$$

To adapt this to computation, $r$ must be expressed in degrees, minutes, &c. then

$$d = \frac{57^\circ.2957795}{\mathrm{P}} \times \mathrm{R}.$$

2. If two observers under the same meridian, but at a great distance from one another, observe the zenith distances of the same planet, when it passes the meridian on the same day, they can from thence determine the horizontal parallax.

Let L and L′ be the two latitudes, $z$ and $z'$ the observed zenith distances, then

$$\mathrm{P} = \frac{z + z' - (\mathrm{L} \pm \mathrm{L}')}{\sin. z + \sin. z'}$$

This formula was employed by Lacaille, at the Cape of Good Hope, and Wargentin, at Stockholm, for finding the parallax of Mars. It cannot be successfully applied to the Sun, or to Jupiter, Saturn, or the Georgian; for where the parallax does not exceed 10 or 12 seconds, the probable errors of observation will bear so large a proportion to it, as materially to affect the certainty of the result.

The moon, however, whose parallaxes are considerable, is a proper instance for the method, though in that case it will require some modification; as we must take into consideration the spheroidical figure of the earth; thus

Let R be the radius of the equator, $r$ and $r'$ the radii of the earth at the two places of observation, $z$ and $z'$ the zenith distances found as before, but corrected for the ∠'s between the vertical and the radius, then the horizontal parallax at the equator is

$$\frac{z + z' - (L \pm L')}{r \sin.\ z + r \sin.\ z'} \times R.$$

3. The parallax of a planet in R. A. being found by observation, to find its horizontal parallax.

Let $\epsilon$ be the R. A. in time, taken out of the meridian, then

$$P = \frac{15\,\epsilon \times \cos.\ \text{dec.}}{\cos.\ \text{lat.} \times \sin.\ \text{hour angle}}.$$

If the R. A. be taken both before and after the meridian, and $h$ and $h'$ be the two hour angles, and $s$ the sum of the parallaxes in R. A. on the east and west of the meridian,

$$P = \frac{15\,s \times \cos.\ \text{dec.}}{\cos.\ \text{lat.} \times (\sin.\ h + \sin.\ h')} \text{ or } =$$

$$\frac{15\,s \times \cos.\ \text{dec.}}{2 \cos.\ \text{lat.} \times \sin.\ \frac{h + h'}{2} \times \cos.\ \frac{h - h'}{2}}$$

4. The greatest horizontal parallax of the sun and planets.

| | | | | | |
|---|---|---|---|---|---|
| Sun | 8″,75 | Venus | 29″,16 | Jupiter | 2″,08 |
| Mercury | 14″,58 | Mars | 17″,50 | Saturn | 1″,027 |
| | | | | Georgian | 0,″415. |

For Sun's parallax in altitude—*see Sun.*

5. Parallax of the fixed stars.

If the *annual* parallax does not exceed 1″, the distances of the fixed stars cannot be less than 206265 times the radius of the earth's orbit. It is probable, however, that the parallax of a star of the second magnitude is not more than $\frac{1}{5}$ of a second; and of a star of the sixth magnitude, not more than $\frac{1}{20}$ or $\frac{1}{30}$ of that quantity.

**PENDULUMS,** *oscillation of, &c.—(Wood, Whewell, &c.)*

1. Let T = time of vibration of a simple pendulum in a cycloidal arc, L = length, F = accelerating force, $g$ = force of gravity = 32⅙ feet, $\pi = 3.14159$, &c., $n$ = number of vibrations in a given time T′, then

$$T = \sqrt{\frac{\pi^2 L}{F}},\text{ or in case of gravity } T = \sqrt{\frac{\pi^2 L}{g}}.$$

$$\text{and } n = \sqrt{\frac{F T'^2}{\pi^2 L}},\text{ or in case of gravity } n = \sqrt{\frac{g T'^2}{\pi^2 L}}.$$

*Cor.* Hence if $x$ = space fallen through by gravity in 1″ in any latitude, and L = length of a seconds pendulum, then if $x$ be given, $L = \frac{2x}{\pi^2}$; and if L be given, $x = \frac{\pi^2 L}{2}$.

By help of this last formula $x$ is found more exactly than can be done by direct experiment. In the latitude of London L = 39.126 inches, hence $x$ = 16.09 feet.

2. To find the vibration of a pendulum in a circular arc, let $a$ = ver. sin. of ½ arc of vibration, $r$ radius of the circle; then

$$T = \sqrt{\frac{\pi^2 r}{g}} \times \left(1 + \frac{a}{8r} + \frac{9a^2}{256 r^2} + \&c.\right) =,$$ when $a$ or the arc is very small, $\sqrt{\frac{\pi^2 r}{g}}$ = time of vibration in a cycloid. Hence the formulæ above given are applicable to bodies vibrating in very small circular arcs.

3. If a pendulum vibrating in a circular arc keeps true time whilst oscillating through $\delta$ degrees on each side of the vertical; then when it oscillates through D degrees, the seconds lost in 24 hours, if D is greater than $\delta$,

$$= \frac{5}{3}\left(D^2 - \delta^2\right)$$

or if D is less than $\delta$, time gained

$$= \frac{5}{3}\left(\delta^2 - D^2\right)$$

4. To correct the going of a clock.

Let L = present length of the pendulum, $t''$ = No. of seconds gained or lost in the time $T''$, $x$ = quantity by which the pendulum must be altered; then

$$x = \pm \frac{2\,L\,t''}{T''} \text{ nearly.}$$

5. Let $x$ = height of a mountain upon which a seconds pendulum loses $n''$ per hour; then

$$x = n + \frac{n}{9} \text{ miles nearly.}$$

6. If the force of gravity be slightly altered, to find the number of seconds gained or lost in a day by a seconds pendulum.

Let $g$ = force of gravity when pendulum vibrates seconds,

N = No. of seconds in a day,

$g\,(1 + k)$ = force of gravity when slightly increased,

$t$ = seconds gained in consequence, then

$$t = \frac{N\,k}{2}.$$

7. Given the number of vibrations ($v$) of a pendulum in air, to find the number V in a vacuum.—*(Galbraith.)*

Let $m$ be the spec. grav. of the pendulum, that of air being 1; then

$$V = v\left(1 + \frac{1}{2\,m} + \frac{1}{8\,m^2} + \&c.\right)$$

8. If $n'$ be the number of oscillations performed in 24 hours by the experimental pendulum, $n$ the true number, $e$ the expansion for a change of 1° Fahrenheit, $t$ the standard temperature, and $t'$ the observed; then

$$n = n' + \tfrac{1}{2}\,n'.e\,(t' - t)$$

9. To reduce the length of the pendulum from any height to the level of the sea, the true length being denoted by $l$, the observed by $l'$, the height above the sea by $a$, and the radius of the earth by $r$; then

$$l = l' + \frac{2\,a\,l'}{r}.$$

Some allow one-third for the effect of the dense strata immediately under the pendulum, in which case

$$l = l' + \frac{4\,a\,l'}{3\,r}.$$

In a similar manner $v = v' + \dfrac{2\,v'\,a}{3\,r}$.

10. To find the length of the seconds pendulum at the level of the sea, in any latitude $\lambda$.

$$\text{Length} = 39.0117150 + 0.2102732 \sin.^2 \lambda.$$

TABLE,

*Of the length of the seconds Pendulum in vacuo, at the level of the sea, by Sir G. Shuckburgh's standard (see Weights and Measures), observed at the following places, by Capt. Kater's method, and with his apparatus.*

| | | *Latitude.* | | | *Length of Pendulum.* |
|---|---|---|---|---|---|
| Captain Sabine ....... | Melville Island ....... | 74 | 47 | 13,4 | 39,207 |
| Do. .............. | Hare Isl. Baffin's Bay | 70 | 26 | 17 | 39,1984 |
| Captain Kater ...... | Unst ...................... | 60 | 45 | 28,01 | 39,17146 |
| Captain Sabine ....... | Brassa, Shetland ..... | 60 | 10 | 0 | 39,16929 |
| Captain Kater ........ | Portsay .................. | 57 | 40 | 58,65 | 39,16159 |
| Do. .............. | Leith Fort ............. | 55 | 58 | 40,8 | 39,15554 |
| Do. .............. | Clifton .................. | 53 | 27 | 43,12 | 39,14600 |
| Do. .............. | Arbury Hill ........... | 52 | 12 | 55,32 | 39,14250 |
| Do. .............. | London .................. | 51 | 31 | 8,40 | 39,13929 |
| Do. .............. | Shanklin Farm ....... | 50 | 37 | 23,94 | 39,13614 |
| Captain Basil Hall | San Blas ................ | 21 | 30 | 24 | 39,03776 |
| Mr Goldingham ...... | Madras .................. | 13 | 4 | 9 | 39,026302 |
| Captain Basil Hall | Galapagos .............. | 0 | 32 | 19 | 39,01717 |
| Do. ............ | Rio de Janeiro ........ | 22 | 55 | 22 S | 39,04381 |
| Sir T. Brisbane ....... | Paramatta ............... | 33 | 48 | 43 S | 39,07696 |
| Captain Sabine ....... | St Thomas ............. | 0 | 24 | 41 N | 39,02074 |
| Do. .............. | Maranham ............. | 2 | 31 | 43 S | 39,01214 |
| Do. .............. | Ascension .............. | 7 | 55 | 48 S | 39,02410 |
| Do. .............. | Sierra Leone ......... | 8 | 29 | 28 N | 39,01997 |
| Do. .............. | Trinidad ................ | 10 | 38 | 56 N | 39,01884 |
| Do. .............. | Bahia ..................... | 12 | 59 | 21 S | 39,02425 |
| Do. .............. | Jamaica .................. | 17 | 56 | 7 N | 39,03510 |
| Do. .............. | New York ............. | 40 | 42 | 43 N | 39,10168 |
| Do. .............. | Drontheim ............ | 63 | 25 | 54 N | 39,17156 |
| Do. .............. | Hammerfest .......... | 70 | 10 | 5 N | 39,19519 |
| Do. .............. | Greenland ............. | 74 | 32 | 19 N | 39,20335 |
| Do. .............. | Spitzbergen ........... | 79 | 49 | 58 N | 39,21469 |

For the ellipticity of the earth as deduced from these experiments—*see Earth, figure of.*

*The following Table shews the seconds gained in one day by a pendulum vibrating seconds at the Equator, in different latitudes, when it remains of the same length.—(Vince.)*

| Lat. | Seconds gained. | Lat. | Seconds gained. |
|---|---|---|---|
| $5^0$ | $1''$,7 | $50^0$ | $134''$,0 |
| 10 | 6,9 | 55 | 153,2 |
| 15 | 15,3 | 60 | 171,2 |
| 20 | 26,7 | 65 | 187,5 |
| 25 | 40,8 | 70 | 201,6 |
| 30 | 57,1 | 75 | 213,0 |
| 35 | 75,1 | 80 | 221,4 |
| 40 | 94,3 | 85 | 226,5 |
| 45 | 114,1 | 90 | 228,3 |

**PERCUSSION,** *Centre of.—See Centre.*

**PERMUTATIONS** *and Combinations.*

*Permutations.*

1. The number of permutations that can be formed out of $n$ quantities,

Taken 2 and 2 together ......... $= n.\ (n-1.)$

Taken 3 and 3 together ......... $= n.\ (n-1.)\ (n-2.)$

&c. &c.

Taken $r$ and $r$ together ......... $= n.\ (n-1.)\ (n-2) \ldots (n-r+1.)$

Thus the number of changes that can be made with 3 bells out of 8 $=$ 8. 7. 6. $= 336$.

If $n = r$ i.e. if the permutations respect all the quantities at once, then (since $n - r = o$) the number will be $n.\ (n-1.)\ (n-2) \ldots\ldots 3.\ 2.\ 1.$ Thus the number of permutations which can be formed ont of the letters composing the word *virtue*, are 6. 5. 4. 3. 2. 1 $= 720$.

2. The number of permutations that can be made out of $n$ things where there are $p$ of one kind, $r$ of another, $q$ of another, &c.

$$= \frac{n.\ (n-1.)\ (n-2.)\ (n-3)\ \ldots\ldots\ 2.\ 1}{1.\ 2.\ 3 \ldots p \times 1.\ 2.\ 3 \ldots\ldots r \times 1.\ 2.\ 3 \ldots q}.$$

Thus the number of permutations that can be made of the letters in the word *Bacchanalia* (since *a* occurs four times, *c* twice)

$$= \frac{11.\ 10.\ 9.\ 8.\ 7.\ 6.\ 4.\ 3.\ 2.\ 1}{1.\ 2.\ 3.\ 4 \times 1.\ 2} = 831600.$$

### *Combinations.*

1. The number of combinations that can be formed out of $n$ things,

Taken 2 and 2 together $= n.\,\frac{n-1}{2}$

—— 3 and 3 —— $= n.\,\frac{n-1}{2}.\,\frac{n-2}{3}$

&c. &c.

—— $r$ and $r$ —— $= n.\,\frac{n-1}{2}.\,\frac{n-2}{3}\,\ldots\ldots\,\frac{n-r+1}{r}$

Thus how many combinations can be made of two letters out of the 26 letters of the alphabet,

$$\text{No.} = 26.\,\frac{25}{2} = 325.$$

2. The total number of combinations of $n$ things taken 1 and 1; 2 and 2; 3 and 3, &c. together,

$$= 2^n - 1.$$

Thus all the possible combinations that can be made of a common suit of 13 cards taking them by one's, two's, three's, &c. at a time $= 2^{13} - 1 = 8191$.

3. All the possible permutations and combinations that any number of things can be made to undergo when taken by two's, three's, &c., up to the whole number of things given, is expressed by the sum of the geometrical series $n + n_2 + n^3 + n^4 \ldots\ldots\ldots\ldots + n^n$; and $\therefore$

$$= \frac{n^n - 1}{n - 1} \times n.$$

Thus the whole number of permutations and combinations that can be made of the 4 letters $a$, $b$, $c$, $d$, when they are taken by two's, three's, and four's

$$= \frac{4^4 - 1}{4 - 1} \times 4 = 340.$$

4. Supposing there are $m$ sets of things, one set containing $n$ things, another $p$, another $q$, &c., then the total number of combinations that can be formed by taking one from each set

$$= n \times p \times q \ldots\ldots\ldots \text{ to } m \text{ factors.}$$

Thus suppose there are 4 companies, in one of which there are 6 men, in another 8, in each of the other two 9, then the number of changes that can be made by taking one out of each company $= 6\ 8.\ 9.\ 9 = 3888$.

**PIPES,** *leaden and iron, weight of.—(Gregory.)*

Let $l$ be the length in feet, $d$ the interior diameter, and $t$ the thickness both in inches and parts of an inch, W the weight in hundred weights; then,

In a leaden pipe, $W = ,1382\ l\,t\,(d + t.)$

In a cast iron pipe, $W = ,0876\ l\,t\,(d + t.)$

**PIPES** *for conveying Water.—See Fluids, pressure of.*

**PLANE** *inclined.—See Inclined Plane.*

**PLANETS,** *Elements, &c. of.*

What are usually called the elements of a planet's orbit are in number seven.

1. The longitude of the ascending node of the orbit.
2. The inclination of the orbit to the plane of the Ecliptic.
3. The mean motion of the planet round the sun.
4. The mean distance of the planet from the sun.
5. The eccentricity of the orbit.
6. The longitude of the aphelion.
7. The epoch at which the planet is in the aphelion.

*Elements and general view of the Planetary System.—(Laplace, Maskelyne, Vince, Playfair.)*

| *Names of the Planets.* | *Sidereal Periods of the Planets.* | *Mean distances or ½ axes of the orbits.* | *Ratio of the eccentricities to the ½ axes at the beginning of* 1801. | *Mean longitudes reckoned from the mean Equinox at the epoch of the mean noon of Jan.* 1, 1801, *Greenwich.* |
|---|---|---|---|---|
| | d. | | | o. ′ ″ |
| Mercury | 87.969258 | 0.387098 | 0.205514 | 166. 0. 48.2 |
| Venus ... | 224.700824 | 0.723332 | 0.006853 | 11. 33. 16.1 |
| Earth .... | 365.256384 | 1.000000 | 0.016853 | 100. 39. 10 |
| Mars..... | 686.979619 | 1.523694 | 0.093134 | 64. 22. 57.5 |
| Vesta .... | 1335.205 ... | 2.373000 | 0.093220 | 267. 31. 49 |
| Juno ..... | 1590.998 ... | 2·667163 | 0.254944 | 290. 37. 16 |
| Ceres .... | 1681.539 ... | 2,767406 | 0.078349 | 264. 51. 34 |
| Pallas .... | 1681.709 ... | 2.767592 | 0,245384 | 252. 43. 32 |
| Jupiter .. | 4332.596308 | 5.202791 | 0.048178 | 112. 15. 7 |
| Saturn ... | 10758.969640 | 9.538770 | 0.056168 | 135. 21. 32 |
| Georgian | 30688.712687 | 19.183305 | 0.016670 | 177. 47. 38 |

| *Names of the Planets.* | *Mean longitudes of the Perihelion for the same epoch as before.* | *Inclinations of orbits to the Ecliptic, for Jan. 1, 1801.* | *Longitudes of the ascending nodes on the Ecliptic, Jan. 1, 1801.* |
|---|---|---|---|
| | °. ′ ″ | ° ′ ″ | ° ′ ″ |
| Mercury | 74. 21. 46 | 7. 0. 1 | 45. 57. 31 |
| Venus ... | 128. 37. 0.8 | 3. 23. 32 | 74. 52. 38.6 |
| Earth .... | 99. 30. 5 | 0. 0. 0 | 0. .0 0 |
| Mars ...... | 332. 24. 24 | 1. 51. 3.6 | 48. 14. 38 |
| Vesta .... | 249. 43. 0 | 7. 8. 46 | 171. 6. 37 |
| Juno ...... | 53. 18. 41 | 13. 3. 28 | 103. 0. 6 |
| Ceres ..... | 146. 39. 39 | 10. 37. 34 | 80. 55. 2 |
| Pallas .... | 121. 14. 1 | 34. 37. 7.6 | 172. 32. 35 |
| Jupiter .. | 11. 8. 35 | 1. 18. 51 | 98. 25. 34 |
| Saturn ... | 89. 8. 58 | 2. 29. 34.8 | 111. 55. 46 |
| Georgian | 167. 21. 42 | 0. 46. 26 | 72. 51. 14 |

| *Names of the Planets.* | *Mean diameter in English miles.* | *Mean dist. from the sun in mil. of miles.* | *Mean appar. diam. as seen from the earth.* | *Mean diameter as seen from the sun.* | *Apparent diam. of sun as seen from each.* | *Diurnal rotation on their own axes.* | *Inclination of axes to orbits.* |
|---|---|---|---|---|---|---|---|
| The Sun | 883246 | ...... | 32′ 1″.5 | ...... | ...... | 25*d* 14*h* 8*m* 0*s* | 82° 44′ 0″ |
| Mercury | 3224 | 37 | 10 | 16″ | 80′ | 1 0 5 28 | ......... |
| Venus ... | 7687 | 68 | 58 | 30 | 45,7 | 0 23 20 54 | ......... |
| Earth ... | 7912 | 95 | ...... | 17.2 | 32 | 1 0 .0 0 | 66 32 |
| Mars .... | 4189 | 144 | 27 | 10 | 21,33 | 0 24 39 22 | 59 22 |
| Ceres .... | 163 | 263 | 1 | ...... | ...... | .............. | ......... |
| Pallas .... | 80 | 265 | 0.5 | ...... | ...... | .............. | ......... |
| Juno ...... | 1425 | 252 | 3 | ...... | ...... | .............. | ......... |
| Vesta ..... | 238 | 225 | 0.5 | ...... | ...... | .............. | ......... |
| Jupiter... | 89170 | 490 | 39 | 37 | 6,15 | 0 9 55 37 | 90 near. |
| Saturn ... | 79042 | 900 | 18 | 16 | 3,37 | 0 10 16 2 | 60 prob. |
| Georgian | 35112 | 1800 | 3.5 | 4 | 1,64 | .............. | ......... |

| *Names of the Planets.* | *Place of Aphelia in Jan.* 1800. | *Secular motions of the aphelia: progressive.* | *Secular motions of nodes.* | *Greatest Equation of the Centre.* | *Elongation when stationary.* | *Arc of retrogradation.* | *Time of retrogradation.* |
|---|---|---|---|---|---|---|---|
| Sun ....... | ............ | ......... | ......... | ......... | ...... | ...... | ... |
| Mercury | 8s 14° 20′ 50″ | 1° 33′ 45″ | 1° 12′ 10″ | 23° 40′ 0″ | 18° 00′ | 13° 30′ | 23 *d.* |
| Venus ... | 10 7 59 1 | 1 21 0 | 0 51 40 | 0 47 20 | 28 48 | 16 12 | 42 |
| Earth .... | 9 8 40 12 | 0 19 35 | ......... | 1 55 30,9 | ...... | ...... | ... |
| Mars ..... | 5 2 24 4 | 1 51 40 | 0 46 40 | 10 40 40 | 136 48 | 16 12 | 73 |
| Ceres .... | 4 25 57 15 | ......... | ......... | 9 20 8 | ...... | ...... | ... |
| Pallas .... | 10 1 7 0 | ......... | ......... | 28 25 0 | ...... | ...... | ... |
| Juno ...... | 7 29 49 33 | ......... | ......... | ......... | ...... | ...... | ... |
| Vesta .... | 2 9 42 53 | ......... | ......... | ......... | ...... | ...... | ... |
| Jupiter .. | 6 11 8 20 | 1 34 33 | 0 59 30 | 5 30 38 | 115 12 | 9 54 | 121 |
| Saturn ... | 8 29 4 11 | 1 50 7 | 0 55 30 | 6 26 42 | 108 54 | 6 18 | 139 |
| Georgian | 11 16 30 31 | 1 29 2 | 1 44 35 | 5 27 16 | 103 30 | 3 36 | 151 |

| *Names of the Planets.* | *Synodic revolution.* | *Densities.* | *Quantities of matter.* | *Gravity on surface.* | *Intensities of light and heat.* |
|---|---|---|---|---|---|
| Sun ....... | ...... | 0,25226 | 333928 | 27,7 | ...... |
| Mercury | 118 *d.* | 2,5833 | 0,16536 | 1,0333 | 6,25 |
| Venus ... | 584 | 1,024 | 0,88993 | 0,9771 | 2,04 |
| Earth .... | ...... | 1 | 1 | 1 | 1 |
| Mars ..... | 780 | 0,6563 | 0,08752 | 0,3355 | 0,44375 |
| Jupiter .. | 399 | 0,20093 | 312,101 | 2,3287 | 0,036875 |
| Saturn ... | 378 | 0,10349 | 97,762 | 1,0154 | 0,01106 |
| Georgian | 370 | 0,21805 | 16,837 | 0,9285 | 0,00276 |

**For the telescopic appearance of the Planets**—*see Telescope.*

**PLANETS,** *disturbances occasioned by their mutual action upon each other.—(Playfair.)*

**The orbit of every planet, by the action of the other planets, is changed in all its elements but two; the mean motion, and the mean distance from the sun. Thus in Mercury and Venus the line of the nodes, the**

inclination to the Ecliptic, the line of the apsides, the eccentricity, and consequently the greatest equation of the centre all vary. In Mars the eccentricity, the lines of the apsides and nodes, vary by the action of Venus, the Earth, and Jupiter; as also his *place* in his orbit, which is not the case with Mercury and Venus, in consequence of their nearness to the Sun. In Jupiter and Saturn, the place in their orbit, the motion of the apsides, and the change of eccentricity, are chiefly produced by their action on each other; but in the disturbance of the inclination the other planets have a sensible effect. Uranus, from his great distance, suffers no disturbance in his motion but from Jupiter and Saturn.

Two interesting results are obtained from the investigation of the planetary disturbances. 1. That both in the system of primary and secondary planets, two elements of every orbit remain secure against all disturbances, the mean distance, and the mean motion, *i. e.* the transverse axis of the orbit, and the periodic time. 2. That all the inequalities in the planetary motions are periodical, and after a certain time run through the same series of changes. This accurate compensation of the planetary inequalities arises from three conditions; 1st. that the eccentricities of the orbits are small; 2d. that the planets all move in the same direction, *i. e.* from west to east; 3d. that the planes of their orbits are but little inclined to one another.

**PLANET,** *time of its passing over the meridian.—See Time.*

**POLAR** *Seas.—See Seas, Polar.*

**POLYGONS** *regular, to find the area of.*

Let $s$ represent the length of one of the equal sides, $n$ the number of them; then

$$\text{Area} = s^2 \times \frac{n}{4} \tan.\left(\frac{90\,n - 180^\circ}{n}\right).$$

Hence the following Table:—

Trigon $= s^2 \times 0.4330127$
Tetragon $= s^2 \times 1.0000000$
Pentagon $= s^2 \times 1.7204774$
Hexagon $= s^2 \times 2.5980762$
Heptagon $= s^2 \times 3.6339124$
Octagon $= s^2 \times 4.8284271$
Nonagon $= s^2 \times 6.1818242$
Decagon $= s^2 \times 7.6942088$
Undecagon $= s^2 \times 9.3656399$
Dodecagon $= s^2 \times 11.1961524$

**POPULATION,** *increase of.—(Bridge.)*

Of the method of finding the increase of population in any country, under given circumstances of births and mortality.

Let P represent the population of a country at any given period; $\frac{1}{m}$ the fractional part of the population which die in a year (or ratio of mortality;) $\frac{1}{b}$ the proportion of births in a year; then, if A represent the state of the population at the end of $n$ years,

$$A = P\left(1 + \frac{m-b}{mb}\right)^n.$$

$$\text{Or Log. } A = \text{Log. } P + n \times \log. \left(1 + \frac{m-b}{mb}\right).$$

Of the quantities A, P, $m$, $b$, $n$, any four being given, the fifth may be found.

*Ex.* 1. Suppose the population of Great Britain, in the year 1800, to have been ten millions; that $\frac{1}{40}$th part die annually; and the number of births are $\frac{1}{30}$th, and that no emigration takes place during the present century; What will be the state of its population in the year 1900?

Here $A = 22930000$.

*Ex.* 2. Suppose the population of France in the year 1792 to have been 27000000; the *ratio of mortality,* during the 18th century, to have been $\frac{1}{30}$th, and the number of births $\frac{1}{26}$th; What was the state of its population in the year 1700?

Here $P = 16861396$.

*Ex.* 3. Suppose the population of North America to have been five millions in the year 1800; in how many years will it amount to 16 millions, taking the ratio of mortality at $\frac{1}{45}$th, and the annual proportion of births at $\frac{1}{24}$th?

Here $n = 60.3$ years.

*Ex.* 4. The population of a province, in the year 1760, was estimated at 500000 persons; in the year 1800 it amounted to 720900 persons; from the bills of mortality it appeared, that, upon an average, $\frac{1}{50}$th part of the population had died annually; no register was kept of the births; What was the annual proportion of *them* during that period?

Here $b = 34.4$.

The annual proportion of births was about $\frac{1}{34}$th.

*Supposed Population of the World.—(Enc. Brit. Suppl.)*

| | |
|---|---|
| Europe | 185,000,000 |
| Asia (with Australia and Polynesia) | 270,000,000 |
| Africa | 55,000,000 |
| America | 40,000,000 |
| | 550,000,000 |

POWERS *of numbers.—See Involution.*

POWERS *Mechanical.—See Mechanical Powers.*

PRECESSION *of the Equinoxes.—(Woodhouse, Vince, Playfair.)*

I. The mean annual precession $= 50'',34$, which gives nearly 1° for the precession in 71¼ years, or about 25745 years for the entire revolution of the pole of the Equator round that of the Ecliptic. The part of the precession arising from the action of the sun $= 15'',3$, that from the moon $= 35''$. If the effect of the sun be reduced to $12'',5$; that of the moon will be triple of it, which is agreeable to the latest results deduced from the theory of the tides.

The precession affects the situation of stars in Declination or North Polar distance, and in right ascension; hence the following Formulæ.

*Annual Precession in Declination.*

This $= 50'',34 \times$ sin. obliquity $\times$ cos. star's R. A.

*Cor.* When the right ascension (R. A.) is between 90° and 270°, the declination is diminished by the effect of precession. And when the R. A. is between 0° and 90°, or between 270° and 360°, the declination is increased.

*Annual Precession in R. A.*

This $= 50'',34 \times$ (cos. I + sin. I $\times$ sin. star's R. A. $\times$ tan. star's declination) where I $=$ obliquity of ecliptic. In this expression the first part, $50'',34$ cos. I is common to all stars.

*Cor.* The precession in R. A. is nothing when the angle of position is a right ∠; it is also positive when that ∠ is acute, and negative when obtuse.

II. *Precession Solar, inequality of.*

The mean annual precession has been stated at $50'',34$; but it cannot have been equably produced. For the sun is sometimes in the Equator, when its force causing precession is nothing; at other times more than 23° distant, when its force is greatest. Hence the sun's action in producing precession must continually vary from the Equinox in March to the solstice in June. The correction due to this solar inequality is called the semi-annual Solar Equation. In consequence of this solar inequality, the pole of the earth describes, half yearly according to the order of the signs, round the place of the mean pole a circle whose radius = 0″.4345.

The solar inequality affects the precession of the stars in longitude, declination, and right ascension, also the obliquity of the ecliptic; hence the following formulæ.

*Equation of precession in longitude.*

This = 1″, 1 × sin. 2 sun's longitude.

Substitute this expression for 50″,34 in the above formulæ for precession, and we shall have the equations of precession in declination and R. A.

*Correction of the obliquity.*

This = 0″,4345 × cos. 2 sun's longitude.

The variation in the obliquity of the ecliptic arising from the sun, is called the *correction* of the obliquity; that from the moon is called the *equation* of the obliquity.

III. *Precession, lunar inequality of.*

The lunar inequality of precession is called *Nutation*, to distinguish it from the solar inequality. In consequence of the lunar action, the true pole of the earth describes about the place of the mean pole, in 18 years 7 months, contrary to the order of the signs, an ellipse of which the major axis = 19″,2, and minor axis = 15″.

The nutation affects the precession of the stars in longitude, declinanation, and R. A. and also the obliquity of the ecliptic; hence the following formulæ.

*Equation of the Equinoxes in R. A.*

The variation in the precession, or in the equinoctial points, usually called the *Equation of the Equinoxes in R. A.* is

17″,2 sin. longitude moon's node.

This affects the longitude of all the stars equally.

*Nutation in declination.*

Let A be the R. A. of a star, D its declination, N the longitude of the moon's node; then nutation in declination =

1″,1 sin. (A + N) + 8″,5 sin. (A − N).

*Nutation in Right Ascension.*

This $= 8'',5$ tan. dec. cos. $(N - A) + 1'',1$ tan. dec. cos. $(N + A)$ and if to this be added the equation of the equinoxes, the whole effect of nutation will $=$

$$\pm 8,''5 \text{ tan. dec. cos. } (N - A) + 1'',1 \text{ tan. dec. cos. } (N + A) + 17'',2 \text{ sin. } N.$$

*Equation of the obliquity.*

This $= 9'',63$ cos. N.

PRESSURE *of Earth against walls.—See Earth, pressure of.*

PRESSURE *of Fluids.—See Fluids.*

PRESSURE, *centre of.—See Centre.*

PRISM.—*See Refraction.*

PROGRESSION, *Arithmetical, Geometrical, and Harmonical.*

I. *Arithmetical Progression.*

All the cases of Arithmetical progression may be solved by the following formulæ :—

1. Let $a$ = first term, $l$ = last, $b$ = common difference, $n$ = number of terms, $s$ = sum of the series ; then

Of the quantities $a$, $l$, $b$, $n$, any three being given, the other may be found by the equation

$$l = a + \overline{n - 1}.\, b.$$

2. Of the quantities $a$, $b$, $n$, $s$, any three being given, the other may be found by the equation

$$s = (2\,a + \overline{n - 1}.\, b).\ \frac{n}{2}.$$

3. Of the quantities $a$, $l$, $n$, $s$, any three being given, the other may be found by the equation

$$s = (a + l).\ \frac{n}{2}.$$

II. *Geometrical Progression.*

1. All the cases of Geometrical Progression may be solved by the following formulæ :—

Let $a$ = first term, $l$ = last, $r$ = common ratio, $n$ = number of terms, $s$ = sum of the series ; then

Of the quantities $a$, $l$, $r$, $n$, any three being given, the other may be found by the equation

$$l = a r^{n-1}$$

2. Of the quantities $a$, $r$, $n$, $s$, any three being given, the other may be found by the equation

$$s = \frac{ar^n - a}{r - 1}.$$

3. Of the quantities $a$, $l$, $r$, $s$, any three being given, the other may be found by the equation

$$s = \frac{lr - a}{r - 1}.$$

4. When $n$, or the number of terms is infinite, then of the quantities $a$, $r$, $s$, any two being given, the other may be found by the equation

$$s = \frac{a}{1 - r}.$$

III. *Harmonical Progression.*

1. Let $a$, $b$, $c$ be in Harmonical Progression; then $a : c :: a - b : b - c$.

2. Let $a$, $b$, $c$, &c. be as before, then

$$\frac{1}{a},\ \frac{1}{b},\ \frac{1}{c},\ \text{\&c. are in arithmetical progression.}$$

3. Let $a$ and $b$ be the two first terms of an Harmonical Progression, to continue the series.

$$a,\ b,\ \frac{ab}{2a - b},\ \frac{ab}{3a - 2b},\ \frac{ab}{4a - 3b},\ \text{\&c.}$$

4. To find an harmonic mean ($x$) between two quantities $a$ and $b$.

$$x = \frac{2ab}{a + b}.$$

5. If between two quantities $a$ and $b$, an harmonic mean $x$, and an arithmetical mean $y$, be inserted,

$$a : x :: y : b.$$

6. If between two quantities $a$ and $b$ an arithmetic mean $x$, a geometric mean $y$, and an harmonical $z$, be inserted

$$x : y :: y : z.$$

7. If a fourth proportional be found to three quantities in Arithmetical progression, the three last terms are in Harmonical progression.

PROJECTILES *in a vacuum*.—(*Whewell.*)

Formulæ for finding the range, altitude, and time of flight, of bodies projected along planes inclined to the horizon.

1. Let $r$ = range, A = greatest altitude, $t$ = time of flight, $v$ = velo-

city of projection, $h$ = height due to this velocity, $\alpha = \angle$ of projection above the horizontal plane, $\iota$ = elevation of the plane above the horizon, $g = 32\frac{1}{6}$ feet; then we have the following equations.

$$r = \frac{2v^2}{g} \cdot \frac{\sin.(\alpha - \iota)\cos.\alpha}{\cos.^2\iota} = 4h\frac{\sin.(\alpha - \iota)\cos.\alpha}{\cos.^2\iota}.$$

$$A = \frac{v^2}{2g} \cdot \frac{\sin.^2(\alpha - \iota)}{\cos.^2\iota} = h\frac{\sin.^2(\alpha - \iota)}{\cos.^2\iota}.$$

$$t = \frac{2v}{g} \cdot \frac{\sin.(\alpha - \iota)}{\cos.\iota} = \sqrt{\frac{2h}{g}} \cdot \frac{2\sin.(\alpha - \iota)}{\cos.\iota}.$$

$$\text{Greatest range} = \frac{2h}{1 + \sin.\iota}.$$

2. When $\iota = o$; $r$ will be the *horizontal* range, and the above equations will become

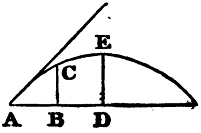
$$r = \frac{v^2}{g} \cdot \sin.2\alpha = 2h\sin.2\alpha.$$

$$A = \frac{v^2}{2g} \cdot \sin.^2\alpha = h\sin.^2\alpha.$$

$$t = \frac{2v}{g} \cdot \sin.\alpha = \sqrt{\frac{2h}{g}} \cdot 2\sin.\alpha.$$

$$\text{Greatest range} = 2h.$$

3. The curve described by a projectile is a parabola, the principal parameter of which $= 4h\cos.^2\alpha$, and the velocity at any point is that acquired by falling from the directrix.



4. To find an equation to the curve, referred to horizontal and vertical co-ordinates.

Let $AB = x$, $BC = y$, $t$ = any time; then

$$x = vt\cos.\alpha.$$

$$y = vt\sin.\alpha - \frac{gt^2}{2} \text{ and eliminating } t.$$

$$y = x\tan.\alpha - \frac{gx^2}{2v^2\cos.^2\alpha},$$

the equation to the curve.

*Cor.* 1. If, as before, $h = \frac{v^2}{2g}$,

$$y = x \tan.\ \alpha - \frac{x^2}{4\,h \cos.^2 \alpha}.$$

*Cor.* 2. To find where the curve meets the horizontal plane, we must put $y = o$, $\therefore\ x \tan.\ \alpha - \frac{x^2}{4\,h \cos.^2 \alpha} = o$, $\therefore\ x = 4\,h \tan.\ \alpha \cos.^2 \alpha = 4\,h \sin.\ \alpha \cos.\ \alpha = 2\,h \sin.\ 2\,\alpha$, which agrees with Art. 2.

*Cor.* 3. If $v$ does not enter the conditions of the problem, we have, by eliminating $v$,

$$y = x \tan.\ \alpha - \frac{g\,t^2}{2}.$$

*Cor.* 4. To find the ∠ which the curve makes with the horizon at any point.

Let $\varphi$ be this angle, $\tan.\ \varphi = \frac{d\,y}{d\,x}$, and differentiating the value of $y$,

$$\tan.\ \varphi = \tan.\ \alpha - \frac{x}{2\,h \cos.^2 \alpha}.$$

*Ex.* 1. Let a body be projected from the top of a tower horizontally with a velocity acquired in falling down its height; at what distance from the base will it strike the horizon?

$$y = x \tan.\ \alpha - \frac{g\,x^2}{2\,v^2\,(\cos.)^2\,\alpha}.$$

Here if $a =$ altitude of tower, $y = -a$, $\alpha = o$, and $v^2 = 2\,g\,a$, $\therefore$

$-a = -\frac{x^2}{4\,a}$, and $x = 2\,a$.

*Ex.* 2. A body is projected at an ∠ of 45°, with a velocity of 50 feet per second; find its horizontal range.

$$y = x \tan.\ \alpha - \frac{g\,x^2}{2\,v^2\,(\cos.)^2\,\alpha}.$$

Here $\alpha = 45^0$, $v = 50$, $\therefore$ when $y = o$

$$x = \frac{2500}{g}.$$

*Ex.* 3. A projectile is thrown across a plain 120 feet wide, to strike a mark 30 feet high, the velocity of projection being that acquired down 80 feet; required the ∠ of projection.

$$y = x \tan.\ \alpha - \frac{g\,x^2}{2\,v^2\,(\cos.)^2\,\alpha}.$$

Here $y = 30$, $x = 120$, $v^2 = 160\,g$, $\therefore$

$$1 = 4 \tan.\ \alpha - \frac{3}{2\,(\cos.)^2\,\alpha}$$

$$\therefore (\tan.)^2\,\alpha - \frac{8}{3} \tan.\ \alpha = -\frac{5}{3},$$

$$\text{and } \tan.\ \alpha = 1 \text{ or } \frac{5}{3}, \text{ and } \alpha = 45^0.$$

*Ex.* 4. A body projected from the top of a tower at an ∠ of 45o above the horizontal direction, fell in 5″ at a distance from the bottom equal to its altitude; required the altitude.

$$y = x \tan.\ \alpha - \frac{g\,t^2}{2}.$$

Let $a$ = height, then $\alpha = 45^0$, $t = 5$, and $y = -a$,

$$\therefore -a = a \tan.\ 45^0 - \frac{g}{2}.\ 25,$$

$$\therefore a = 200.$$

**PROJECTILES,** *resistance of air to.—See Gunnery.*

**PROJECTION,** *principles of.—(Vince.)*

I. *Orthographic Projection.*

1. The figure of a straight line is a straight line in the projection.

2. The figure of the projection of a circle is an ellipse, of which the minor axis is the cosine of inclination of the circle to the plane of projection. Hence if the circle be parallel to the plane of projection, the projection will be a circle equal to it. If the circle be perpendicular to the plane of projection, the circle is projected into its diameter; any arc, reckoned from its intersection with the plane, into its versed sine; and the remainder of the quadrant into the sine of that remainder, or into the cosine of the first mentioned arc.

3. In this projection the area of the circle : the area of the ellipse into which it is projected :: radius : cosine of inclination of the plane of the body to the plane of projection; hence the area of the circle will be diminished in the ratio of radius : the cos. of this inclination. And this is true whatever be the form of the projected body. Also the projection is not similar to the body. Hence equal parts upon the surface of a sphere will not be projected into parts either equal or similar.

This projection is not convenient for maps, but is used in the construction of solar eclipses.

## II. *Stereographic Projection.*

1. The projection of an arc, measured from the pole, is equal to the tangent of half that arc.

2. The projection of every circle is a circle.

3. The projection of all circles parallel to the plane of projection will be concentric circles, the radii of which are the tangents of ½ the distances of the circles from the pole.

4. The projection of every great circle passing through the pole is a straight line.

5. The radius of projection of any other great circle is the secant of the angle between the plane of the circle and the plane of projection.

From these Arts. it appears, that the projection of the parts of the sphere will not properly represent, in magnitude and situation, the parts themselves.

6. If the place of the eye be the pole of the earth, the meridians will be projected into straight lines (Art. 4); and the parallels to the equator will be projected into circles (Art. 3). This is called the *Polar Projection.*

7. If the eye be placed in the equator 90° distant from the point from which the longitude is reckoned, the projection of the radius of any meridian will be the secant of its longitude (Art. 5). And the radius of projection of the parallels of latitude is the cotangent of their latitude. This is called an *Equatorial Projection.*

The stereographic projection is chiefly used in delineating maps of the world.

## III. *Mercator's Projection.*

1. In this projection the meridians are parallel lines, the degrees of longitude are all equal; the parallels of latitude are also parallel lines, but unequal, a degree of latitude being to a degree of longitude :: rad. : cos. latitude, and ∴ the length of a degree of longitude being constant, the length of a degree of latitude will be inversely as the cosine of latitude, and will ∴ increase in going towards the pole.

2. To find the length of the meridian on this projection for any number of degrees of latitude.

Let $x$ = required length, $r$ = earth's radius then

$$x = r \times \text{h. l. } \frac{\text{cot. } \frac{1}{2} \text{ comp. latitude}}{r}.$$

If ∴ we take the latitude = 1°, 2°, 3° ........... 90° we can construct a

Table shewing the length of the meridian on the projection for every degree of latitude; in like manner it may be constructed for every minute. Such a table is called a table of *Meridional Parts.*

This projection is of great use in navigation, on account of its being constructed by right lines only; the rhumb lines or lines of azimuth being also straight lines.

Suppose for example a ship wants to go from any place A to B laid down upon Mercator's map, and it is required to find the rhumb or point of the compass it must sail upon; we have only to join A B, and that is the rhumb. Now to determine what rhumb this is, there is always in these maps one or more points, from which are drawn 32 straight lines, representing the 32 points of the compass. Apply ∴ one edge of a parallel ruler to the line A B, and bring the other edge over the point from which the lines of the compass are drawn, and it immediately gives the direction in which the ship must sail.

## PULLEY.

1. In the single fixed pulley, there is an equilibrium when the power and weight are equal.

2. In the single moveable pulley whose strings are parallel, $P : W :: 1 : 2$.

3. In a system where the same string passes round any number of pullies, and the parts of it between the pullies are parallel, $P : W :: 1 : n$, $n$ being the number of strings at the lower block.

*Cor.* If we consider the weight of the pullies, it is only requisite to add the weight of the lower block; hence if $a$ be this block,

$$W = n P - a.$$

4. In general in the single moveable pulley, P : W :: rad. : 2 cosine of the angle which either string makes with the direction in which the weight acts; or :: sin. ½ angle which the two strings make with each other : sin. of the whole angle.

5. In a system where each pulley hangs by a separate string, and the strings are parallel, $P : W :: 1 : 2^n$ where $n$ is the number of moveable pullies.

*Cor.* 1. Hence $W = 2^n P$. If the weight of the pullies be taken into the account, and $a =$ weight of each, $W = 2^n P - a (2^n - 1)$; hence the weight W is less as $a$ is greater.

*Cor.* 2. When the strings are not parallel, $P : W :: (\text{rad.})^n : 2^n \times$

cos. $\alpha$ × cos. $\beta$ × cos. $\gamma$ &c. where $\alpha$, $\beta$, $\gamma$, &c. are the angles which the strings make with the direction in which the weight acts in each case.

6. In a system of $n$ pullies each hanging by a separate string, where the strings are attached to the weight, $P : W :: 1 : 2^n - 1$.

*Cor.* Supposing the weight of each pulley $= a$, then the part of the weight sustained by the pullies $= a \times (2^n - n - 1)$; and $\therefore W = (2^n - 1) P + (2^n - n - 1) a$.

**PULLEY,** *on the ascent or descent of bodies over.*

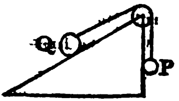
1. If two bodies P and Q are connected by a string and hung over a fixed pulley, the accelerating force, supposing P the heaviest, is $g \times \frac{P-Q}{P+Q}$. Substitute this for F in the formulæ for the rectilinear descent of bodies *(see Motion)* and we get

$$v = \frac{P-Q}{P+Q} \times g t.$$

$$s = \frac{P-Q}{P+Q} \times \frac{g t^2}{2}.$$

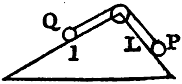
$$s = \frac{P+Q}{P-Q} + \frac{v^2}{2 g}.$$

2. If two bodies P and Q are connected together by a cord going over a fixed pulley, and one of them Q descends down an inclined plane, we have the moving force of $Q = Q \times \frac{H}{L}$; hence the moving force of Q when connected with $P = \frac{QH}{L} - P = \frac{QH - PL}{L}$ and accelerating force $=$ $g \times \frac{QH - PL}{L \times \overline{P+Q}}$.



If P draws up Q, accelerating force $= g \times \frac{PL - QH}{L \times \overline{P+Q}}$, which may be substituted for F as in the last Art.

Let both bodies P and Q move upon inclined planes, whose lengths are L and $l$ respectively, and having a common altitude H, and let Q be the descending body; then moving force of Q



$= \frac{QH}{l}$; do. $P = \frac{PH}{L}$; $\therefore$ the moving force of the system $=$ $\frac{LQ - lP}{Ll} \times H$, and accelerating force of $Q = g \times \frac{\overline{LQ - lP} \times H}{Ll \times \overline{P + Q}}$, which may be substituted as before.

**PUMP.** *Air Pump.*

1. If $b$ represent the capacity of one of the barrels, and $r$ that of the receiver together with the pipes and gages connected with it; then the quantity of air extracted after every turn : the quantity before that turn :: $b : 2b + r$. And the quantity left in : the quantity before :: $b + r : 2b + r$.

*Cor.* Hence if P represent the quantity of air in the machine before the first turn, the quantity left in after $n$ turns is

$$P. \left(\frac{b+r}{2b+r}\right)^n.$$

And the quantity exhausted is $P - P. \left(\frac{b+r}{2b+r}\right)^n =$

$$P. \frac{(2b+r)^n - (b+r)^n}{(2b+r)^n}$$

2. The density of the air in the receiver at first : the density after $t$ turns :: $(2b + r)^t : (b + r)^t$.

3. When the density of the air is diminished in the ratio of $n : 1$, the number of turns $t = \frac{\log. n}{\log. \overline{2b+r} - \log. \overline{b+r}}$.

4. As the air is exhausted, the mercury will rise in the gage, and the defects of the mercury in the gage from the standard altitude, after each successive turn, form a geometric series, the ratio of whose terms is $2b + r : b + r$. And the ascents of the mercury at each successive turn form a geometric series, the ratio of whose terms is $2b + r : b + r$.

**PUMP** *condensing, or condenser.*

If $b$ represent the capacity of the barrel of the syringe, and $r$ that of the receiver, then after $t$ descents of the sucker, the density of the air in the receiver, will be to the density at first in the ratio of $r + tb : r$.

**PUMP,** *common or sucking.*

1. In the common pump the force necessary to overcome the resistance experienced by the piston, in ascending, is equal to the weight of a co-

lumn of water, having the same base as the piston, and an altitude equal to that of the surface of the water in the body of the pump above that in the reservoir.

2. In a sucking pump, if the height of the lower or fixed valve above the surface of the water $= a$, the length of the stroke of the piston $= b$, and the height of a column of water in equilibrium with the pressure of the atmosphere $= h$, the height to which the water is raised by the first stroke is

$$\frac{a+b+h-\sqrt{(a+b+h)^2-4bh}}{2}.$$

3. The same notation being retained, and $c$ being put for $a+b$ or the greatest height to which the piston ascends, $b$ must be greater than $\frac{c^2}{4h}$ otherwise the water will not rise above the piston.

4. Height to which water will rise in a vacuum in different states of the barometer.

| *Barom. in inches.* | *Height of water in feet.* |
|---|---|
| 28 | 31.66 |
| 28½ | 32.23 |
| 29 | 32.79 |
| 29½ | 33.36 |
| 30 | 33.92 |
| 30½ | 34.49 |
| 31 | 35.05 |

Hence the valve of the piston in the common pump must be nearer to the surface of the water in the reservoir than 33 feet, otherwise the water can never rise above it.

PYROMETER, *Wedgwood's, for measuring very high temperatures.*

The scale of this Pyrometer, or the point marked 0 commences at red heat fully visible in day light, and is equivalent to 1077½° of Fahrenheit's scale, and one degree of the former is = 130° of the latter. The extremity of Wedgwood's scale is 240°, but the highest heat he measured with it is 160°. It appears ∴ that this pyrometer includes an extent of about 32000 of Fahrenheit's degrees, or about 54 times as much as that between the boiling and freezing points of mercury, by which mercurial ones are naturally limited; that if the scale be produced downward in the same manner as Fahrenheit's has been supposed to be produced upward for an ideal standard, the freezing point of water would fall nearly 8° below 0 of Wedgwood's and the freezing point of mercury a little below 8½, and that there are 8° from the freezing of water to full ignition.

# Q

**QUADRATRIX** *of Dinostrates, Equation to.*

$y \sqrt{r^2 - s^2} = s\,(r - x)$, where $r =$ radius, and $s$ the sine of the circ. arc, by the help of which the curve is generated.

The radius of the generating circle is a mean proportional between the quadrantal arc and the base of the quadratrix.

If with the base of the quadratrix as radius, there be described a quadrantal arc, this will be equal in length to the radius of the generating circle.

**QUADRATURE** *of Curves.—See Area.*

# R

**RADIUS** *vector of a planet's orbit.—See Anomaly.*

**RAIN,** *quantity of at different places.*

*Mean annual quantity of rain for 30 years, as observed at the apartments of the Royal Society. The gage is placed 75 feet 6 inches above the ground.*

| YEARS. | INCHES. | YEARS. | INCHES. |
|---|---|---|---|
| 1790 | 16.052 | 1800 | 18.925 |
| 1791 | 15.310 | 1801 | 19.197 |
| 1792 | 19.489 | 1802 | 13.946 |
| 1793 | 17.128 | 1803 | 17.922 |
| 1794 | 18.466 | 1804 | 20.973 |
| 1795 | 16.864 | 1805 | 20.396 |
| 1796 | 14.779 | 1806 | 20.427 |
| 1797 | 22.697 | 1807 | 14.206 |
| 1798 | 19.411 | 1808 | 18.475 |
| 1799 | 19.662 | 1809 | 20.711 |

| YEARS. | INCHES. | YEARS. | INCHES. |
|---|---|---|---|
| 1810 and 1811 | — | 1817 | 15.299 |
| 1812 | 18.348 | 1818 | 11.636 |
| 1813 | 15 953 | 1819 | 13.727 |
| 1814 | 16.367 | 1820 | 18.381 |
| 1815 | 12.968 | 1821 | 23.567 |
| 1816 | 15.174 | | |

| | |
|---|---|
| Average of the 30 years | 17.548 |
| Greatest mean quantity during this period | 23.567 |
| Least do. | 11.636 |

*Mean quantity of rain for each month during the above period of 30 years.*

| | INCHES. | | INCHES. |
|---|---|---|---|
| January | 1.253 | July | 1.979 |
| February | 1.004 | August | 1.489 |
| March | 0.884 | September | 1.564 |
| April | 1.269 | October | 1.712 |
| May | 1.476 | November | 1.985 |
| June | 1.411 | December | 1.520 |

It appears from observation, that the quantity of rain, as shewn by two gages, is not materially influenced by the height of the places above the level of the sea, provided the heights of the gages above the ground are equal; but it is a singular fact, which has not been satisfactorily accounted for, that it is very considerably affected by the height of the gages above the surface of the earth, though all other circumstances are the same. This will appear by a comparison of the following results, given in the Philosophical Transactions.

Quantity of rain observed by Mr Daines Barrington, for upwards of four months in 1770, as shewn by two gages, the one placed upon Mount Rennig, in Wales, the other on the plain below at about half a mile distant; the perpendicular height of the mountain being 1350 feet, and each gage being at the same height above the surface of the ground.

| | INCHES. |
|---|---|
| Bottom of mountain | 8.766 |
| Top of mountain | 8.165 |

Quantity of rain observed by Dr Heberden, from July 7, 1766, to July 7, 1767, as shewn by three gages, one placed below the top of a house, a second upon the top of a house, and the third upon Westminster Abbey.

| | Inches. |
|---|---|
| Lowest gage | 22.608 |
| Middle gage | 18.139 |
| Highest | 12.099 |

The same result was obtained from the two gages belonging to the Royal Society, the one placed 75 feet 6 inches above the ground, the other, a few feet distant from the other and 11 feet 6 inches lower.

*Mean annual quantity of rain, as shewn by the two gages.*

| Year. | Lower gage in inches. | Higher. |
|---|---|---|
| 1812 | 22.03 | 18.348 |
| 1813 | 18.296 | 15.953 |
| 1814 | 20.728 | 16.367 |

These facts should be attended to, in order to prevent any inaccurate conclusions from a comparison of different gages.

Estimate by Humboldt of the quantity of rain in different latitudes.

| *Latitude.* | *Eng. inches.* | *Latitude.* | *Eng. inches.* |
|---|---|---|---|
| 0° | 96 | 45° | 29 |
| 19 | 80 | 60 | 17 |

Professor Leslie has given the following empirical rule for the annual deposit of rain and dew in any latitude.

Quantity = 75 (1 — sin. lat.) + 8 = depth in inches.

*Annual fall of rain at different places, according to Dalton and others. —(Young's Nat. Phil.)*

| | Inches. |
|---|---|
| Granada, Antilles | 126 |
| Cape François, St Domingo | 120 |
| Calcutta | 81 |
| Bombay | 64 |
| Charlestown | 50.9 |
| Pisa | 43.2 |
| Rome | 39.0 |
| Venice | 36.1 |
| Padua | 34.5 |
| Zurich | 33.1 |
| Madeira | 31.0 |
| Leyden | 30.2 |
| Lower California | 00. |

[illegible]

| | INCHES. |
|---|---|
| Hague | 28.4 |
| Algiers | 27.0 |
| Utrecht | 24.7 |
| Lisle | 24.0 |
| Dublin | 22.2 |
| Edinburgh | 22 |
| Berlin | 20.6 |
| Petersburgh | 17.2 |
| Upsal | 16.7 |
| Keswick, Cumberland, 7 years | 67.0 |
| Kendal, Westmorland, 25 years | 53.9 |
| Garsdale, Westmorland, 3 years | 52.3 |
| Lancaster, 20 years | 39.7 |
| Townley, Lancashire, 15 years | 41.5 |
| Dover, 5 years | 37.5 |
| Liverpool, 18 years | 34.4 |
| Manchester, 33 years | 36.1 |
| Bristol, 3 years | 29.2 |
| Chatsworth, Derbyshire, 15 years | 27.8 |
| Barrowby, near Leeds, 6 years | 27.5 |
| Fyfield, Hampshire, 7 years | 25.9 |
| Norwich, 13 years | 25.5 |
| Lyndon, Rutlandshire, 21 years | 24.3 |
| Near Oundle, Northamptonshire, 14 years | 23.0 |
| South Lambeth, 9 years | 22.7 |
| Dalton's mean for all England | 31.3 |
| Dalton's mean, for rain and dew together, for all England | 36.0 |
| M. Cotte's mean annual quantity of rain falling at 147 places from N. lat. 11°. to 60°. | 34.7 |

The superficies of the globe consists of 170,981,012 square miles; supposing therefore that the mean annual quantity of rain for the whole globe to be 34 inches, the quantity of rain falling annually will amount to somewhat more than 91,751 cubic miles of water, which must be supplied by evaporation from the surface of the earth and sea.—*See Evaporation.* The dry land amounting to 52,745,253 square miles, the quantity of rain falling on it will amount to 30,960 cubic miles. The quantity of water running annually into the sea is estimated at 13,140 cubic miles, a quantity of water equal to which must be supplied by evaporation from the *sea*, otherwise the land would be soon completely drained of its moisture.

The area of England and Wales = 46,450 square miles, taking therefore Dalton's mean at 36 inches, we shall have the annual quantity of rain and dew falling in England and Wales = 28 cubic miles of water.

RAINBOW.—*(Wood.)*

1. If a ray of light refracted into a sphere, emerge from it after any given number of reflections, to find the angle contained between the directions in which it is incident and emergent.

Let $\phi$ and $\phi'$ = angles of incidence and refraction, $p$ = number of reflections, then the deviation, or inclination of the emergent to the incident ray is

$$2\phi - 2(p+1)\phi' \text{ or } 2(p+1)\phi' - 2\phi.$$

*Cor.* In the primary rainbow $p = 1$, $\therefore$ deviation $= 4\phi' - 2\phi$; in the secondary bow $p = 2$; $\therefore$ deviation $= 6\phi' - 2\phi$, or its supplement.

2. The rays of any colour will fall most copiously on the eye, when the rays emerge parallel, in which case only they are efficient.

3. When rays emerge parallel, the increment of the angle included between the incident and emergent rays = 0, and tangent of incidence : tangent of refraction :: $p + 1 : 1$.

*Cor.* Hence it is easy to shew that if the ratio of refraction = $m$, $\cos.\ \phi = \sqrt{\frac{m^2 - 1}{p^2 + 2p}}$.

*Ex.* 1. If the pencil be parallel red rays incident upon a sphere of water, and suffer two refractions and one reflection, as in the primary bow, $m = \frac{4}{3}$, and $p = 1$ $\therefore \cos.\ \phi = \sqrt{\frac{7}{27}}$ when the rays emerge parallel, or $\phi = 59^\circ.\ 23'$; also $\phi' = 40^\circ.\ 12'$; $\therefore$ deviation, or angle between the incident and emergent rays, $= 4\phi' - 2\phi = 42^\circ.\ 2'$.

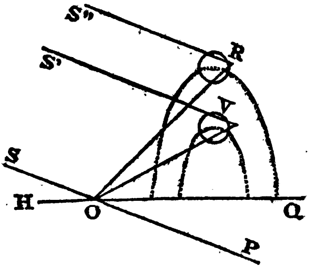
When the violet rays are thus incident and emergent, $\phi = 58^\circ.\ 40'$, $\phi' = 39^\circ.\ 24'$, and deviation $= 40^\circ.\ 16'$.

*Ex.* 2. If the pencil be parallel red rays incident upon a sphere of water, and suffer two refractions and two reflections, as in the secondary bow, $p = 2$, and $\therefore$ when the rays emerge parallel, $\phi = 71^\circ.\ 50'$; $\phi' =$

$45^0. 27'$, $\therefore 6\phi' - 2\phi = 129^0. 2'$, the supplement of which, or the deviation, $= 50^0. 58'$.

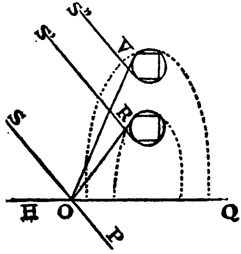
When violet rays are thus incident, $\phi = 71^0. 26'$, $\phi' = 44^0. 47'$, and the deviation $= 54^0. 10'$.

### 4. Construction of the primary and secondary Rainbow.



The red rays we have seen are efficacious when the ∠ between the incident and emergent rays $= 42^0. 2'$, and the violet rays when the same $\angle = 40^0. 16'$; hence if H Q be the horizon, S, S′, S″ rays proceeding from the sun, O the eye of the spectator, and the ∠ P O R ($=$ ∠ S″ R O) be taken $= 42^0. 2'$ the drop R will transmit the red rays to the eye; and if P O V ($=$ S′ V O) be taken $= 40_0. 16'$ the drop V will transmit the violet rays. The drops betwixt R and V will transmit to the eye the other colours in their proper order.

If O R and O V revolve about the axis O P, every drop of water in the surface of the cones thus described will respectively transmit to the eye a small parallel pencil of red and violet rays; and thus a red and violet arc, whose radii (measured by the angles which they subtend at the eye) are $42^0. 2'$, and $40^0. 16'$ respectively, will appear in the falling rain opposite to the sun; and the same may be said of the other colours.



The parallel pencils of red &c. rays which emerge from other drops fall above or below the eye.

The secondary rainbow is formed by two refractions and two reflections. In this case, as we have seen, the violet rays are efficacious when the ∠ contained by the incident and emergent rays $= 54^0. 10'$, and the red rays when the same $\angle = 50^0. 58'$. Hence as in the primary bow, if ∠ P O V $= 54^0. 10'$, the drop V will transmit the violet rays to the eye; and if P O R $= 50^0. 58'$ the drop R will transmit the red rays.

Hence the colours in the two bows lie in a contrary order, the red forming the exterior ring of the primary, and the interior ring of the secondary bow.

5. To find the altitude and breadth of the rainbow.

In the primary, the altitude of the highest point of the red arc = 42°. 2′ — sun's altitude; and of the violet = 40°. 16′ — sun's altitude. Hence the breadth of the bow, supposing the sun a point = 1°. 46′; this breadth must, however, be increased by 30′ the sun's apparent diameter, and ∴ the true breadth = 2°. 16′.

In the same manner it may be shewn that the altitude of the highest point of the secondary = 54°. 10′ — sun's altitude; and breadth = 3°. 42′.

*Cor.* When the sun is in the horizon, the altitude of the bow is equal to its radius; if the sun's altitude equal or exceed 42°. 2′, there can be no primary bow; and if it equal or exceed 54°. 10′, there can be no secondary.

6. Given the radius of an arc of any colour in the primary rainbow, to find the ratio of the sine of incidence to the sine of refraction, when rays of that colour pass out of air into water.

The radius of the arc $= 4\,\phi' - 2\,\phi$; let the tangent of $2\,\phi' - \phi$, half this angle, be $a$, $z$ the tangent of $\phi'$; then

$$2\,z^3 - 3\,a\,z^2 - a = o.$$

The value of $z$ being thus obtained, the angles $\phi'$ and $\phi$ and consequently their sines may be found from the tables.

RECIPROCALS *of numbers.—See Involution.*

RECIPROCAL *Spiral.—See Spiral.*

RECTIFICATION *of Curves.*

Let $z$ = curve, $x$ and $y$ the abscissa and ordinate; then

$$z = \text{fl.}\ \sqrt{d\,x^2 + d\,y^2} = \text{fl.}\ d\,x\sqrt{1 + \frac{d\,y^2}{d\,x^2}}.$$

*Ex.* 1. In the semicubical parabola, where $a\,x^2 = y^3$,

$$z = \frac{(9y + 4\,a)^{\frac{3}{2}}}{27\,a^{\frac{1}{2}}} - \frac{8\,a}{27}.$$

2. In the common parabola, $z = \frac{1}{2\,b} \times (y^4 + b^2\,y^2)^{\frac{1}{2}} + \frac{1}{2}\,b$

$$\times\ \text{h. l.}\ \frac{y + \sqrt{y^2 + b^2}}{b}.$$

3. In a circular arc, whose tangent is $t$,

$$x = t - \frac{t^3}{3a^2} + \frac{t^5}{5a^4} - \frac{t^7}{7a^6} + \&c.$$

For rectification of Spirals—*see Spiral.*

**REFLEXION** *in Optics.—(Coddington.)*

1. *Reflexion at plane surfaces.*

1. To find the direction in which a ray of light, emanating from a given point, takes after reflexion at a plane mirror.

Let the ray proceed from a point Q, and a perpendicular Q C be drawn to the surface of the reflector, and let the ray after reflexion cut Q C produced in $q$; then will Q and $q$ be on opposite sides of C, and Q C will $=$ C $q$.

2. To find the same when the ray is reflected alternately by two plane mirrors inclined to each other at any given angle.

Let $\phi_1$ be the ∠ of incidence at the first reflexion

$\phi_2$ ——————— second

$\vdots$ $\qquad\qquad\qquad\qquad$ $\vdots$

$\phi_n$ ——————— $n^{th}$

$\iota$ the inclination of the two mirrors; then we shall have this series of equations,

$$\begin{array}{l} \phi_1 - \phi_2 = \iota \\ \phi_2 - \phi_3 = \iota \\ \phi_3 - \phi_4 = \iota \\ \vdots \quad \vdots \\ \hline \phi_1 - \phi_n = (n-1)\,\iota \end{array}$$

$$\text{or} \quad \phi_n = \phi_1 - (n-1)\,\iota$$

If now $\phi_1$ be any multiple of $\iota$, as $(n-1)\,\iota$, we shall have somewhere $\phi_n = 0$, i.e. some reflected ray will be perpendicular to one of the mirrors, and these of course will end the series of reflexions. If $\phi_1$ be not a multiple of $\iota$, some value of $n$ will make $(n-1)\,\iota$ greater than $\phi_1$, and then $\phi_n$ will be negative. This shews that the ray will at length be turned back upon itself in a direction contrary to what it at first proceeded in.

To find the angle between the 1st incident and last reflected ray, let

$q_1$ represent the first incident ray and $q_2$, $q_3$ ...... the rays after the 1st, 2d, 3d, &c. reflexions, and let the ∠'s. between them be denoted by $(q_1 q_2)$ $(q_1 q_3)$ &c. and we shall have

$$(q_1\ q_3) = (q_3\ q_5) = \dots\dots\dots (q_{n-1}\ q_{n+1}) = 2\,\iota$$

and $(q_1\ q_{2n+m}) = 2\,n\,\iota$, provided $m$ be an odd number.

$$\text{Also } (q_1\ q_4) = 2\,\phi_\prime - 2\,\iota$$

$$(q_1\ q_6) = 2\,\phi_\prime - 4\,\iota$$

$$\vdots \qquad \vdots$$

$$(q_1\ q_{2m}) = 2\,\phi_\prime - (2\,m - 2)\,\iota$$

II. *Reflexion at spherical surfaces.*

1. Rays meeting in a point being incident on a spherical reflecting surface; to determine the directions of the reflected rays.

Let $r$ = radius of the surface, $q$ and $q'$ = distance of the foci of incident and reflected rays from the *centre* of the reflector, then when the incident rays are nearly coincident with the axis,

$$\frac{1}{q'} = \frac{1}{q} + \frac{2}{r}.$$

If $q$ is infinite, or the rays parallel,

$$\frac{1}{q} = \frac{2}{r} \text{ or } q' = \frac{r}{2}.$$

This is technically called the *principal focal distance* of the reflector, and if we call it $f$, we have $f = \frac{r}{2}$, and ∴ by substituting in the first equation,

$$\frac{1}{q'} = \frac{1}{q} + \frac{1}{f}.$$

These formulæ may be obtained in another form, which is often more convenient, thus:—

Let $\Delta$ and $\Delta'$ = distance of the foci of incident and reflected rays from the *surface* of the mirror, then $\frac{1}{\Delta} + \frac{1}{\Delta'} = \frac{2}{r}$; $r$ being negative if the mirror be convex.

If $\Delta$ is infinite, or the rays parallel,

$$\frac{1}{\Delta'} = \frac{2}{r}, \text{ or } \Delta' = \frac{r}{2} = f$$

and ∴ as before $\frac{1}{\Delta} + \frac{1}{\Delta'} = \frac{1}{f}$.

*Cor.* If E be the centre of the mirror, Q and $q$ the foci of incident and reflected rays, F the principal focus of parallel rays,

$$Fq : FE :: FE : FQ$$

where Q and $q$ lie on the same side of F, move in opposite directions, and meet at the centre and surface of the reflector.

III. *Reflexion, images produced by.*

1. When an object is placed between two parallel *plane* mirrors, a row of images is formed, which are gradually fainter as they are more remote, and at length become invisible.

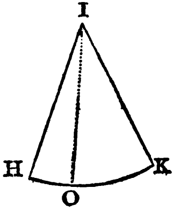
Now let A and B represent the two mirrors, O the object between them, and let $O'$ be the image of O at the mirror A, $O''$ the image of $O'$ at the second mirror B, $O'''$ the image of $O''$ at the mirror A &c.

Also let $O_{\prime}$ be the image of O at the second mirror B, $O_{\prime\prime}$ the image of $O_{\prime}$ at A, $O_{\prime\prime\prime}$ of $O_{\prime\prime}$ at B &c. and let it be required to find the distances $OO'$, $OO''$ ............ $OO_{\prime}$, $OO_{\prime\prime}$ &c.

Put $OA = a$. $OB = b$, $AB = c$ or $a + b$. Then

$$\left.\begin{array}{l} OO' = 2a \\ OO'' = 2c \\ OO''' = 2c + 2a \\ \vdots \quad \vdots \end{array}\right\} \text{Again} \left\{\begin{array}{l} OO_{\prime} = 2b \\ OO_{\prime\prime} = 2c \\ OO_{\prime\prime\prime} = 2c + 2b \\ \vdots \quad \vdots \end{array}\right.$$

2. Suppose now that the mirrors, instead of being parallel, are inclined to each other, in this case the number of images will be limited, and will evidently lie in the circumference of a circle, whose centre is the intersection of the two planes, and radius the distance of the object from that intersection.



Now let H I and K I be the mirrors, O the object, then as before there will be two series of images

$O'$, $O''$, $O'''$ &c. and $O_{\prime}$, $O_{\prime\prime}$, $O_{\prime\prime\prime}$ &c.

to determine the distances $OO'$, $OO''$ &c. measured along the circumference of the circle, put H I O or H O $= \theta$, O I K or O K $= \theta'$, H I K or H K $= \iota$, then

$$\left.\begin{array}{l} O\,O' = 2\,\theta \\ O\,O'' = 2\,\iota \\ O\,O''' = 2\,\iota + 2\,\theta \\ O\,O'''' = 2\,\iota + 2\,\theta + 2\,\theta' = 4\,\iota \\ \vdots \qquad \vdots \end{array}\right\}\text{Again}\left\{\begin{array}{l} O\,O_{,} = 2\,\theta' \\ O\,O_{,,} = 2\,\iota \\ O\,O_{,,,} = 2\,\iota + 2\,\theta' \\ \vdots \qquad \vdots \end{array}\right.$$

and the number of images in the first series is the least whole number greater than $\frac{\pi - \theta}{\iota}$; and the number of images in the other series is the least whole number greater than $\frac{\pi - \theta'}{\iota}$.

If $\iota$ be a measure of $\pi$, the whole number of images is $\frac{2\,\pi}{\iota}$; and in this case two images of the different series coincide.

3. If the object placed before a *spherical* reflector be a circular arc concentric with it, the image will also be a circular arc concentric with and similar to the object, and its position and magnitude may be determined by the proportion

$$F\,q : F\,E :: F\,E : F\,Q.$$

4. If the object placed before a spherical reflector be a straight line, the image is a conic section; and is a parabola, ellipse, or hyperbola, according as the distance of the object from the centre of the mirror is equal to, greater, or less than, half its radius.

## REFRACTION *in Optics.—(Coddington.)*

### I. *Refraction at plane surfaces.*

1. Given the direction in which a ray falls on a plane surface bounding a refracting medium; to find the direction of the refracted ray.

Let the ray proceed from the point Q, and a perpendicular Q C be drawn to the surface of the refracting medium, and let the ray after refraction cut Q C or Q C produced in $q$; put $\Delta$ and $\Delta' =$ C Q and C$q$; $\theta$ and $\theta' = \angle$'s. of incidence and refraction; $m$ the ratio of the sine of incidence : sine of refraction, usually called the *ratio of refraction*; then

$$\Delta' = m\,\Delta\left(1 + \frac{m^2 - 1}{2\,m^2}\,\tan.\,\theta^2\right)$$

or when $\theta$ is small, as it is usually supposed to be,

$$\Delta' = m\,\Delta \text{ nearly.}$$

2. Let a ray pass through a refracting substance bounded by two parallel plane surfaces; to determine its direction after emergence.

Let D = distance of the foci of incident and emergent rays, T the thickness of the medium, then

$$D = \frac{m-1}{m} T.$$

3. To determine the refraction which a ray experiences in passing through a medium bounded by planes not parallel; for example a triangular prism of glass,

Let $\iota$ = vertical angle of the prism.
$\phi = \angle$ of incidence.
$\psi = \angle$ of emergence.
$\delta = \angle$ of deviation of the incident and emergent rays; then

$$\delta = \phi + \psi - \iota,$$

or if the ∠ of the prism and the ∠'s. of incidence and emergence be exceedingly small,

$$\delta = (m-1)\,\iota.$$

*Cor.* The ∠ of deviation is a minimum when the incident and emergent rays make equal ∠'s. with the sides of the prism.

II. *Refraction at spherical surfaces.*

1. A ray of light is refracted at a spherical surface bounding two different media; given the point where it meets the axis, required the point where the refracted ray meets the axis.

Let $r$ = radius of the spherical surface,
$\Delta$ and $\Delta'$ = distances of Q and $q$ from the refracting surface; then we may tabulate the results as follows:—

| *Case.* | *Refracting Medium.* | *Surface.* | *Equation.* |
|---|---|---|---|
| 1 | Denser | Concave | $\frac{1}{\Delta'} = \frac{m-1}{m\,r} + \frac{1}{m\,\Delta}$ |
| 2 | Denser | Convex | $\frac{1}{\Delta'} = -\frac{m-1}{m\,r} + \frac{1}{m\,\Delta}$ |
| 3 | Rarer | Concave | $\frac{1}{\Delta'} = -\frac{m-1}{r} + \frac{m}{\Delta}$ |
| 4 | Rarer | Convex | $\frac{1}{\Delta'} = \frac{m-1}{r} + \frac{m}{\Delta}$ |

In order to find the *principal focal distance*, which we call $f$ *(see Reflexion* II*)* we have of course only to make $\Delta$ infinite in the equations just given; we have then

$$\textit{Case 1. } \frac{1}{f} = \frac{m-1}{m\,r}, \text{ or } f = \frac{m}{m-1}\,r.$$

$$\textit{Case 2. } \frac{1}{f} = -\frac{m-1}{m\,r}, \text{ or } f = -\frac{m}{m-1}\,r.$$

$$\textit{Case 3. } \frac{1}{f} = -\frac{m-1}{r}, \text{ or } f = -\frac{1}{m-1}\,r.$$

$$\textit{Case 4. } \frac{1}{f} = \frac{m-1}{r} \text{ or } f = \frac{1}{m-1}\,r.$$

It is important to observe that in all cases the distance of the principal focus from the surface, is to its distance from the centre, as the sine of incidence to the sine of refraction.

If we introduce the distance $f$ into the formulæ, we shall have in

$$\text{Cases 1 \& 2, } \frac{1}{\Delta'} = \frac{1}{f} + \frac{1}{m\,\Delta},$$

$$\text{3 \& 4, } \frac{1}{\Delta'} = \frac{1}{f} + \frac{m}{\Delta}.$$

The following are corresponding values of $\Delta$ and $\Delta'$, in different positions of the conjugate foci Q and $q$.

$$\textit{Case 1.} \begin{cases} \Delta = \infty,\ r,\ 0,\ -\dfrac{r}{m-1},\ \infty \\[2ex] \Delta' = \dfrac{m\,r}{m-1},\ r,\ 0,\ \infty,\ \dfrac{m\,r}{m-1} \end{cases}$$

$$\textit{Case 2.} \begin{cases} \Delta = \infty,\ \dfrac{r}{m-1},\ 0,\ -r.\ \infty \\[2ex] \Delta' = -\dfrac{m\,r}{m-1},\ \infty,\ 0,\ -r,\ \dfrac{m\,r}{m-1} \end{cases}$$

$$\textit{Case 3.} \begin{cases} \Delta = \infty,\ \dfrac{m\,r}{m-1},\ r,\ 0,\ \infty \\[2ex] \Delta' = -\dfrac{r}{m-1},\ -\infty,\ r,\ 0,\ -\dfrac{r}{m-1} \end{cases}$$

$$\textit{Case 4.}\begin{cases}\Delta = \infty,\ 0,\ -r,\ -\dfrac{mr}{m-1},\ \infty \\[2ex] \Delta' = \dfrac{r}{m-1},\ 0,\ -r,\ \infty,\ \dfrac{r}{m-1}\end{cases}$$

2. To find the direction of a ray after refraction through a lens.

The method here is to consider a ray refracted at the first surface, as incident on the second, and there again refracted.

Let $\Delta''$ be the distance of the focus after the second refraction, $r'$ the radius of the second surface, $t$ the thickness of the lens, the other symbols as above; then

$$\frac{1}{\Delta''} = (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right) + \frac{1}{\Delta}.$$

To find the *principal* focal distance F, put $\Delta$ infinite in the above expression, i.e. suppose the rays parallel, and we have

$$\frac{1}{F} = (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right)$$

and then $\dfrac{1}{\Delta''} = \dfrac{1}{F} + \dfrac{1}{\Delta}.$

If we put $\dfrac{1}{\varrho}$ for $\dfrac{1}{r} - \dfrac{1}{r'}$ in the above equation,

$$\frac{1}{F} = \frac{m-1}{\varrho} \text{ or } F = \frac{\varrho}{m-1}.$$

Hence arise different values of $\dfrac{1}{F}$ according as $\dfrac{1}{r} - \dfrac{1}{r'}$ is positive or negative.

In the *concavo-convex* lens, either $r$ is less than $r'$ and F positive; or when the lens is turned the contrary way and $r$ greater than $r'$ they are both negative, we have then

$$\frac{1}{F} = (m-1)\left(\frac{1}{r'} - \frac{1}{r}\right)$$

In the *meniscus*, either $r$ is greater than $r'$, both being positive, and then

$$\frac{1}{F} = -(m-1)\left(\frac{1}{r'} - \frac{1}{r}\right)$$

or $r$ is less than $r'$ and both are negative, so that

$$\frac{1}{F} = -(m-1)\left(\frac{1}{r} - \frac{1}{r'}\right)$$

In the *double concave* lens, $r'$ is negative,

$$\frac{1}{F} = (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$$

In the *double convex*, $r$ is negative,

$$\frac{1}{F} = -(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$$

In the *plano-concave*, either $r'$ is infinite, or $r$ is infinite and $r'$ negative; therefore putting $r$ for the single radius

$$\frac{1}{F} = \frac{m-1}{r},\ F = \frac{r}{m-1}.$$

In the *plano-convex*

$$\frac{1}{F} = -\frac{m-1}{r},\ F = -\frac{r}{m-1}.$$

When in the *double concave* or *double convex* lens the radii are equal,

$$\frac{1}{F} = \pm (m-1)\,\frac{2}{r},\ \text{or } F = \pm\frac{r}{2.(m-1)}.$$

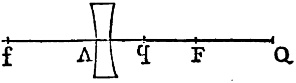
$$\text{Since } \frac{1}{\Delta''} = \frac{1}{F} + \frac{1}{\Delta}$$

$\Delta'' = \dfrac{\Delta F}{\Delta + F}$. Hence the following useful proportions, it being understood that F and $f$ are the principal foci of rays coming in a contrary direction.

$A q : A Q :: A F : A Q + A F$

or $Q f : f A :: Q A : A q$

$\& \therefore Q f : Q A :: Q A : Q q$



The following are corresponding values of $\Delta$ and $\Delta''$ for a *concave lens* :—

$$\infty \ldots 2F \ldots F \ldots \frac{F}{2} \ldots 0 \ldots -\frac{F}{2} \ldots -F \ldots -2F \ldots -3F \ldots -\infty$$

$$F \ldots \frac{2}{3}F \ldots \frac{F}{2} \ldots \frac{F}{3} \ldots 0 \ldots -F \ldots \infty \ldots 2F \ldots \frac{3}{2}F \ldots F$$

The following are for a *convex* one :—

$$\infty, 2\,\mathrm{F}, \mathrm{F}, \frac{9}{10}\,\mathrm{F}, \frac{\mathrm{F}}{2}, \left(\frac{\mathrm{F}}{n}\right), 0, -\frac{\mathrm{F}}{2}, \left(-\frac{\mathrm{F}}{n}\right), -\mathrm{F}, -3\,\mathrm{F}. -\infty$$

$$-\mathrm{F}, -2\,\mathrm{F}, \infty, 9\,\mathrm{F}, \mathrm{F}, \left(\frac{\mathrm{F}}{n-1}\right), 0, -\frac{\mathrm{F}}{3}, \left(-\frac{\mathrm{F}}{n+1}\right), -\frac{\mathrm{F}}{2}, -\frac{3}{4}\mathrm{F}, -\mathrm{F}.$$

If it be not thought proper to neglect $t$ the thickness of the lens, then in the case of parallel incident rays the equation is

$$\frac{1}{\mathrm{F}} = -(m-1)\left\{\frac{1}{r'} - \frac{1}{r}\left(1 + \frac{m-1}{m}\,\frac{t}{r}\right)^{-1}\right\}$$

3. If the lens be a sphere.

Put $q$ = distance of the focus of incident rays from the centre ; $q'$ = do. of focus of rays after the first refraction ; $q''$ = do. after two refractions ; then

$$\frac{1}{q''} = -2\,\frac{m-1}{m\,r} + \frac{1}{q}.$$

*Cor.* 1. To find the *principal* focus, suppose $q$ infinite, or $\frac{1}{q} = o$ ;

$\therefore \mathrm{F} = -\frac{m\,r}{2\,(m-1)}$. The negative sign meaning that the focus is on the opposite side from whence the light proceeds.

*Cor.* 2. If the sphere be glass, $m = \frac{3}{2}$, and $\mathrm{F} = \frac{3}{2}\,r$. If water, $m = \frac{4}{3}$, and $\mathrm{F} = 2\,r$.

4. If there be a compound lens, or a system of lenses placed close to each other, whose principal focal distances are $\mathrm{F}, \mathrm{F}', \mathrm{F}'' \ldots\ldots \mathrm{F}^{(n)}$, and $\mathrm{F}_n$ is the distance of the focus after all the refractions ; then

$$\frac{1}{\mathrm{F}_n} = \frac{m-1}{\varrho} + \frac{m'-1}{\varrho'} + \frac{m''-1}{\varrho''} \ldots\ldots + \frac{m^{(n)}-1}{\varrho^{(n)}}$$

$$\text{or} = \frac{1}{\mathrm{F}} + \frac{1}{\mathrm{F}'} + \frac{1}{\mathrm{F}''} \ldots\ldots + \frac{1}{\mathrm{F}^{(n)}}.$$

Hence if, with Mr Herschel, we call the reciprocal quantity $\frac{1}{\mathrm{F}}$ the *power* of a lens, we have the following enunciation,

" *The power of any system of lenses is the sum of the powers of the component lenses.*"

### III. *Refraction, images produced by.*

1. The image of a straight line, formed by a plane refracting surface, is a straight line: and if $a$ be the perpendicular distance of any point of the object from the surface, and $m$ the ratio of refraction, the distance of its image $= ma$.

*Cor.* In the case of water, $m = \frac{3}{4}$. Thus the image of the bed of a river is nearer to the surface than the bed itself by $\frac{1}{4}$ of the whole depth.

2. If the object placed before a lens or a sphere be a circular arc concentric with it, the image will also be a circular arc concentric with and similar to the object.

3. The image of a straight line formed by a lens or sphere is the arc of a conic section.

4. The sun's image formed by a lens is a circle, and nearly in the principal focus: and the density of his rays when viewed with a reflector or refractor varies as $\frac{\text{area of aperture} \times \text{power}}{(\text{Focal length})^2}$.

### TABLE,

*Of the refractive and dispersive power of different substances, with their densities compared with that of water, which is taken as the unit.*

The substances marked (*) are *combustible.*

The refraction is supposed to take place between the given substance and a vacuum.

| *Substance.* | *Ratio of refraction* | *Dispersive power.* | *Density.* |
|---|---|---|---|
| Chromate of lead (strongest) ...... | 2.974 | 0.4 | 5.8 |
| Realgar ...................................... | 2.549 | 0.267 | 3.4 |
| Chromate of lead (weakest) ........ | 2.503 | 0.262 | 5.8 |
| * Diamond .................................. | 2.45 | 0.038 | 3.521 |
| * Sulphur (native) ....................... | 2.115 | | 2.033 |
| Carbonate of lead (strongest) ...... | 2.084 | } 0.091 | 6.071 |
| ——— weakest ........................ | 1.813 | | 4.000 |
| Garnet ........................................ | 1.815 | 0.033 | 3.213 |
| Axinite ....................................... | 1.735 | 0.030 | |

| *Substance.* | *Ratio of refraction* | *Dispersive power.* | *Density.* |
|---|---|---|---|
| Calcareous Spar (strongest) ....... | 1.665 | 0.04 | 2.715 |
| ——— weakest ........................ | 1.519 | | |
| * Oil of Cassia ........................... | 1.641 | 0.139 | |
| Flint glass .................................. | 1.616 | 0.048 | 3.329 |
| ——— another kind ................ | 1.590 | | |
| Rock crystal ............................... | 1.562 | 0.026 | 2.653 |
| Rock salt .................................... | 1.557 | 0.053 | 2.130 |
| Canada balsam ........................... | 1.549 | 0.045 | |
| Crown glass ............................... | 1.544 | 0.036 | 2.642 |
| Selenite ...................................... | 1.536 | 0.037 | 2.322 |
| Plate glass ................................. | 1.527 | 0.032 | 2.488 |
| Gum arabic ................................ | 1.512 | 0.036 | 1.452 |
| * Oil of almonds ........................ | 1.483 | | 0.917 |
| * Oil of turpentine ..................... | 1.475 | 0.042 | 0.869 |
| Borax ......................................... | 1.475 | 0.030 | 1.718 |
| Sulphuric acid ........................... | 1.410 | 0.031 | 1.850 |
| Fluor spar .................................. | 1.436 | 0.022 | 3.168 |
| Nitric acid .................................. | 1.406 | 0.015 | 1.217 |
| Muriatic acid .............................. | 1.374 | 0.013 | 1.194 |
| * Alcohol ..................................... | 1.374 | 0.029 | 0.825 |
| White of egg .............................. | 1.361 | 0.037 | 1.090 |
| Salt water ................................... | 1.313 | | 1.026 |
| Water ........................................... | 1.336 | 0.035 | 1.000 |
| Ice ................................................ | 1.307 | | 0.930 |
| | | | |
| Air ................................................ | 1.00029 | | 0.0013 |
| Oxygen ........................................ | 1.00028 | | 0.0014 |
| * Hydrogen ................................... | 1.00014 | | 0.0001 |
| Nitrogen ...................................... | 1.00029 | | 0.0012 |
| Carbonic acid gas ...................... | 1.00045 | | 0.0018 |

**REFRACTION**, *terrestrial.—(Vince, Playfair, &c.)*

1. To determine it, let E = apparent elevation of a mountain from a point in the plain below; D = apparent depression of that point from the top of the mountain observed at the same moment; A = ∠ subtended at the earth's centre by the distance between them; then

$$\text{Refraction} = \frac{A + E - D}{2}.$$

The terrestrial refraction found by this theorem, when the elevation is not very great, varies from $\frac{1}{4}$ to $\frac{1}{24}$ of the ∠ A, but in the mean state of the atmosphere $= \frac{1}{14}$ of A, which, in taking the elevation of any object, must be subtracted from the observed ∠ E to give the correct elevation. Also the radius of curvature of the ray varies from twice to 12 times the earth's radius, but in the mean state of the atmosphere = 7 times earth's radius. When the ray is not horizontal it $= \frac{\text{7 times earth's radius}}{\text{sin. appar. zen. dist.}}$.

2. But in determining the height of a mountain, a correction may be made at once both for the curvature of the earth and for refraction thus. Let L = horizontal distance of the object in English miles, then the correction for curvature in feet is $\frac{2L^2}{3}$, *(see Levelling)* and for refraction is $\frac{2L^2}{21}$; ∴

$$\frac{2L^2}{3} - \frac{2L^2}{21} = \frac{4L^2}{7}$$ = feet which must be added to computed height, and it will give correct height both for curvature and refraction.

3. To determine the most distant point on the earth's surface that can be seen from the top of a given height with and without refraction.

Let $h$ = given height in miles, $r$ = earth's radius, then in the mean state of the atmosphere, the distance of the farthest visible point $= \sqrt{\frac{7rh}{3}}$; and distance, if there was no refraction, $= \sqrt{2rh}$; ∴ distance which the eye can reach with refraction : do. without :: $\sqrt{7}$ : $\sqrt{6}$ :: 14 : 13 nearly.

*Cor.* $\sqrt{\frac{7r}{2}} = 96.1$ miles, ∴ the distance of the farthest visible point in miles, allowing for refraction, $= 96.1\sqrt{h}$. Or by the last Art. if $h'$ = height in *feet*, $\frac{4L^2}{7} = h'$, ∴ $L = \frac{\sqrt{7h'}}{2}$.

4D: $d^2 = \frac{100}{76} h$,

$d$ = naut. miles ; $h$ = height in feet.

R E

By the last formula the following Table was computed :—

TABLE,

*Shewing the distance of the farthest visible point in miles that can be seen from the top of a given height, taking into account the effect of refraction.*

| Height in feet. | Dist. in miles. | Height in feet. | Dist. in miles. |
|---|---|---|---|
| 5 | 2.96 | 500 | 29.5 |
| 10 | 4.18 | 700 | 35.0 |
| 15 | 5.12 | 1000 | 41.8 |
| 20 | 5.91 | 1500 | 51 |
| 25 | 6.61 | 2000 | 59 |
| 30 | 7.25 | 2500 | 66 |
| 35 | 7.82 | 3000 | 72 |
| 40 | 8.37 | 3500 | 78 |
| 50 | 9.35 | 4000 | 83 |
| 60 | 10.25 | 5000 | 94 |
| 70 | 11.1 | 6000 | 102 |
| 100 | 13.2 | 7000 | 110 |
| 150 | 16.2 | 8000 | 118 |
| 200 | 18.7 | 9000 | 125 |
| 250 | 20.9 | 10000 | 132 |
| 300 | 22.9 | 15000 | 162 |
| 400 | 26.4 | 20000 | 187 |

*Ex.* 1. The topmast of a ship 50 feet high was just visible to a spectator situated 20 feet above the level of the sea; required the distance of the ship.

| | *Feet.* | | *Miles.* |
|---|---|---|---|
| By table | 50 | give | 9.35 |
| | 20 | do. | 5.91 |
| Required distance | | | 15.26 |

*Ex.* 2. The summit of Mouna Roa (whose height is supposed 15,000 feet) was observed at 180 miles distance; required the height of the observer.

| | *Miles.* |
|---|---|
| Observed distance | 180 |
| Distance due to 15000 feet | 162 |
| Difference | 18 |

which answers to a little less than 200 feet altitude.

TABLE *of distances at which mountains are said to have been observed.*

| | AUTHORITIES. | MILES. |
|---|---|---|
| Himalaya mountains | Sir W. Jones | 244 |
| Mount Ararat | Bruce | 240 |
| Mouna Roa, Sandwich Isles (53 leagues) | | 189 |
| Chimborazo (47 leagues) | | 160 |
| Peak of Teneriffe from Cape of Lanzerota | | 135 |
| Do. from ship's deck | | 115 |
| Peak of Azores | Humboldt | 126 |
| Temaheud | Morier | 100 |
| Mount Athos | Dr Clarke | 100 |
| Adam's Peak | | 95 |
| Ghaut at the back of Tellichery | | 94 |
| Golden Mount from ship's deck | | 93 |
| Pulo Pera from the top of Penang | | 75 |
| Ghaut at Cape Comorin | | 73 |
| Pulo Penang from ship's deck | | 53 |

The last six observations, and that of the Peak of Teneriffe, were made by a writer in the Calcutta Monthly Journal.

REFRACTION *of the heavenly bodies.—(Vince, Maddy.)*

1. The refraction of a star in the zenith is nothing, is greatest in the horizon, and at considerable altitudes is nearly as the tangent of the zenith distance. Or more nearly as tan. $(z-3r)$, if $z=$ zenith distance, and $r$ the refraction found by the common rule.

*Cor.* Refraction $= 57'' \times \tan.(z-3r.)$

2. To determine the refraction of a star by observation.

Observe the altitude and azimuth of a star of a known declination at the same moment: from the azimuth, the polar distance, and the complement of latitude, compute the altitude; the difference between this and the observed altitude is the refraction.

3. To determine how much the apparent time of rising and setting of a star is affected by refraction,

$$\text{Time} = \frac{\cancel{\text{hor.}}\ \text{refract. at hor.}}{15^0 \times \text{cos. lat.} \times \text{sin. star's azim.}}$$

Hence the time is least, when the star is in the equator. Or if $l=$ latitude, $\delta=$ star's declination, $r=$ ~~hor~~ refraction at horizon.

$$\text{Time} = \frac{r}{15^{\circ}\sqrt{\left(\cos.(l+\delta).\cos.(l-\delta)\right)}}$$

4. Twilight is occasioned by the refraction and reflexion of the sun's rays passing through the atmosphere, and continues till the sun descends about 18° degrees below the horizon.

To find the duration of twilight.

Let $h$ and $h'$ be the hour angles corresponding to the beginning and end of twilight, $l$ the latitude, and $\delta$ the sun's declination; then

$$\cos.\,h = -\tan.\,l.\,\tan.\,\delta$$

$$\cos.\,h' = -\sin.\,18^{0}.\,\sec.\,l.\,\sec.\,\delta - \tan.\,l.\,\tan.\,\delta$$

hence $h' - h$ may be deduced.

*Cor.* Twilight will continue all night, if $l + \delta$ be greater than 72°.

To find the time of year when twilight is shortest.

$$\sin.\,\delta = -\tan.\,9^{0}.\,\sin.\,l$$

$$\text{and } \sin.\,h = \sin.\,9^{0}.\,\sec.\,l$$

The first equation gives the sun's declination, or the time when the twilight is shortest; and the second gives the duration of it.

*Ex.* In latitude 52°, the time of shortest twilight will fall about March 2, and October 11; and the duration will be about 1*h*. 58*m*.

5. The refraction varies with the state of the barometer and thermometer.

*Dr Maskelyne's Formula.*

Let $a$ = height of barometer in inches, $h$ = height of Fahrenheit's thermometer, $z$ = zenith distance, $r = 57'' \tan.\, z$; then

$$\text{Refraction} = \frac{a}{29.6} \times \tan.\,(z - 3\,r) \times 57'' \times \frac{400}{350 + h}.$$

*Dr Young's Formula.*

$$.0002825 = v.\,\frac{r}{s} + (2.47 + .5\,v^{2})\,\frac{r^{2}}{s^{2}} + 3600\,v\,\frac{r^{3}}{s^{3}} + 3600\,(1.235 + .25\,v^{2})\,\frac{r^{4}}{s^{4}}\ \ldots\ldots$$

$r$ being the refraction, $v$ the sine of altitude, and $s$ the cosine.

From this last formula, the following Table, taken from the Nautical Almanack for 1827, is computed.

## TABLE OF REFRACTIONS.

| *App. Altitu.* | *Refr. B. 30 Th. 50°* | *Diff. for 1′ Alt.* | *Diff. for + 1 B.* | *Diff. for -1° Fa* | *App. Altitu.* | *Refr. B 30 Th 50°* | *Diff. for 1′ Alt.* | *Diff. for + 1 B.* | *Diff. for -1° Fa* |
|---|---|---|---|---|---|---|---|---|---|
| D.M. | M.S. | S. | S. | S. | D.M. | M.S. | S. | S. | S. |
| 0. 0 | 33.51 | 11,7 | 74 | 8,1 | 4. 0 | 11.52 | 2,2 | 24,1 | 1,70 |
| 5 | 32.53 | 11,3 | 71 | 7,6 | 10 | 11.30 | 2,1 | 23,4 | 1,64 |
| 10 | 31.58 | 10,9 | 69 | 7,3 | 20 | 11.10 | 2,0 | 22,7 | 1,58 |
| 15 | 31. 5 | 10,5 | 67 | 7,0 | 30 | 10.50 | 1,9 | 22,0 | 1,53 |
| 20 | 30 13 | 10,1 | 65 | 6,7 | 40 | 10.32 | 1,8 | 21,3 | 1,48 |
| 25 | 29.24 | 9,7 | 63 | 6,4 | 50 | 10.15 | 1,7 | 20,7 | 1,43 |
| 30 | 28.37 | 9,4 | 61 | 6,1 | 5. 0 | 9.58 | 1,6 | 20,1 | 1,38 |
| 35 | 27.51 | 9,0 | 59 | 5,9 | 10 | 9.42 | 1,5 | 19,6 | 1,34 |
| 40 | 27. 6 | 8,7 | 58 | 5,6 | 20 | 9.27 | 1,5 | 19,1 | 1,30 |
| 45 | 26.24 | 8,4 | 56 | 5,4 | 30 | 9 11 | 1,4 | 18,6 | 1,26 |
| 50 | 25.43 | 8,0 | 55 | 5,1 | 40 | 8.58 | 1,3 | 18,1 | 1,22 |
| 55 | 25. 3 | 7,7 | 53 | 4,9 | 50 | 8.45 | 1,3 | 17,6 | 1,19 |
| 1. 0 | 24.25 | 7,4 | 52 | 4,7 | 6. 0 | 8.32 | 1,2 | 17,2 | 1,15 |
| 5 | 23.48 | 7,1 | 50 | 4,6 | 10 | 8.20 | 1,2 | 16,8 | 1,11 |
| 10 | 23.13 | 6,9 | 49 | 4,5 | 20 | 8. 9 | 1,1 | 16,4 | 1,09 |
| 15 | 22.40 | 6,6 | 48 | 4,4 | 30 | 7.58 | 1,1 | 16,0 | 1,06 |
| 20 | 22. 6 | 6,3 | 46 | 4,2 | 40 | 7.47 | 1,0 | 15,7 | 1,03 |
| 25 | 21.37 | 6,1 | 45 | 4,0 | 50 | 7.37 | 1,0 | 15,3 | 1,00 |
| 30 | 21. 7 | 5,9 | 44 | 3,9 | 7. 0 | 7.27 | 1,0 | 15,0 | ,98 |
| 35 | 20.38 | 5,7 | 43 | 3,8 | 10 | 7.17 | ,9 | 14,6 | ,95 |
| 40 | 20.10 | 5,5 | 42 | 3,6 | 20 | 7. 8 | ,9 | 14,3 | ,93 |
| 45 | 19.43 | 5,3 | 40 | 3,5 | 30 | 6.59 | ,8 | 14,1 | ,91 |
| 50 | 19.17 | 5,1 | 39 | 3,4 | 40 | 6.51 | ,8 | 13,8 | ,89 |
| 55 | 18.52 | 4,9 | 38 | 3,3 | 50 | 6.43 | ,8 | 13,5 | ,87 |
| 2. 0 | 18.29 | 4,8 | 38 | 3,2 | 8. 0 | 6.35 | ,7 | 13,3 | ,85 |
| 5 | 18. 5 | 4,6 | 37 | 3,1 | 10 | 6.28 | ,7 | 13,1 | ,83 |
| 10 | 17.43 | 4,4 | 36 | 3,0 | 20 | 6.21 | ,7 | 12,8 | ,82 |
| 15 | 17.21 | 4,3 | 36 | 2,9 | 30 | 6.14 | ,7 | 12,6 | ,80 |
| 20 | 17. 0 | 4,1 | 35 | 2,8 | 40 | 6. 7 | ,7 | 12,3 | ,79 |
| 25 | 16.40 | 4,0 | 34 | 2,8 | 50 | 6. 0 | ,6 | 12,1 | ,77 |
| 30 | 16.21 | 3,9 | 34 | 2,7 | 9. 0 | 5.54 | ,6 | 11,9 | ,76 |
| 35 | 16. 2 | 3,7 | 33 | 2,7 | 10 | 5.47 | ,6 | 11,7 | ,74 |
| 40 | 15.43 | 3,6 | 32 | 2,6 | 20 | 5.41 | ,6 | 11,5 | ,73 |
| 45 | 15.25 | 3,5 | 32 | 2,5 | 30 | 5.36 | ,6 | 11,3 | ,71 |
| 50 | 15. 8 | 3,4 | 31 | 2,4 | 40 | 5.30 | ,5 | 11,1 | ,71 |
| 55 | 14.51 | 3,3 | 30 | 2,3 | 50 | 5.25 | ,5 | 11,0 | ,70 |
| 3. 0 | 14.35 | 3,2 | 30 | 2,3 | 10. 0 | 5,20 | ,5 | 10,8 | ,69 |
| 5 | 14.19 | 3,1 | 29 | 2,2 | 10 | 5.15 | ,5 | 10,6 | ,67 |
| 10 | 14. 4 | 3,0 | 29 | 2,2 | 20 | 5.10 | ,5 | 10,4 | ,65 |
| 15 | 13.50 | 2,9 | 28 | 2,1 | 30 | 5. 5 | ,5 | 10,2 | ,64 |
| 20 | 13.35 | 2,8 | 28 | 2,1 | 40 | 5. 0 | ,5 | 10,1 | ,63 |
| 25 | 13.21 | 2,7 | 27 | 2,0 | 50 | 4.56 | ,4 | 9,9 | ,62 |
| 30 | 13. 7 | 2,7 | 27 | 2,0 | 11. 0 | 4.51 | ,4 | 9,8 | ,60 |
| 35 | 12.53 | 2,6 | 26 | 2,0 | 10 | 4.47 | ,4 | 9,6 | ,59 |
| 40 | 12.41 | 2,5 | 26 | 1,9 | 20 | 4.43 | ,4 | 9,5 | ,58 |
| 45 | 12.28 | 2,4 | 25 | 1,9 | 30 | 4.39 | ,4 | 9,4 | ,57 |
| 50 | 12.16 | 2,4 | 25 | 1,9 | 40 | 4.35 | ,4 | 9,2 | ,56 |
| 55 | 12. 3 | 2,3 | 25 | 1,8 | 50 | 4.31 | ,4 | 9,1 | ,55 |

## TABLE OF REFRACTIONS.

| *App. Altitu.* | *Refr. B. 30 Th. 50°* | *Diff. for 1' Alt.* | *Diff. for +1 B.* | *Diff. for -1° Fa* | *App. Altitu.* | *Refr. B 30 Th. 50°* | *Diff. for 1' Alt.* | *Diff. for +1 B.* | *Diff. for -1° Fa* |
|---|---|---|---|---|---|---|---|---|---|
| D. M | M. S. | S. | S. | S. | D. | M. S. | S. | S. | S. |
| 12. 0 | 4.28,1 | ,38 | 9,00 | ,556 | 42 | 1. 4,6 | ,038 | 2,16 | ,130 |
| 10 | 4.24,4 | ,37 | 8,86 | ,548 | 43 | 1. 2,4 | ,035 | 2,09 | ,125 |
| 20 | 4.20,8 | ,36 | 8,74 | ,541 | 44 | 1. 0,3 | ,034 | 2,02 | ,120 |
| 30 | 4.17,3 | ,35 | 8,63 | ,533 | 45 | 58,1 | ,034 | 1,94 | ,117 |
| 40 | 4.13,9 | ,33 | 8,51 | ,524 | 46 | 56,1 | ,033 | 1,88 | ,112 |
| 50 | 4.10,7 | ,32 | 8,41 | ,517 | 47 | 54,2 | ,032 | 1,81 | ,108 |
| 13. 0 | 4. 7,5 | ,31 | 8,30 | ,509 | 48 | 52,3 | ,031 | 1,75 | ,104 |
| 10 | 4. 4,4 | ,31 | 8,20 | ,503 | 49 | 50,5 | ,030 | 1,69 | ,101 |
| 20 | 4. 1,4 | ,30 | 8,10 | ,496 | 50 | 48,8 | ,029 | 1,63 | ,097 |
| 30 | 3.58,4 | ,30 | 8,00 | ,490 | 51 | 47,1 | ,028 | 1,58 | ,094 |
| 40 | 3.55,5 | ,29 | 7,89 | ,482 | 52 | 45,4 | ,027 | 1,52 | ,090 |
| 50 | 3.52,6 | ,29 | 7,79 | ,476 | 53 | 43,8 | ,026 | 1,47 | ,088 |
| 14. 0 | 3.49,9 | ,28 | 7,70 | ,469 | 54 | 42,2 | ,026 | 1,41 | ,085 |
| 10 | 3.47,1 | ,28 | 7,61 | ,464 | 55 | 40,8 | ,025 | 1,36 | ,082 |
| 20 | 3.44,4 | ,27 | 7,52 | ,458 | 56 | 39,3 | ,025 | 1,31 | ,079 |
| 30 | 3.41,8 | ,26 | 7,43 | ,453 | 57 | 37,8 | ,025 | 1,26 | ,076 |
| 40 | 3.39,2 | ,26 | 7,34 | ,448 | 58 | 36,4 | ,024 | 1,22 | ,073 |
| 50 | 3.36,7 | ,25 | 7,26 | ,444 | 59 | 35,0 | ,024 | 1,17 | ,070 |
| 15. 0 | 3.34,3 | ,24 | 7,18 | ,439 | 60 | 33,6 | ,023 | 1,12 | ,067 |
| 30 | 3.27,3 | ,22 | 6,95 | ,424 | 61 | 32,3 | ,022 | 1,08 | ,065 |
| 16. 0 | 3.20,6 | ,21 | 6,73 | ,411 | 62 | 31,0 | ,022 | 1,04 | ,062 |
| 30 | 3.14,4 | ,20 | 6,51 | ,399 | 63 | 29,7 | ,021 | ,99 | ,060 |
| 17. 0 | 3. 8,5 | ,19 | 6,31 | ,386 | 64 | 28,4 | ,021 | ,95 | ,057 |
| 30 | 3. 2,9 | ,18 | 6,12 | ,374 | 65 | 27,2 | ,020 | ,91 | ,055 |
| 18. 0 | 2.57,6 | ,17 | 5,93 | ,362 | 66 | 25,9 | ,020 | ,87 | ,052 |
| 19. 0 | 2.47,7 | ,16 | 5,61 | ,340 | 67 | 24,7 | ,020 | ,83 | ,050 |
| 20 | 2.38,7 | ,15 | 5,31 | ,322 | 68 | 23,5 | ,020 | ,79 | ,047 |
| 21 | 2.30,5 | ,13 | 5,04 | ,305 | 69 | 22,4 | ,020 | ,75 | ,045 |
| 22 | 2.23,2 | ,12 | 4,79 | ,290 | 70 | 21,2 | ,020 | ,71 | ,043 |
| 23 | 2.16,5 | ,11 | 4,57 | ,276 | 71 | 19,9 | ,020 | ,67 | ,040 |
| 24 | 2.10,1 | ,10 | 4,35 | ,264 | 72 | 18,8 | ,019 | ,63 | ,038 |
| 25 | 2. 4,2 | ,09 | 4,16 | ,252 | 73 | 17,7 | ,018 | ,59 | ,036 |
| 26 | 1.58,8 | ,09 | 3,97 | ,241 | 74 | 16,6 | ,018 | ,56 | ,033 |
| 27 | 1.53,8 | ,08 | 3,81 | ,230 | 75 | 15,5 | ,018 | ,52 | ,031 |
| 28 | 1.49,1 | ,08 | 3,65 | ,219 | 76 | 14,4 | ,018 | ,48 | ,029 |
| 29 | 1.44,7 | ,07 | 3,50 | ,209 | 77 | 13,4 | ,017 | ,45 | ,027 |
| 30 | 1.40,5 | ,07 | 3,36 | ,201 | 78 | 12,3 | ,017 | ,41 | ,025 |
| 31 | 1.36,6 | ,06 | 3,23 | ,193 | 79 | 11,2 | ,017 | ,38 | ,023 |
| 32 | 1.33,0 | ,06 | 3,11 | ,186 | 80 | 10,2 | ,017 | ,34 | ,021 |
| 33 | 1.29,5 | ,06 | 2,99 | ,179 | 81 | 9,2 | ,017 | ,31 | ,018 |
| 34 | 1.26,1 | ,05 | 2,88 | ,173 | 82 | 8,2 | ,017 | ,27 | ,016 |
| 35 | 1.23,0 | ,05 | 2,78 | ,167 | 83 | 7,1 | ,017 | ,24 | ,014 |
| 36 | 1.20,0 | ,05 | 2,68 | ,161 | 84 | 6,1 | ,017 | ,20 | ,012 |
| 37 | 1.17,1 | ,05 | 2,58 | ,155 | 85 | 5,1 | ,017 | ,17 | ,010 |
| 38 | 1.14,4 | ,05 | 2,49 | ,149 | 86 | 4,1 | ,017 | ,14 | ,008 |
| 39 | 1.11,8 | ,04 | 2,40 | ,144 | 87 | 3,1 | ,017 | ,10 | ,006 |
| 40 | 1. 9,3 | ,04 | 2,32 | ,139 | 88 | 2,0 | ,017 | ,07 | ,004 |
| 41 | 1. 6,9 | ,04 | 2,24 | ,134 | 89 | 1,0 | ,017 | ,03 | ,002 |

### *Explanation of the Table of Refractions.*

The apparent altitude being found in the first column, the second shows the refraction when the barometer stands at 30 inches, which is its mean height on the level of the sea, and the thermometer at 50° of Fahrenheit. The third column contains the difference to be subtracted or added for every minute of altitude, reckoned from the nearest number in the first column. The fourth shows the number of seconds to be added for every inch that the height of the barometer exceeds 30, or to be subtracted for each inch that it wants of 30; and the last contains the number of seconds to be subtracted for each degree that the thermometer stands above 50°, or to be added for each degree that its height wants of 50°.

*Ex.* At 7°. 18′. 13″. Bar. 29.87. Ther. 66°. required refraction.

| | | | |
|---|---|---|---|
| Alt. 7°. 20′. R. 7′. 8″ | Diff. Alt. ″,9 | B. 14″, 3 | Th. ″,93 |
| + 1,62 | 1′. 47″ = 1′. 8 | − .13 | − 16 |
| 7. 9,62 | + 1,62 | 1,86 | 14,88 |
| 16,74 | | | 1,86 |
| Ref. = 6. 52,88 | | | 16,74 |

REFRANGIBILITY *of light.—See Light.*

RESISTANCE *of air to Projectiles.—See Gunnery.*

RESISTANCE *of Fluids.—See Fluids.*

RIVER.—*(Du Buat, Robison.)*

1. Let V = velocity of the stream per second in inches, R the quotient arising from the division of the section of the stream, expressed in square inches, by its perimeter *minus* the superficial breadth of the stream in linear inches, S the slope the numerator being unity, i.e. the quotient arising from dividing the length of the stream, supposing it extended in a straight line, by the difference of level of its two extremities, or let it be the cotangent of the inclination or slope;—then the section and velocity being both supposed uniform,

$$V = \sqrt{R - \frac{1}{10}}\left(\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,\text{h. l.}\left(S + \frac{16}{10}\right)} - \frac{3}{10}\right)$$

When R and S are very great

$$V = R^{\frac{1}{2}}\left(\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,\text{h. l. } S} - \frac{3}{10}\right) \text{ nearly.}$$

The slope remaining the same, the velocities are as $\sqrt{R - \frac{1}{10}}$ or as $\sqrt{R}$, when R is very great.

The velocity will become nothing by making the declivity so small that $\frac{307}{S^{\frac{1}{2}} - \frac{1}{2}\,\text{h.l.}\left(S + \frac{16}{10}\right)} - \frac{3}{10} = 0$; but if $\frac{1}{S}$ is less than $\frac{1}{500000}$ or than $\frac{1}{10}$th of an inch to an English mile, the water will have sensible motion.

In the above formula R is called the *radius of the section.*

2. In a river the greatest velocity is at the surface and in the middle of the stream, from which it diminishes towards the bottom and sides, where it is least; and it has been found by experiment, that if $v$ = velocity of the stream in the middle in inches, then the velocity at the bottom is

$$v - 2\sqrt{v} + 1.$$

3. The mean velocity, or that with which (were the whole stream to move) the discharge would be the same with the real discharge, is equal to half the sum of the greatest and least velocities, as computed in the last Prop. Hence the mean velocity $= v - \sqrt{v} + \frac{1}{2}$.

4. Suppose that a river having a rectangular bed is increased by the junction of another river equal to itself, the declivity remaining the same; required the increase of depth.

Let the breadth of the river $= b$, the depth before the junction $= d$, and after it $= x$; then

$x^3 - \frac{3\,d^3}{b + 2\,d}\,x = \frac{4\,b\,d^3}{b + 2\,d}$, a cubic equation which can always be resolved by Cardan's rule.

5. To find the fall of water under bridges, let the breadth of the river in feet $= b$; the breadth between the piers $= c$; the velocity in a second $= v$; $g = 32\frac{1}{6}$ feet; then the fall of the river will be

$$\left\{\left(\frac{25\,b}{21\,a}\right)^2 - 1\right\}\frac{v^2}{2g}.$$

Thus at London bridge $b = 926$, $c = 236$, reduced by the piles to $196\frac{3}{5}$, $v = 3\frac{1}{6}$, hence the fall is 4.739; by observation 4.75.—(*Young's. Nat. Phil.*)

6. When the sections of a river vary, the quantity of water remaining the same, the mean velocities are inversely as the areas of the sections.

7. The following Table abridged from Dr Robison serves at once to compare the surface, bottom, and mean velocities in rivers according to the principles of Arts. 2, 3.—(*Gregory.*)

| *Velocity in Inches.* | | | *Velocity in Inches.* | | |
|---|---|---|---|---|---|
| *Surface.* | *Bottom.* | *Mean.* | *Surface.* | *Bottom.* | *Mean.* |
| 4 | 1 | 2.5 | 56 | 42.016 | 49.008 |
| 8 | 3.342 | 5.67 | 60 | 45.509 | 52.754 |
| 12 | 6.071 | 9.036 | 64 | 49.0 | 56.5 |
| 16 | 9.0 | 12.5 | 68 | 52.505 | 60.252 |
| 20 | 12.055 | 16.027 | 72 | 56.025 | 64.012 |
| 24 | 15.194 | 19.597 | 76 | 59.568 | 67.784 |
| 28 | 18.421 | 23.210 | 80 | 63.107 | 71.553 |
| 32 | 21.678 | 26.839 | 84 | 66.651 | 75.325 |
| 36 | 25.0 | 30.5 | 88 | 70.224 | 79.112 |
| 40 | 28.245 | 34.172 | 92 | 73.788 | 8[illegible].891 |
| 44 | 31.742 | 37.871 | 96 | 77.370 | 86.685 |
| 48 | 35.151 | 41.570 | 100 | 81.0 | 90.5 |
| 52 | 38.564 | 45.282 | | | |

8. Eytelwein, a German mathematician, gives the following formula for the mean velocity of the stream of a canal. Let $v$ be the mean velocity of the current in English feet, $a$ the area of the vertical section of the stream, $p$ the perimeter of the section, or sum of the bottom and two sides, $l$ the length of the bed of the canal corresponding to the fall $h$, all in feet; then

$$v = -0.109 + \sqrt{9582\,\frac{a\,h}{p\,l} + 0.0111}$$

9. To find experimentally the velocity of the water in a river, and the quantity which flows down in a given time, observe a place where the banks of the river are steep and nearly parallel, and by taking the depth at various places in crossing make a true section of the river. Stretch a string at right ∠'s. over it, and at a small distance another parallel to the first. Then take an apple, orange, or a pint or quart bottle partly filled with water so as just to swim in it, and throw it into the water above the strings. Observe when it comes under the first string by means of a quarter second pendulum or a stop watch, and observe also when it arrives at the second string. By this means the velocity of the upper surface, which in practice may *frequently* be taken for that of the whole, will be obtained. The section of the river at the second string must then be ascertained by taking various depths as before, and the mean of the two will be obtained by adding both together and taking half the sum for the mean section. Then the area of the mean section in square feet being multiplied by the distance between the strings in feet will give the contents of the water in solid feet which passed from

one string to the other during the time of observation; and this by the rule of three may be adapted to any other portion of time. This operation may often be greatly abridged by noticing the arrival of the floating body opposite to two stations on the shore, especially when it is not convenient to stretch a string across. Where a time piece is not at hand the observer may easily construct a quarter second or other pendulum.

RIVERS, *proportional lengths of, and supposed quantity of water discharged per annum.—(Ency. Brit. Suppl.)*

| | RIVERS. | LENGTH. | QY. OF WATER. |
|---|---|---|---|
| EUROPE | Thames | 1 | 1 |
| | Rhine | 4⅓ | 13 |
| | Loire | 4 | 10 |
| | Po | 2¼ | 6 |
| | Elbe | 4⅓ | 8 |
| | Vistula | 4¼ | 12 |
| | Danube | 9¾ | 65 |
| | Dneiper | 7¾ | 36 |
| | Don | 7⅓ | 38 |
| ASIA … | Wolga | 14 | 80 |
| | Euphrates | 9¾ | 60 |
| | Indus | 11⅓ | 133 |
| | Ganges | 10 | 148 |
| | Kang-tse or Great river of China | 21⅓ | 258 |
| | Amour, Chinese Tartary | 16 | 166 |
| | Lena, Asiatic Russia | 13⅓ | 125 |
| | Oby do. | 15 | 179 |
| AFRICA | Nile | 18⅓ | 250 |
| AMERICA | St Lawrence including Lakes | 22⅓ | 112 |
| | Mississippi | 19 | 338 |
| | Plata | 13⅓ | 490 |
| | Amazon, not including Araguay | 22¾ | 1280 |

To deduce the approximate lengths of the rivers in miles from the proportional lengths we may multiply the latter by 180. To convert the proportional discharge into known measures we may multiply by 1800 to obtain the number of *cubic feet per second*, or by .4 or $\frac{4}{10}$ to find the annual discharge in *cubic miles.*

*Proportional lengths according to Major Rennell.*

| | | |
|---|---|---|
| EUROPE | Thames | 1 |
| | Rhine | 5¼ |
| | Danube | 7 |
| | Volga | 9½ |
| ASIA ... | Indus | 6¾ |
| | Euphrates | 8½ |
| | Ganges | 9½ |
| | Burrampooter | 9½ |
| | Ava River | 9½ |
| | Jenicei | 10 |
| | Obi | 10½ |
| | Amour | 11 |
| | Lena | 11½ |
| | Hoang-Ho | 13½ |
| | Kian Ku | 15½ |
| AFRICA | Nile | 12¼ |
| AMERICA | Mississippi | 8 |
| | Amazon | 15¾ |

**ROOFS**, *equilibrium of.—(Whewell.)*

1. A roof A C A′, consisting of beams forming an isosceles triangle with its base horizontal, supports a given weight at its vertex C: the weights of the beams being also given; it is required to find the horizontal force at A and A′.

Let B be the weight of the beam A C, C the weight at C, $\alpha$ the angle which A C makes with the horizon, H the horizontal pressure at A; then

$$H = \frac{B + C}{2 \tan. \alpha}.$$

If there is no beam joining A A′, this horizontal pressure H must be counteracted by the supports on which the ends A, A′ are placed.

If the roof A C A′ support a covering of uniform thickness, the formula will still be true including in the weight B, the weight of that portion of the covering which rests upon the beam.

The weight C at the point C may arise from a longitudinal beam perpendicular to the plane A A′ C.

2. Any number of given beams, arranged as sides of a polygon, in a vertical plane, support each other, and support also given weights at the ∠.s; it is required to find the horizontal pressure at the points of support.

Let B and $B_{\prime}$ be the weights of two contiguous beams, $\alpha$ and $\alpha_{\prime}$ the angles they make with the horizon, and C the given weight at the ∠, or point of junction; then

$$H = \frac{\frac{1}{2}(B + B_{\prime}) + C}{\text{tan. } \alpha - \text{tan. } \alpha_{\prime}}.$$

This horizontal pressure is the same at all the angles.

*Cor.* If we suppose the weights of the beams $= o$, $H = \dfrac{C}{\text{tan. } \alpha - \text{tan. } \alpha_{\prime}}$; If we suppose no weights, except the beams,

$$H = \frac{\frac{1}{2}(B + B_{\prime})}{\tan \alpha - \tan \alpha_{\prime}}.$$

3. To find the position of the beams, having given their weights $B_1$, $B_2$, $B_3$ &c. the weights $C_1$, $C_2$, $C_3$ &c. and the position of two of them.

By the last Prop. we have the following equations, $\alpha_1$, $\alpha_2$, $\alpha_3$ being the ∠.s which the beams make with the horizon.

$$H (\text{tan. } \alpha_1 - \text{tan. } \alpha_2) = \tfrac{1}{2} (B_1 + B_2) + C_1$$

$$H (\text{tan. } \alpha_2 - \text{tan. } \alpha_3) = \tfrac{1}{2} (B_2 + B_3) + C_3$$

&c. &c.

If there be $n$ beams there will be $n - 1$ weights $C_1$, $C_2$ &c. and $n - 1$ equations. The number of unknown quantities is $n + 1$, viz. the $n$ tangents, tan. $\alpha_1$, tan. $\alpha_2$ &c. and the pressure H. Hence if we know two of the ∠.s $\alpha_1$, $\alpha_2$, we can find the rest.

ROOTS *of numbers.—See Involution.*

ROPES, *rigidity of.—See Friction.*

ROTATION *of bodies about a fixed or moveable axis.*

The following Proposition is of the greatest use in Mechanics, and is general under the circumstances there mentioned, whether bodies move in right lines or have a rotatory motion. It applies with peculiar facility to the investigation of the motion of revolving bodies, and by the help of it the most difficult problems admit of a simple and easy solution.

*Prop.* If a system of bodies be connected together and supported at any point which is not the centre of gravity, and then left to descend by that part of their weight which is not supported; $2g$ multiplied into the sum of all the products of each body into the space it has perpendicularly descended will be equal to the sum of all the products of each body into the square of its velocity, $g$ being $= 32\frac{1}{6}$ feet.—*(Mr Dawson, Sedbergh.)*

A demonstration of this Prop. may be seen in Leybourn's Mathematical Repository.

*Ex.* 1. Let a cylinder whose weight = W, moveable about a horizontal axis passing through the centre, be put in motion by a weight P attached to a string wound round it; required the force accelerating the body P, and the space descended in $t$ seconds.

Let $s =$ space perpendicularly descended by P, $v =$ velocity acquired in the time $t$, $r =$ radius of the cylinder, $x =$ distance of the centre of gyration from the centre of the cylinder; then by the Prop.

$$2g \times Ps = Pv^2 + W \times v^2 \frac{x^2}{r^2} = Pv^2 + W \times \frac{v^2}{2}$$

$$\text{but } s = \frac{tv}{2}, \therefore v = \frac{2gPt}{2P+W}, \text{ or if } t = 1$$

$$v = \frac{2gP}{2P+W} = \text{accelerating force.}$$

To find $s$, put $v^2 = \frac{4s^2}{t^2}$ in the leading equation, and we shall have

$$s = \frac{gPt^2}{2P+W}.$$

*Ex.* 2. A given cylinder with a thread wound round it is suffered to unwrap itself and descend; required the time of its descent through a given space.

The same notation being retained

$$2g \times Ws = Wv^2 + Wv^2 \times \frac{x^2}{r^2} = W \times \left(v^2 + \frac{v^2}{2}\right)$$

$$= \frac{3Wv^2}{2} \text{ but } v = \frac{2s}{t},$$

$$\therefore t = \sqrt{\frac{3s}{g}}.$$

*Ex.* 3. P and W are hung over a fixed pulley, to find how far P will descend in $t''$.

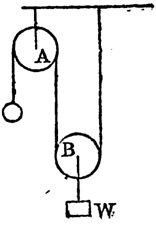
Let $r$ = radius of pulley, $w$ = its weight, $x$ = distance of the centre of gyration from its centre; then

$$2g \times (P - W)\,s = (P + W)\,v^2 + w\,v^2 \times \frac{x^2}{r^2}$$

$$= (P + W)\,v^2 + w.\,\frac{v^2}{2}, \text{ but } v = \frac{2\,s}{t},$$

$$\therefore\ s = \frac{g\,t^2\,(P - W)}{2\,P + 2\,W + w}.$$

*Ex.* 4. Let A and B represent a single fixed and moveable pulley as represented in the annexed figure; required the space which the descending weight P describes in a given time.



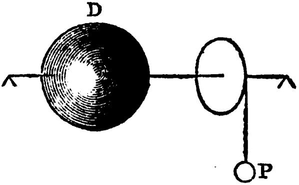
Let $w$ = weight of each pulley, $v$ = velocity of P, then $\frac{v}{2}$ = velocity of W; also $\frac{v}{\sqrt{2}}$ = velocity of the centre of gyration of A, and $\frac{v}{2\sqrt{2}}$ = velocity of the same centre in B; then

$$2g \times \left(P\,s - W.\,\frac{s}{2}\right) = P\,v^2 + W \times \frac{v^2}{4} + w \times \frac{v^2}{2} + w \times \frac{v^2}{8}$$

$$= P\,v^2 + W \times \frac{v^2}{4} + w \times \frac{5\,v^2}{8}, \text{ but } v = \frac{2\,s}{t},\ \therefore$$

$$s = \frac{2\,g\,t^2 \times (2\,P - W)}{8\,P + 2\,W + 5\,w}$$

*Ex.* 5. A sphere D, whose radius is $\rho$ and weight W, is put in motion by a weight P acting by means of a string going over a wheel whose radius is $r$: required the velocity acquired in the time $t$.



Let $v$ = velocity of P, $s$ the space descended by P in $t''$, $x$ = distance of the centre of gyration of the sphere from its centre then

$$2\,g \times P\,s = P\,v^2 + W\,v^2 \times \frac{x^2}{r^2};\ \text{but } x = \rho \times \sqrt{\frac{2}{5}},$$

$$\therefore 2g \times Ps = Pv^2 + Wv^2 \times \frac{2\varrho^2}{5r^2}$$

$$\text{but } s = \frac{tv}{2}, \ \therefore v = \frac{5gPtr^2}{5r^2P + 2W\varrho^2}.$$

Or by substituting $\frac{2s}{t}$ for $v$; $s$ or $t$ may be found.

*Ex.* 6. Let a weight P, fastened to a string going over a wheel, by its descent cause two weights W, W′ to be wound up on two axles. Required the velocity of P after it has descended $t''$; the radii of the wheel and of the two axles being $r$, $\varrho$, $\varrho'$.

Here

$$2g \times \left( Ps - W \times \frac{s\varrho}{r} - W' \times \frac{s\varrho'}{r} \right) = Pv^2 + Wv^2 \times \frac{\varrho^2}{r^2} + W'v^2 \times \frac{\varrho'^2}{r^2}$$

$$\text{or } 2g \times (Pr^2 - Wr\varrho - W'r\varrho') \times \frac{tv}{2} = (Pr^2 + W\varrho^2 + W'\varrho'^2)\, v^2$$

$$\therefore v = gt \times \frac{Pr^2 - Wr\varrho - W'r\varrho'}{Pr^2 + W\varrho^2 + W'\varrho'^2}.$$

Here the weight of the wheel and axles are not taken into the account.

*Ex.* 7. The force which accelerates the centre of gravity of a sphere, while it rolls down an inclined plane, is to the force by which it would be accelerated, were the sphere to slide, in the ratio of 5 to 7.

Let W = weight of the sphere, $s$ = space descended along the plane, $v$ = velocity generated in time $t$ when the sphere rolls, $v'$ = do. when it slides, then since the distance of the centre of gyration $= r\sqrt{\frac{2}{5}}$, we have, when the sphere rolls,

$$2g \times W \times s \times \frac{H}{L} = Wv^2 + W \times v^2 \times \frac{2}{5}.$$

$$\text{or } gtv \times \frac{H}{L} = \frac{7}{5}\, v^2.$$

When the sphere slides,

$$gtv \times \frac{H}{L} = v'^2;$$

$$\therefore v : v' :: 5 : 7.$$

# S

**SATELLITES.**—*(Vince, Playfair.)*

1. *Of Jupiter.*

Jupiter's satellites were discovered by Galileo in 1610. The times of their rotation are the same with the periodic times round the primary. Occultations happen to the first and second satellites at every revolution; the third very rarely escapes an occultation, but the fourth more frequently, by reason of its distance. The three first are eclipsed in every revolution, the fourth not always. In the first satellite we never can see both the immersion and emersion; the other three satellites *may* have both visible, but it depends on the position of the earth. The first satellite is the most proper for finding the longitude, its tables being the most correct. The observer should be settled at his telescope three minutes before the expected time of an immersion of the first satellite, six or eight minutes before that of the second or third; and at least a quarter of an hour before that of the fourth. If the longitude be different from that of Greenwich, allowance must be made for it. The telescopes proper for observing these eclipses are reflecting ones of 18 inches or 2 feet, or the 46 inch achromatic with three object glasses.

There is a singular analogy between the three first satellites, discovered by Laplace, viz. that if $m'$, $m''$, $m'''$, are the mean motions of the 1st, 2d, and 3d, satellites of Jupiter,

$$m' + 2\,m''' = 3\,m''.$$

Also if $L'$, $L''$ $L'''$, are the mean longitudes of these satellites,

$$L' - 3\,L'' + 2\,L''' = 180^{\circ}.$$

The last equation shews that the three satellites can never be eclipsed at the same time.

*Table of the Satellites.*

| | I. | II. | III. | IV. |
|---|---|---|---|---|
| *Sidereal Revolution.* | *d. h. m. s.*<br>1 18 27 33 | *d. h. m. s.*<br>3 13 13 42 | *d. h. m. s.*<br>7 3 42 33 | *d. h. m. s.*<br>16 16 32 8 |
| *Synodic Revolution.* | 1 18 28 36 | 3 13 17 54 | 7 3 59 36 | 16 18 5 7 |
| *Mean distance, the radius of the Planet being* 1. | 5.812964 | 9.248679 | 14.752401 | 25.946860 |
| *Mass.* | .0000173281 | .0000232355 | .0000884972 | .0000426591 |
| *Greatest duration of an eclipse.* | *h m.*<br>2 16 | *h. m.*<br>2 54 | *h. m.*<br>3 34 | *h. m.*<br>4 48 |

## TABLE

**Of the apparent distances of Jupiter's satellites from its limb at the time of an eclipse, in ½ diameters of Jupiter and decimal parts, for every tenth day of Jupiter's distance from opposition or conjunction.** *Note.*—**Before the oppositions of Jupiter the immersions and emersions happen to the west of Jupiter; after opposition they happen to the east; in an astronomical telescope the appearance will be contrary.**

| *Distance of Jupiter from opposition to the Sun.* | *Distance of the Satellites from Jupiter's limb at the eclipses, in semidiameters of Jupiter.* | | | | *Distance of Jupiter from conjunction with the Sun.* | *Distance of the Satellites from Jupiter's limb at the eclipses, in semidiameters of Jupiter.* | | | |
|---|---|---|---|---|---|---|---|---|---|
| *Days.* | I. | II. | III. | IV. | *Days.* | I. | II. | III. | IV. |
| 10 | 0.20 | 0.33 | 0.50 | 0.85 | 10 | 0.15 | 0.25 | 0.35 | 0.56 |
| 20 | 0.40 | 0.66 | 1.05 | 1.66 | 20 | 0.30 | 0.45 | 0.70 | 1.25 |
| 30 | 0.60 | 0.95 | 1.50 | 2.65 | 30 | 0.40 | 0.67 | 1.05 | 1.70 |
| 40 | 0.75 | 1.20 | 1.90 | 3.35 | 40 | 0.55 | 0.90 | 1.40 | 2.50 |
| 50 | 0.90 | 1.40 | 2.25 | 3.95 | 50 | 0.70 | 1.00 | 1.80 | 3.20 |
| 60 | 1.00 | 1.60 | 2.50 | 4.40 | 60 | 0.80 | 1.25 | 2.00 | 3.50 |
| 70 | 1.05 | 1.70 | 2.66 | 4.70 | 70 | 0.90 | 1.40 | 2.25 | 3.95 |
| 80 | 1.10 | 1.75 | 2.75 | 4.85 | 80 | 1.00 | 1.55 | 2.45 | 4.33 |
| 90 | 1.10 | 1.75 | 2.75 | 4.85 | 90 | 1.05 | 1.66 | 2.60 | 4.60 |
| 100 | 1.10 | 1.70 | 2.70 | 4.80 | 100 | 1.10 | 1.75 | 2.70 | 4.80 |

The difficulty of observing the immersions, and particularly the emersions of Jupiter's satellites, may be attributed to the observer not having his eye well directed to the spot at which the satellite first issues from the shadow. The discordancies will be materially diminished by the above Table, particularly if a diagram be formed from it, representing the disk of Jupiter at the several times mentioned in the Table, and the proportional distances of the several satellites as there expressed.

2. *Saturn's satellites.*

The 4th satellite of Saturn was discovered by Huygens, in 1655; and the 1st, 2d, 3d, and 5th by Cassini, within the years 1671 and 1684. Herschel discovered two others in 1789 interior to the other five, but which, to prevent confusion, are called the 6th and 7th, the 7th being the innermost. They revolve nearly all in the same plane, inclined to Saturn's orbit at an ∠ of about 30°; hence they are eclipsed seldomer than Jupiter's. The 5th satellite (like those of Jupiter) revolves round its axis in the same time as round Saturn; a remarkable instance of analogy among the secondary planets.

*Table of the Satellites.*

| | VII. | VI. | I. | II. | III. | IV. | V. |
|---|---|---|---|---|---|---|---|
| *Sidereal Revolution.* | *h. m. s.*<br>22.37.30 | *d.h.m.s*<br>1.8.53.9 | 1.21.18.26 | 2.17.44.51 | 4.12.25.11 | 15.22.41.14 | 79.7.54.37 |
| *Synodic Revolution.* | | | 1.21.18.55 | 2.17.45.51 | 4.12.27.55 | 15.23.15.23 | 79.22.3.13 |
| ***Mean dist.*** *rad. of planet being* 1. | 3.080 | 3.952 | 4.893 | 6.268 | 8.754 | 20.295 | 59.154 |

3. ***Satellites of the Georgian Planet.***

These six satellites were discovered by Dr Herschel, in 1787 and 1788. They all move in a plane which is nearly ***perpendicular*** to the plane of the planet's orbit, and ***contrary to the order of the signs.***

*Table of the Satellites.*

| | I. | II. | III. | IV. | V. | VI. |
|---|---|---|---|---|---|---|
| ***Sidereal Revolution.*** | ***d. h. m. s.***<br>5.21.25.20 | 8.16.57.47 | 10.23.3.59 | 13.10.56.30 | 38.1.48.0 | 107.16.39.56 |
| ***Mean dist. rad. of planet being*** 1. | 13.120 | 17.022 | 19.845 | 22.752 | 45.507 | 91.008 |

SATURN. ***For its elements, &c.—See Planets elements of. And for its satellites—See Satellites.***

***Saturn's ring.—( Vince.)***

Galileo announced his discovery of Saturn's ring in 1610. Dr Herschel and others have since ascertained that it consists of two concentric rings, situated in one plane, which is probably not much inclined to the equator of the planet. The dimensions of the rings are as follows :—

| | *Miles.* |
|---|---|
| Inside diameter smaller ring | 146345 |
| Outside do. | 184393 |
| Inside diameter larger ring | 190248 |
| Outside do. | 204883 |
| Breadth of inner ring | 20000 |
| ——— of outer ring | 7200 |
| Space between rings | 2839 |
| ——— between planet and ring | 70277 |
| Mean thickness of ring | 4500 |

Time of rotation ………………… 10*h*. 32*m*. 15,4*s*.

When Saturn's geocentric longitude is 5*s*. 20°. or 11*s*. 20°, his ring is invisible to us. When he is in 2*s*. 20°. or 8*s*. 20°. we may see it to most advantage; its minor axis is then nearly half its major. But the following Table will shew both the apparent figure of the ring, and of the orbits of the six first satellites, at all times, and as seen either from the sun or the earth.

N.B. When the geocentric latitude and longitude are taken, we get the appearance as seen from the earth; the heliocentric latitude and longitude being assumed, gives the appearance as seen from the sun.

*For the ring and six first Satellites.*

Arg. Long. Sat. + 13°. 43′. 30″

| Deg. | O. VI.<br>− + | I. VII.<br>− + | II. VIII.<br>− + | Deg. |
|---|---|---|---|---|
| 0 | 0,000 | 0,260 | 0,451 | 30 |
| 3 | 0,027 | 0,284 | 0,464 | 27 |
| 6 | 0,054 | 0,306 | 0,476 | 24 |
| 9 | 0,081 | 0,328 | 0,486 | 21 |
| 12 | 0,108 | 0,348 | 0,495 | 18 |
| 15 | 0,135 | 0,368 | 0,503 | 15 |
| 18 | 0,161 | 0,384 | 0,509 | 12 |
| 21 | 0,187 | 0,405 | 0,514 | 9 |
| 24 | 0,212 | 0.421 | 0,518 | 6 |
| 27 | 0,236 | 0,437 | 0,520 | 3 |
| 30 | 0,260 | 0,451 | 0,521 | 0 |
| | + −<br>XI. V. | + −<br>X. IV. | + −<br>IX. III. | |

To the quantity taken from the Tables, apply the latitude of Saturn expressed in minutes divided by 4000, with the sign —, when the latitude is north, and +, when it is south; and the result gives the minor axis of the ring or of the orbits, the major axis being unity.

*Ex.* On April 22, 1767, the geocentric latitude of Saturn was 1°. 10′ south, and longitude 2*s*. 16°. 55′; hence for the ring and six first satellites.

2s. 16°. 55'.
13 43
3 0 38 ......... — 0.521
$\frac{70}{4000}$ ......... + 0.017
Minor axis ......... — 0,504

The sign + shews that that half of the ring, or of the orbits, which is most distant, is more *north* than the centre of Saturn, and the sign — shews it to be more south.

SCREW.

When there is an equilibrium upon the screw, P : W :: the distance between two contiguous threads, measured in the direction of the axis : the circumference of the circle which the power describes.

Hence if $d$ = distance between the threads, $a$ = radius of the circle described by the power, $P : W :: d : 2 \pi a$ ; $\therefore P = \frac{W d}{2 \pi a}$, from which equation any three of the four quantities P, W, $a$, $d$ being given, the fourth may be found.

In the endless screw, which works in, and turns a dented wheel, let $a$ = length of the lever, R = radius of the wheel, $r$ = do. of the axle, the rest as before; then

$$P : W :: dr : 2 \pi a R;$$
$$\therefore P = \frac{W d r}{2 \pi a R}.$$

SEA WATER, *specific gravity of.*

Table of the specific gravity of sea water in various parts of the globe, as ascertained by Dr Marcet.

| | | | |
|---|---|---|---|
| Arctic Ocean | 1.02664 | Sea of Marmora | 1.01915 |
| Northern hemisphere | 1.02829 | Black Sea | 1.01418 |
| Equator | 1.02777 | White Sea | 1.01901 |
| Southern hemisphere | 1.02882 | Baltic | 1.01523 |
| Yellow Sea | 1.02291 | Ice—sea waters | 1.00057 |
| Mediterranean | 1.02930 | Lake of Ourmia | 1.16507 |

Dead Sea 1.21100

From the preceding facts, Dr Marcet concludes,

1. That the Southern Ocean contains more salt than the Northern in the ratio of 1.02919 to 1.02757,

2. That the mean spec. grav. of sea water near the equator is 1,02777; intermediate between that of the N. and S. hemispheres.

3. That there is no notable difference in sea waters under different meridians.

4. That there is no satisfactory evidence that the sea at great depths is more salt than at the surface.

5. That the sea in general contains more salt where it is deepest, and most remote from land, and that its saltness is always diminished in the vicinity of large masses of ice.

6. That small inland seas, though communicating with the ocean, are much less salt than the open ocean.

7. But that the Mediterranean contains rather larger proportions of salt than the ocean.

SEA WATER, *saline contents of.*

Sea water contains in solution, muriate of soda, sulphate of soda, muriate of lime, and muriate of magnesia; Dr Wollaston has also ascertained that it contains potash, though in a proportion less than $\frac{1}{2000}$th part of sea water at its average density. The following analysis of sea water, brought from the middle of the North Atlantic, as given by Dr Marcet, may serve as a specimen. The quantity operated upon was 500 grains:—

| | |
|---|---|
| Muriate of soda | 13,3 grs. |
| Sulphate of soda | 2,33 |
| Muriate of lime | 0,975 |
| Muriate of magnesia | 4,955 |
| | 21,460 |

Analysis of the water of the Dead Sea, by Dr Marcet; the quantity operated upon being 100 grains.

| | |
|---|---|
| Muriate of lime | 3,920 grs. |
| Muriate of magnesia | 10,246 |
| Muriate of soda | 10,360 |
| Sulphate of lime | 0,054 |
| | 24,580 |

SEA WATER, *temperature of.*

In Baffin's Bay, the Mediterranean Sea, and the Tropical Seas, the temperature of the sea diminishes with the depth, according to the observations of Phipps, Ross, Parry, Sabine, Saussure, Ellis, and Peron, but this diminution is not subject to any regular law. At the depth of 100 fathoms the difference is sometimes no more than 1°, and sometimes as great as 20°. Sometimes the coldness attains its maximum at 100 fathoms, and sometimes it increases to 400 and 500. Humboldt thinks that, on a mean, the change is about six times more rapid than in the atmosphere, or about 1° in 50 feet; but the facts are too anomalous to be easily brought under any general rule. It is a remarkable fact that in the Arctic or Greenland seas the temperature of the sea increases with the depth. This singular result was first obtained by Mr Scoresby, and has been confirmed by the later observations of Franklin, Beechy, and Fisher.

The following are some of Mr Fisher's results obtained on board the Dorothea:—

| *Latitude.* | *Longitude* | *Depth in Fath.* | *Temp. at Bottom.* | *Temp. of Surface.* |
|---|---|---|---|---|
| Between 79° 50′ & 80. 14. | 11°. 30′. E. | 40 ... | 35°.5 .. | 31°.8 |
| | | 60 ... | 36.0 ... | 32.0 |
| | | 100 ... | 36.3 ... | 32.0 |
| | | 124 ... | 36.7 ... | 33.5 |
| | | 140 ... | 36.5 ... | 32.0 |
| | | 188 ... | 42.5 ... | 33.0 |
| | | 304 ... | 39.0 ... | 31.0 |

And similar results were obtained by Lieut. Beechy and Mr Scoresby. The greatest difference found by Lieut. Parry was 6° at a depth of 246 fathoms; and the greatest obtained by Capt. Sabine was 7½° at a depth of 680 fathoms.

SEAS POLAR.—*(Enc. Brit. Supp.)*

Short chronological notice of the principal navigators, who have explored the Polar seas, from the voyages of Davis to the present time, with the highest latitude reached by each.

| *Year.* | NORTH. | *Highest Lat.* | |
|---|---|---|---|
| 1585 | Davis, three voyages | 72°. 12′ | Davis Strait. |
| 1594 | Barentz, three voyages | 80. 11. | Spitzbergen. |
| 1602 | Weymouth | —— | Resolution Island. |

| *Year.* | NORTH. | *Highest Lat.* |
|---|---|---|
| 1605 | Hall and Knight | 66. 55. W. coast of Greenland. |
| 1606 | Knight | 56. 48. Labrador. |
| 1607 | Hudson, four voyages | 78. 56. E. coast of Greenland. |
| 1611 | Button | 65. 0. Southampton Island.* |
| 1612 | Hall | 67. 0. W. coast of Greenland. |
| 1614 | Gibbon | —— Labrador. |
| 1615 | Baffin, two voyages | 78. 0. Baffin's Bay. |
| 1631 | Fox | —— Fox's Farthest. |
| 1741 | Middleton | 66. 14. Cape Hope. |
| 1746 | Moor and Smith | —— Repulse Bay. |
| 1773 | Phipps and Lutwidge | 80. 48. W. coast of Spitzbergen. |
| 1779 | Cook and Clarke | 70. 41. Behring's Strait. |
| 1787 | Lowenorn | 66. 30. E. coast of Greenland. |
| 1791 | Duncan | —— Chesterfield Inlet. |
| 1806 | Scoresby | 81. 30. Longitude 19° E. |
| 1819 | Parry's second voyage | 75. 35. Melville Island. |
| | Farthest point westward | 74. 26. 25. Long. 113. 46. 43. W. |
| Do. | Capt. Franklin, land expedition | 67. 48. Coppermine River, and 5 or 600 miles to the eastward. |
| 1826 | Franklin & Richardson, do. | —— Mackenzie's River, and from 113 to 149. 38. W. Long. |
| 1827 | Parry's 4th | 81. 5¼. and on the ice to<br>82. 45¼. 20°. E. Long. |
| | SOUTH. | |
| 1774 | Cook | 71. 10. Long. 101 to 110 W. |
| 1822 | Weddell | 74. 15. Long. 34. 16. 45. W. |

No human beings are found in the Southern Ocean below the 55th parallel of latitude, and none beyond the 50th, except on Patagonia and Terra del Fuego.

It is impossible to enter here into any of those points of scientific research which these expeditions have been the means of communicating. It may not, however, be uninteresting to subjoin the result of Captain Parry's observations on the temperature of Melville Island, in 1819 and 1820, as indicating a very extraordinary degree of cold.

| | | *Greatest Temperat.* | *Least.* | *Mean.* |
|---|---|---|---|---|
| 1819 | September | + 37° | − 1° | + 22°.54 |
| | October | + 17.5 | − 28 | − 3.46 |
| | November | + 6 | − 47 | − 20.60 |
| | December | + 6 | − 43 | − 21.79 |
| 1820 | January | − 2 | − 47 | − 30.09 |
| | February | − 17 | − 50 | − 32.19 |
| | March | + 6 | − 40 | − 18.10 |
| | April | + 32 | − 32 | − 8.37 |
| | May | + 47 | − 4 | + 16.66 |
| | June | + 51 | + 28 | + 36.24 |
| | July | + 60 | + 32 | + 42.41 |
| | August | + 45 | + 22 | + 32.68 |
| | | | Annual temperature | + 1.33 |

According to Leslie's Table *(see Atmosphere)* the temperature of Melville Island should have been nearly 36°, whereas it is only $1^{\circ}\frac{1}{5}$.

| | *Inches.* |
|---|---|
| Greatest height of barometer was | 30.86 |
| Least do. | 29.00 |

SEA, *extent of.—See Earth.*

SEASONS, *length of.—See Earth, elements of.*

SECANTS, *figure of.—See Figure.*

SEMIDIURNAL *arcs.—See Arcs Semidiurnal.*

SHIPS, *tonnage of.*

*To find the tonnage of Ships.*

Rule 1.—Multiply the length of the keel, taken within the vessel, or as much as the ship treads upon the ground, by the length of the midship beam, taken also within, from plank to plank, and that product by half the breadth, taken as the depth; then divide the last product by 94, and the quotient will give the tonnage.

If the length of a ship's keel be 80 feet, and the midship-beam 30; required the tonnage. *Ans.* 382.9787 + tons.

Rule 2.—Shipwrights take the dimensions on the outside of the light mark, as the ship swims, being unladen, to find the content of the empty ship. But if the measure of the ship be taken from the light mark to her full draught of water, when laden, it will give the burden of the ship; and then the length, breadth, and depth multiplied together, and the product divided by 100 for men of war (which gives an allowance for guns, anchors, &c. that are all burden but no tonnage) and by 95 for merchant ships, will give the tonnage.

N.B. A hundred solid feet make a ton.

Required the tonnage of a ship, whose length is 300 feet, breadth 50, and depth 30.

*Ans.* $4736\frac{16}{19}$ tons.

Rule 3.—At London, shipwrights multiply the length of the keel by the extreme breadth of the ship, taken from outside to outside, and that product by half the breadth; and this they divide by 94 for merchant ships, and by 100 for men of war; the quotients are the tonnage of the vessels of their respective classes.

Required the tonnage of an eighty gun ship, the length of whose keel is 149 feet 4 inches, and her extreme breadth 49 feet 8 inches.

*Ans.* 1841·86 + tons.

The following method is used in the Royal Navy:—

Rule 4.—Let fall a perpendicular from the foreside of the stern at the height of the hawse holes, and another from the back of the main port at the height of the wing transom; from the distance between these perpendiculars deduct $\frac{3}{5}$ of the extreme breadth, and as many times 2½ inches as there are feet in the height of the wing transom above the upper edge of the keel, the remainder is the length of the keel for tonnage. Then multiply the length of the keel by the extreme breadth, and that product by half the breadth; divide this product by 94 for the tonnage.

Given the length of the keel 68 feet, and the extreme breadth 22; required the tonnage.

*Ans.* $175\frac{6}{91}$ tons.

*Ship-building.*

A man-of-war of 74 guns requires about 3000 loads of timber, of 50 cubic feet each; worth, at £5. a load, £15,000. A tree contains about two loads, and 3000 loads would cover fourteen acres. The value of shipping in general is estimated at £8. or £10. a ton.

It is said that 180,000 pounds of hemp are required for the rigging of a first-rate man-of-war.—(*Young's Nat. Phil.*)

*Note.*—The above calculation of fourteen acres to a 74 gun ship is probably much too low. It will be nearer the truth to suppose each tree to contain only a load and a half of timber, and that every acre contains 35 trees fit for naval purposes; this gives 57 acres of land for a 74 gun ship. *See Report of the Board of Commissioners of Woods and Forests*, 1812.

SHOT, *pile of.*

Shot or shells are usually piled up in a pyramidal form, the base being an equilateral triangle, square, or rectangle.

The following formulæ give the total number of balls in any of these piles :—

$$\text{Triangular pile} = \frac{n.\ (n+1.)\ (n+2.)}{6}.$$

$$\text{Square pile} = \frac{n.\ (n+1.)\ (2\,n+1.)}{6}.$$

$$\text{Rectangular pile} = \frac{m.\ (m+1.)\ (3\,n-m+1)}{6}.$$

Where $n$ in the two first formulæ denotes the number of balls in the side of the base; and in the last $n$ is the number of balls in the length of the base, and $m$ the number of those in the breadth.

SHOT, *weight of.*—(*Hutton.*)

Let W be the weight in pounds, $\delta$ = diameter in inches, then

$$\text{In iron balls, } W = \frac{9}{64} \times \delta^3.$$

$$\text{In leaden, } \quad W = \frac{3}{14} \times \delta^3.$$

In iron shells, if D and $\delta$ be the external and internal diameters, $W = \frac{9}{64} \times (D^3 - \delta^3.)$

SHOT.—*See Gunnery.*

SIDEREAL *time.*—*See Time.*

SINES, *figure of.*—*See Figure.*

SINES, *arithmetic of.*—*See Trigonometry.*

SIPHON, *oscillatory motion of water in.*—(*Playfair.*)

1. Let an inverted siphon, partly filled with water, be composed of three rectilinear tubes of equal diameters, of which the intermediate one is horizontal, and the two others inclined to the horizon at any angles , $\theta'$; and let an oscillatory motion be communicated to the water; re-

quired the time of the water's oscillating in either of the legs from the lowest to the highest points.

Let L = length of the whole canal, $g = 32\frac{1}{6}$ feet; then

$$T = \pi \sqrt{\frac{L}{g \times (\sin. \theta + \sin. \theta')}}.$$

When the two ascending tubes are vertical,

$$T = \pi \sqrt{\frac{L}{2g}}.$$

*Cor.* Hence if the legs are vertical, the time of one oscillation = the time in which a pendulum would vibrate, whose length is ½ L.

2. The vibratory motion of water in the form of waves may be compared to the above reciprocation in a siphon or bent tube. And hence if $a$ be the altitude of a wave, and $b$ half the breadth, the time of one undulation, i.e. the time, from the wave being highest at any point, to its being highest at that point again, is

$$\frac{\pi}{\sqrt{2g}} \sqrt{a + b}.$$

and the space which the wave appears to pass over in a second is

$$\frac{b\sqrt{2g}}{\pi\sqrt{a + b}}.$$

*Cor.* 1. If $a$ be neglected, the velocity of the wave becomes $\frac{\sqrt{2gb}}{\pi}$, which is the velocity as determined by Newton, Princip. lib. 2. Prop. 46.

*Cor.* 2. Hence a pendulum whose length = ½ its distance between any two consecutive highest and lowest points will make two vibrations during the time of one complete undulation; or if the pendulum is four times the preceding, i.e. equal to the distance of any two consecutive waves, the time of one undulation equals the time in which this latter pendulum would perform one vibration.

SLUICES.—*See Fluids.*

SOLAR *inequality.—See Precession.*

SOLAR *mean time.—See Time.*

**SOLIDS** *the five regular, surface and solidity of.*

| *Names.* | *Surface.* | *Solidity.* |
|---|---|---|
| Tetraedron | $s^2 \times 1.7320508$ | $s^3 \times 0.1178513$ |
| Hexaedron | $s^2 \times 6.0000000$ | $s^3 \times 1.0000000$ |
| Octaedron | $s^2 \times 3.4641016$ | $s^3 \times 0.4714045$ |
| Dodecaedron | $s^2 \times 20.6457288$ | $s^3 \times 7.6631189$ |
| Icosaedron | $s^2 \times 8.6602540$ | $s^3 \times 2.1816950$ |

**SOLIDS,** *contents of.*

Let $x$ and $y$ be the abscissa and ordinate of any curve; then if $\pi = 3.14159$ &c.

$$\text{Solid content} = \pi y^2 dx.$$

*Ex.* 1. Content of cylinder $= \pi y^2 x$.

2. Content of cone $= \frac{1}{3} \pi y^2 x = \frac{1}{3}$ of circumscribing cylinder.

3. Content of paraboloid $= \frac{1}{2} \pi y^2 x = \frac{1}{2}$ circumscribing cylinder.

4. Content of sphere $= \frac{2}{3}$ of circumscribing cylinder.

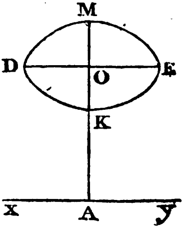
5. Content of spheroid round ax. maj.

$$= \frac{4 \pi b^2 a}{3}. \quad \text{Do. round ax. min.} = \frac{4 \pi a^2 b}{3}.$$

6. Content of pyramid $= \frac{1}{3}$ content of prism of the same base and altitude.

*Guldinus' property.*

Let M D E K be any plane figure revolving about an axis $x y$ in its own plane, then the solid generated is equal to the circumference described by the centre of gravity multiplied into the area of the figure.



*Ex.* Let D M E K be a circle, then the solid will represent the ring of an anchor; in this case if $r$ = radius of circle, and $a$ = A O, the solid $= 2 \pi a \times \pi r^2 = 2 \pi^2 a r^2$.

**SOUND,** *velocity of.*—(*Phil. Trans.* 1823.)

The velocity with which vibrations are propagated through the air, is the same that a heavy body would acquire by falling through half the height of the *homogeneous* atmosphere, or that which the atmosphere would be reduced to, if it were everywhere of the same density, and the same temperature with the air at the surface of the earth.

The height of this homogeneous atmosphere has been computed at 4343 fathoms, when the temperature is that of freezing. If this height be called H, then $v$, the velocity of the aerial vibrations, $= \sqrt{2gH}$. Hence $v = 1057$, which is too small, see infra.

The velocity of sound has been variously given by different philosophers, as appears from the following Table:—

| | Feet. |
|---|---|
| Newton | 968 per second. |
| Roberts | 1300 |
| Boyle | 1200 |
| Walker | 1338 |
| Flamstead, Halley, and Derham | 1142 |
| Florentine Academy | 1148 |
| French Academy | 1172 |

More modern determinations.

| | | |
|---|---|---|
| Millington | 1130 | Chili. |
| Bengenberg | 1005 | Dusseldorf. |
| La Caille | 1106½ | Montmartre. |
| La Place | 1133 | |
| Lacaille | 1130 | |

Flamstead's and Halley's measure, or 1142, is the one generally assumed by English writers.

1092 [illegible]

$v = 1092 \pm d(1.14)$

[illegible]

*Result of Mr Goldingham's elaborate series of experiments at Madras.*

| *Months.* | *Barometer in Inches.* | *Thermometer, Fah.* | *Hygrometer, dry.* | *Velocity of Sound in a Second in Feet.* |
|---|---|---|---|---|
| January, | 30.124 | 79°.05 | 6°.2 | 1101 |
| February, | 30.126 | 78 .84 | 14 .70 | 1117 |
| March, | 30.072 | 82 .30 | 15 .22 | 1134 |
| April, | 30,031 | 85 .79 | 17 .23 | 1145 |
| May, | 29,892 | 88 .11 | 19 .92 | 1151 |
| June, | 29.907 | 87 .10 | 24 .77 | 1157 |
| July, | 29,914 | 86 .65 | 27 .85 | 1164 |
| August, | 29,931 | 85 .02 | 21 .54 | 1163 |
| September, | 29,963 | 84 .49 | 18 .97 | 1152 |
| October, | 30.058. | 84 .33 | 18 .23 | 1128 |
| November, | 30.125 | 81 .35 | 8 .18 | 1101 |
| December, | 30.087 | 79 .37 | 1 .43 | 1099 |

Mr Goldingham concludes, that for each degree of the thermometer 1.2 feet may be allowed in the velocity of sound for a second; for each degree of the hygrometer 1.4 feet; and for $\frac{1}{10}$th of an inch of the barometer 9.2 feet. He concludes that 10 feet per second is the difference of the velocity of sound between a calm and in a moderate breeze, and 21½ feet in a second, or 1275 in a minute, is the difference, when the wind is in the direction of the motion of sound, or opposed to it.—*See Phil. Trans.* 1823.

SPECIFIC *Gravity.—See Gravity specific.*

SPECTACLES.—*See Eye.*

SPHERE, *doctrine of.*

In what is usually called the doctrine of the sphere is merely included the solution of the following problem:—

In a spherical triangle, whose sides are the co-declination D, the co-latitude of the place L, the zenith distance Z, and two of whose angles are the hour angle from noon H, and azimuth $\alpha$; if any three of these quantities be given, the other two may be found by the rules and formulæ of Trigonometry.

For the solution of the several cases—*see Trigonometry spherical.*

SPHERE, *Equations to, when the axes are rectangular.—(Hamilton.)*

Let $r$ = radius, and suppose $x'$, $y'$, $z'$ to be the coordinates of the centre, and $x$, $y$, $z$ those of any point on the surface; then the general equation is

$$(x - x')^2 + (y - y')^2 + (z - z')^2 = r^2.$$

If the origin be at the centre, $x'$, $y'$, and $z'$ each $= o$, and the equation becomes

$$x^2 + y^2 + z^2 = r^2.$$

SPHERICAL *excess.*

Spherical excess in Trigonometry is the excess of the sum of the three angles of any spherical Δ above two right angles. Now in surveying a country where the sides of the Δ's are usually 14 or 15 miles each, the spherical excess, with a fine instrument, is plainly discernable; and in strict accuracy the sides of the Δ's ought to be calculated by the rules of spherical Trigonometry, which would be a most tedious process, where many hundreds of such operations are to be performed. Legendre has therefore furnished us with the following rule, which combines sufficient exactness, with all the conciseness that can be expected, viz. :—

A spherical Δ being proposed, of which the sides are very small with regard to the radius of the sphere, if from each of its angles one-third of the excess of the sum of its three ∠'s above two right ∠'s be subtracted, the angles so diminished may be taken for the ∠'s of a rectilineal Δ, the sides of which are equal in length to those of the proposed spherical triangle.

SPIRALS.—*(Higman, Vince.)*

1. *Spirals, Equations to.*

In the spiral of Archimedes, let $r =$ rad. vect. $\theta = \angle$ traced out by $r$; then

$$r = \frac{b}{2\pi} \cdot \theta, \text{ or } r = a\,\theta; \text{ if } a = \frac{b}{2\pi}.$$

In the reciprocal or hyperbolic spiral,

$$r = \frac{a}{\theta}.$$

In the logarithmic spiral,

$$r = a^{\theta}.$$

In the lituus,

$$r = \frac{a}{\theta^{\frac{1}{2}}}.$$

The spiral of Archimedes, the reciprocal spiral, and the lituus are particular cases of the equation $r = a\,\theta^n$.

If $n$ be $+$, the spirals begin at the pole, and recede to an infinite distance; but if $n$ be $-$, the spirals begin at an infinite distance, and reach the pole after an infinite number of revolutions.

2. *Spirals to draw tangents to.*

$$\text{Subtangent} = \frac{r^2\,d\,\theta}{d\,r}.$$

*Ex.* 1. In the spiral of Archimedes $r = a\,\theta$,

$$\therefore \text{Subtangent} = \frac{r^2}{a}, \text{ and hence } p = \frac{r^2}{\sqrt{a^2+r^2}}.$$

*Ex.* 2. In the reciprocal spiral,

$$\text{Subtangent} = a, \text{ and } p = \frac{a\,r}{\sqrt{a^2+r^2}}.$$

*Ex.* 3. In the logarithmic spiral,

$$\text{Subtangent} = \frac{a\,r}{b} \text{ and } p = a\,r.$$

3. *Spirals to find the areas of.*

$$\text{Area} = \text{fl.}\,\frac{r^2\,d\,\theta}{2}.$$

*Ex.* 1. In the spiral of Archimedes,

$$\text{fl.}\,\frac{r^2\,d\,\theta}{2} = \text{fl.}\,\frac{\pi\,r^2\,d\,r}{b}, \text{ since } \theta = \frac{2\,\pi\,r}{b} \text{ (Art. 1);}$$

$$\therefore \text{Area} = \frac{\pi\,r^3}{3\,b}.$$

*Ex.* 2. In the reciprocal spiral,

$$\text{fl.}\,\frac{r^2\,d\,\theta}{2} = -\,\text{fl.}\,\frac{a\,d\,r}{2}, \text{ since } \theta = \frac{a}{r};$$

$$\therefore \text{Area} = -\,\frac{a\,r}{2} + C.$$

Suppose the area to vanish when $r = b$, then will the area, intercepted between two radii $b$ and $r$, $= \frac{a}{2}\,(b - r)$.

*Ex.* 3. In the logarithmic spiral,

Area between two radii $b$ and $r = \frac{m}{4}\,(r^2 - b^2)$, $m$ being the modulus.

*Ex.* 4. In the lituus,

$$\text{Area} = a^2 \log. \frac{b}{r}.$$

4. *Spirals to find the lengths of.*

$$d z^2 = d r^2 + r^2 d \theta^2.$$

$$\text{or } d z = \frac{r\, d r}{\sqrt{r^2 - p^2}}.$$ ($p$ = perpendicular on the tangent).

*Ex.* 1. In the spiral of Archimedes,

Arc $= \frac{1}{a}$ fl. $d r \sqrt{a^2 + r^2}$, and $\therefore$ = a parabolic arc, whose latus rectum is $2\, a$, and whose ordinate is $r$ *(see Rectification.)*

*Ex.* 2. In the reciprocal spiral,

Arc = arc of a logarithmic curve contained between the ordinates $b$ and $r$; the subtangent of the curve being equal to the subtangent of the spiral.

*Ex.* 3. In the logarithmic spiral,

$$\text{Arc} = \sqrt{(1 + m^2)}\ (r - b)$$

*Ex.* 4. In the involute of a circle,

$$\text{Arc} = \frac{p^2}{2\, a}$$ ($a$ = radius of the circle).

5. *Spirals, curvature of.*

$$\text{Rad. of curv.} = \frac{r\, d r}{d p}.$$

$$\text{Ch. curv.} = \frac{2\, p\, d r}{d p}.$$

*Ex.* 1. In the logarithmic spiral,

Rad. curv. $= \frac{r}{m}$, and ch. curv. $= 2\, r$.

*Ex.* 2. In the spiral of Archimedes,

$$\text{Rad. curv.} = \frac{(r^2 + a^2)^{\frac{3}{2}}}{r^2 + 2\, a^2}$$

*Ex.* 3. In the reciprocal spiral,

$$\text{Ch. curv.} = \frac{2\, r\, (a^2 + r^2)}{a^2}.$$

6. *Spirals, point of contrary flexure in.*

Here the rad. of curvature is either infinite or nothing; $\therefore \frac{r\,dr}{dp} = 0$ or infinity; and $dp$ is infinite or nothing.

*Ex.* Let $r = a\theta^n$, then when $dp = o$, $r = a\,(-n.\overline{n+1})^{\frac{n}{2}}$. Hence in the case of the lituus, where $n = -\frac{1}{2}$, $r = a\sqrt{2}$.

SPRINGS *hot, temperature of a few of the principal.*—*(Ure.)*

| | | | |
|---|---|---|---|
| Matlock | 66° | Borset | 132° |
| Bristol | 74 | Aix | 143 |
| Buxton | 82 | Carlsbad | 165 |
| Bath | 114 | The Geyzers (Iceland) | 212 |
| Berege | 120 | | |

SPRINGS, *temperature of.*—*See Atmosphere.*

SQUARES *minimum, method of.*—*See Equations of Condition.*

SQUARE *roots of numbers.*—*See Involution.*

STANDARD *measures.*—*See Weights and Measures.*

STARS, *Catalogue of*—*(Naut. Alm.)*

*A Catalogue of 60 principal Fixed Stars for Jan. 1, 1823.*

| No. | Names of Stars. | A.R. | An.Var. | N.P.D. | An.Var. |
|---|---|---|---|---|---|
| | | H. M. S. | S. | ° ′ ″ | ″ |
| 1 | γ Pegasi | 0. 4. 8,1 | + 3,08 | 75.48. 2 | —19,9 |
| 2 | α Cassiopeiæ | 0.30.31,3 | 3,31 | 34.26. 6 | —19,7 |
| 3 | Polaris | 0.57.46,5 | 15,01 | 1.38. 8 | —19,4 |
| 4 | α Arietis | 1.57.13,1 | 3,36 | 67.22.44 | —17,2 |
| 5 | α Ceti | 2.53. 2,3 | 3,12 | 86.36.37 | —14,4 |
| 6 | α Persei | 3.11.44,3 | 4,20 | 40.46.39 | —13,3 |
| 7 | Aldebaran | 4.25.46,6 | 3,43 | 73.51.18 | — 7,7 |
| 8 | Capella | 5. 3.37,8 | 4,41 | 44.11.37 | — 4,3 |
| 9 | Rigel | 5. 6. 2,2 | 2,88 | 98.24.48 | — 4,5 |
| 10 | β Tauri | 5.15. 6,8 | 3,78 | 61.33. 7 | — 3,7 |
| 11 | γ Orionis | 5.15.38,7 | 3,20 | 83.49. 8 | — 4,0 |
| 12 | δ | 5.22.58,3 | 3,06 | 90.26.18 | — 3,1 |
| 13 | ε | 5.27.11.3 | 3,03 | 91.19.23 | — 2,6 |
| 14 | ζ | 5.31.50,1 | 3,01 | 92. 2.38 | — 2,4 |
| 15 | α | 5.45.35,6 | 3,25 | 82.38. 4 | — 1,1 |
| 16 | β Aurigæ | 5.46.32,9 | 4,39 | 45. 4.56 | — 1,2 |
| 17 | Sirius | 6.37.20,9 | 2,64 | 106.28.49 | + 4,8 |
| 18 | Castor | 7.23.17,6 | 3,85 | 57.43.59 | + 7,2 |

| No. | Names of Stars. | A.R. | An. Var. | N.P.D. | An. Var. |
|---|---|---|---|---|---|
| | | H. M. S. | s. | ° ′ ″ | ″ |
| 19 | Procyon | 7.30. 2,2 | 3,17 | 84.19.43 | + 8,9 |
| 20 | Pollux | 7.34.28,5 | 8,69 | 61.33.17 | + 8,1 |
| 21 | α Hydræ | 9.18.53,5 | 2,95 | 97.53.44 | + 15,3 |
| 22 | Regulus | 9,58.56,3 | 3,21 | 77.10.16 | + 17,3 |
| 23 | α Ursæ Majoris | 10.52.43,5 | 3,83 | 27.17.44 | + 19,2 |
| 24 | β Leonis | 11.40. 1,7 | 3,07 | 74.26.16 | + 20,1 |
| 25 | γ Ursæ Majoris | 11.44.28,6 | 3,20 | 35.19.15 | + 19,9 |
| 26 | δ | 12. 6.37,2 | 3,00 | 31.59. 0 | + 20,2 |
| 27 | Spica Virginis | 13.15.52,9 | 3,14 | 100.14. 0 | + 18,9 |
| 28 | ζ Ursæ Majoris | 13.16.46,8 | 2,41 | 34. 8.51 | + 19,9 |
| 29 | η | 13.40.33,5 | 2,38 | 39.48. 0 | + 18,2 |
| 30 | α Draconis | 13.59.35,9 | 1,62 | 24.46.31 | + 17,3 |
| 31 | Arcturus | 14. 7.35,6 | 2,73 | 69.53.29 | + 19,0 |
| 32 | ε Bootis | 14.37.15,6 | 2,61 | 62.10.28 | + 15,5 |
| 33 | 2 α Libræ | 14.41. 6,4 | + 3,30 | 105.17.56 | + 15,4 |
| 34 | β Ursæ Minoris | 14 51.19,6 | − 0,32 | 15. 7.16 | + 14,7 |
| 35 | α Cor. Bor. | 15.27.12,0 | + 2,54 | 62.41. 1 | + 12,5 |
| 36 | α Serpentis | 15.35.33,5 | 2,95 | 83. 0.37 | + 11,7 |
| 37 | δ Ophiuchi | 16. 5. 5,0 | 3,13 | 93.13.49 | + 9,9 |
| 38 | Antares | 16.18.34,2 | 8,66 | 116. 1.43 | + 6,7 |
| 39 | α Herculis | 17. 6 35,0 | 2,73 | 75.24. 0 | + 4,6 |
| 40 | β Draconis | 17.26 25,9 | 1,34 | 37.33.49 | + 2,9 |
| 41 | α Ophiuchi | 17.26.43,5 | 2,78 | 77.18.11 | + 3,2 |
| 42 | γ Draconis | 17.52.30,1 | + 1,38 | 38.29.11 | + 0,7 |
| 43 | δ Ursæ Minoris | 18.29.22,3 | - 19,12 | 3.25.11 | − 2,4 |
| 44 | α Lyræ | 18 30.57,0 | + 2,03 | 51.22.31 | − 3,0 |
| 45 | β | 18.43.33,0 | 2,20 | 56.50.12 | − 3,8 |
| 46 | ζ Aquilæ | 18.57.16,8 | 2,75 | 76.23.31 | − 4,6 |
| 47 | δ Draconis | 19.12 29,7 | 0,02 | 22.38.59 | − 6,2 |
| 48 | δ Aquilæ | 19.16.34,6 | 3,00 | 87.13.47 | − 6,6 |
| 49 | γ | 19 37.50,8 | 2,85 | 79.48 39 | − 8,5 |
| 50 | α | 19.42. 8,9 | 2,93 | 81.35.30 | − 8,9 |
| 51 | β | 19.46.37,3 | 2,95 | 84. 1.40 | − 8,6 |
| 52 | 2 α Capricorni | 20. 8.13,7 | 8,34 | 108. 5. 7 | − 10,8 |
| 53 | α Cygni | 20.35.24,2 | 2,05 | 45.20.52 | − 12,5 |
| 54 | 1st 61 Cygni | 20.58.58,6 | 2,77 | 52. 6.55 | − 17,6 |
| 55 | α Cephei | 21.14.21,0 | 1,42 | 28. 9.43 | − 15,0 |
| 56 | β Aquarii | 21.22.14,2 | 3,15 | 96.20.38 | − 15,3 |
| 57 | β Cephei | 21.26.20,4 | 0,81 | 20.12.54 | − 15,7 |
| 58 | α Aquarii | 21.56.41,5 | 3,09 | 91,10.31 | − 17,0 |
| 59 | α Pegasi | 22.55.57.2 | 2,98 | 75.44.42 | − 19,0 |
| 60 | α Andromedæ | 23.59.15,6 | 3,08 | 61.53.12 | − 19,8 |

**STARS** *double.*

### *On Herschel's Catalogue of double Stars.*

The first Catalogue of double stars was made with a Newtonian telescope of not quite seven feet focus, and with only 4½ inches aperture, charged with a power of 222. The second Catalogue with an aperture of six inches and a quarter, with a power of 227, and 460; when the stars were detected, he used a gradual variety of powers from 460 to 6000.

These double stars are divided into several different classes. In the first are placed all those which require a very superior telescope, the utmost clearness of air, and every other favourable circumstance to be seen at all, or well enough to judge of them. Their distance is so extremely small (seldom exceeding two diameters of the largest) that it cannot be accurately measured by the micrometer, but may be more correctly estimated by the eye in measures of their own apparent diameters. It should be observed, that since it will require no common stretch of power and distinctness to see these double stars, it will ∴ not be amiss to go gradually through a few preparatory steps of vision, such as the following:—for instance, when *η* Coron. Borealis (one of the most minute double stars) is proposed to be viewed, let the telescope be some time before directed to *α* Geminorum, or if not in view to either of the following stars, *ζ* Aquarii, *μ* Draconis, *ρ* Herculis, *α* Piscium, or the curious double-double star *ε* Lyræ. These should be kept in view for a considerable time, that the eye may acquire the habit of seeing such objects well and distinctly. The observer may next proceed to the *ξ* Ursæ Majoris, and the beautiful treble star in Monoceros' right foot; after these to *ι* Bootis, which is a fine miniature of *α* Geminorum, to the star preceding *α* Orionis, and to *η* Orionis. By this time both the eye and the telescope will be prepared for a still finer picture, which is *η* Coronæ Borealis. It will be in vain to attempt this latter, if all the former, at least *ι* Bootis, cannot be distinctly perceived to be fairly separated; because it is almost as fine a miniature of *ι* Bootis as that is of *α* Geminorum. To try stars of unequal magnitude, it will be expedient to take them in some such order as the following: *α* Herculis, *ω* Aurigæ, *δ* Geminorum, *k* Cygni, *ε* Persei, and *b* Draconis; from these the observer may proceed to a most beautiful object *ε* Bootis. As the foregoing remarks have suggested the method of seeing how far the power and distinctness of our instruments will reach, we may next add the way of finding how much light we have. The observer may begin with the pole star, and *α* Lyræ, then go to the star south of *ε* Aquilæ, the treble star near *k* Aquilæ, and last of all to the star following *o* Aquilæ. Now if his telescope has not a great deal of good light, he will not be able to see some of the small stars that accompany them.

In the second class of double stars are put all those that are proper for estimations by the eye, or very delicate measures of the micrometer. To compare the distances with the apparent diameters, the power of the telescope should not be much less than 200, as they will otherwise be too close for the purpose. It will be necessary here to notice that the estimation made with one telescope cannot be applied to those made with another, nor can the estimations made with different powers, though with the same telescope, be applied to each other; therefore if we would wish to compare any such observations together with a view to see whether a change in the distance has taken place, it should be done with the very same telescope and power, even with the very same eye-glass or glasses.

In the third class are placed all those double stars that are more than 5 but less than 15″ asunder. In the same manner that the stars in the 1st and 2d classes will serve to try the goodness of the most capital instruments, these will afford objects for telescopes of inferior power, such as magnify from 40 to 100 times. The observer may take them in this or the like order; ζ Ursæ Maj., γ Delphini, γ Arietis, π Bootis, γ Virginis, ι Cassiopeæ, μ Cygni. And if he can see all these he may pass over into the second class, and direct his instrument to some of those that are pointed out as objects for the very best telescopes, where he will soon find the want of superior power.

The 4th, 5th, and 6th classes contain double stars that are from 15 to 30″; from 30″ to 1′, and from 1′ to 2′ or more asunder.—*Phil. Trans. vol.* 72, 35.

For a list of a few of the most remarkable double stars—see *Telescope.*

**STARS** *changeable.*

*Catalogue of twenty-eight changeable stars.—(Baron de Zach.)*

| Names of Stars. | Right Ascension in 1800. | Declination in 1800. |
|---|---|---|
| 46 Andromeda, | 1*h* 11′ | 44° 29′ N. |
| ο Balænæ *Mira,* | 2 09 | 3 54 S. |
| β Persei *Algol,* | 2 55 | 40 11 N. |
| Unicorn, | 6 13 | 3 51 |
| 5 23 γ Canis Major, | 6 55 | 15 21 |
| 16 ψ Leonis, | 9 33 | 14 56 |
| Leo 420 Mayer | 9 37 | 12 21 |
| 16 *c* Virgo, | 12 10 | 4 26 N. |
| 10 Virgo, | 12 28 | 8 06 N. |
| Virgo, | 13 04 | 15 28 S. |
| υ Hydra | 13 19 | 22 15 S. |
| 97 Virgo, | 14 02 | 8 57 S. |
| Bootes, | 14 04 | 12 57 N. |

| | Names of stars. | Right Ascension. in 1800. | Declination in 1800. |
|---|---|---|---|
| | 1 Libra A, | 14 05 | 26 19 S. |
| 15 | Virgo, | 14 37 | 2 58 N. |
| | 50 Northern Crown, | 15 40 | 28 47 N. |
| | 31 Hercules, | 16 24 | 33 57 N. |
| | α Hercules, | 17 06 | 14 38 N. |
| | 59 Sobieski's Shield. | 18 37 | 5 54 S. |
| 20 | β Lyra, | 18 43 | 33 08 N. |
| | 34 σ Sagittarius, | 18 43 | 26 32 S. |
| | χ Swan, | 19 39 | 33 16 N. |
| | Swan No. 895. F. | 19 41 | 32 57 N. |
| | η Antinous, | 19 42 | 0 30 N. |
| 25 | Southern Fish, | 19 43 | 33 08 S. |
| | 34 Swan, near γ | 20 10 | 37 25 N. |
| | δ Cepheus, | 22 22 | 57 28 |
| | Aquarius, | 23 24 | 16 23 |

Of this catalogue of Variable Stars, Nos. 2, 3, 7, 11, 18, 19, 20, 22, 24, and 26, belong to the list of fifteen as given by Mr Pigott. Some of the others belong to the list of those which he suspected to be variable.

**STARS,** *clusters of.—See Nebulæ.*

**STEAM,** *elasticity and density of.—(Encyc. Brit. Sup.)*

Let E be the No. of atmospheres expressing the elasticity, $f$ the temperature reckoned from 212°; then

$$E = (1 + .004 f)^5$$

From hence is obtained the following Table of the elasticities and densities :—

| *Atmospheres.* | *Temperature.* | *Compar. Density.* | *Atmospheres.* | *Temperature.* | *Compar. Density.* |
|---|---|---|---|---|---|
| 1 | 212° | 1.000 | 30 | 456 | 21.834 |
| 2 | 249 | 1.856 | 40 | 485 | 28.210 |
| 3 | 273 | 2.712 | 50 | 509 | 34.388 |
| 4 | 292 | 3.565 | 60 | 529 | 40.404 |
| 5 | 307 | 4.366 | 70 | 517 | 46.265 |
| 6 | 320 | 5.150 | 80 | 562 | 52.093 |
| 7 | 331 | 5.917 | 90 | 577 | 57.766 |
| 8 | 341 | 6.678 | 100 | 590 | 63.371 |
| 9 | 350 | 7.432 | 1000 | 857 | [illegible] |
| 10 | 358 | 8.170 | 2000 | 1105 | [illegible] |
| 15 | 383 | 11.620 | 3000 | 1202 | [illegible] |
| 20 | 417 | 15.232 | | | |

*Note.*—Bernouilli makes the expansive force of gunpowder equal to 10,000 atmospheres; Rumford, from the bursting of a barrel of iron, 50,000, from some more direct experiments from 20,000 to 40,000. The utmost that can be justly inferred from the bursting of the barrel is in reality about 30,000, since the tension could by no means be equal through every part of its substance.—*(Young's Nat. Phil.)*

STEELYARD.—*See Lever.*

STILE *new.*—*See Calendar.*

STRENGTH *animal.*—*See Animal strength.*

SUBNORMAL, *formula for.*

Let $x$ and $y$ = abscissa and ordinate of any curve; then

$$\text{Subnormal} = \frac{y\,dy}{dx}.$$

$$\text{and normal} = y \times \frac{\sqrt{dx^2 + dy^2}}{dx} = y\sqrt{1 + \frac{dy^2}{dx^2}}.$$

*Ex.* Let the curve be the common parabola, then subnormal $= \frac{L}{2}$, and normal $= \sqrt{y^2 + \frac{L^2}{4}}$, where L = lat. rect.

SUBTANGENT.—*See Tangent.*

SUN *eclipses of.*—*See Eclipse.*

SUN *elements of.*—*See Planets elements of.*

SUN, *table of mean right ascension of.*—*See Time.*

SUN, *time of passing Meridian.*—*See Time.*

## SUN, *Right Ascension and Declination of.*

### TABLE I.

### Sun's Right Ascension for every Day in the Year 182[illegible].

| *Days.* | *January.* | *February.* | *March.* | *April.* | *May.* | *June.* |
|---|---|---|---|---|---|---|
| | h m s | h m s | h m s | h m s | h m s | h m s |
| 1 | 18 44 5 | 20 56 36 | 22 49 40 | 0 43 11 | 2 34 27 | 4 37 12 |
| 2 | 18 48 30 | 21 0 41 | 22 53 24 | 0 46 49 | 2 38 16 | 4 41 18 |
| 3 | 18 52 55 | 21 4 45 | 22 57 7 | 0 50 27 | 2 42 6 | 4 45 24 |
| 4 | 18 57 19 | 21 8 47 | 23 0 51 | 0 54 6 | 2 45 56 | 4 49 31 |
| 5 | 19 1 43 | 21 12 50 | 23 4 33 | 0 57 45 | 2 49 47 | 4 53 38 |
| 6 | 19 6 7 | 21 16 51 | 23 8 16 | 1 1 24 | 2 53 38 | 4 57 45 |
| 7 | 19 10 30 | 21 20 51 | 23 11 57 | 1 5 3 | 2 57 30 | 5 1 52 |
| 8 | 19 14 52 | 21 24 51 | 23 15 30 | 1 8 42 | 3 1 28 | 5 6 0 |
| 9 | 19 19 14 | 21 28 50 | 23 19 20 | 1 12 22 | 3 5 16 | 5 10 8 |
| 10 | 19 23 36 | 21 32 48 | 23 23 1 | 1 16 2 | 3 9 10 | 5 14 17 |
| 11 | 19 27 57 | 21 36 45 | 23 26 41 | 1 19 42 | 3 18 4 | 5 18 25 |
| 12 | 19 32 17 | 21 40 42 | 23 30 22 | 1 23 23 | 3 16 59 | 5 22 34 |
| 13 | 19 36 37 | 21 44 38 | 23 34 1 | 1 27 3 | 3 20 55 | 5 26 43 |
| 14 | 19 40 57 | 21 48 33 | 23 37 41 | 1 30 45 | 3 24 51 | 5 30 52 |
| 15 | 19 45 15 | 21 52 27 | 23 41 21 | 1 34 26 | 3 28 48 | 5 35 2 |
| 16 | 19 49 33 | 21 56 21 | 23 45 0 | 1 38 8 | 3 32 45 | 5 39 11 |
| 17 | 19 58 50 | 22 0 14 | 23 48 39 | 1 41 50 | 3 36 43 | 5 43 20 |
| 18 | 19 58 7 | 22 4 6 | 23 52 18 | 1 45 33 | 3 40 42 | 5 47 30 |
| 19 | 20 2 23 | 22 7 57 | 23 55 56 | 1 49 16 | 3 44 40 | 5 51 39 |
| 20 | 20 6 38 | 22 11 48 | 23 50 35 | 1 52 59 | 3 48 40 | 5 55 49 |
| 21 | 20 10 52 | 22 15 38 | 0 3 13 | 1 56 43 | 3 52 40 | 5 59 59 |
| 22 | 20 15 6 | 22 19 27 | 0 6 51 | 2 0 27 | 3 56 41 | 6 4 8 |
| 23 | 20 19 19 | 22 23 16 | 0 10 29 | 2 4 12 | 4 0 42 | 6 8 18 |
| 24 | 20 23 31 | 22 27 4 | 0 14 7 | 2 7 57 | 4 4 43 | 6 12 27 |
| 25 | 20 27 42 | 22 30 52 | 0 17 45 | 2 11 43 | 4 8 45 | 6 16 36 |
| 26 | 20 31 52 | 22 34 38 | 0 21 48 | 2 15 29 | 4 12 48 | 6 20 45 |
| 27 | 20 36 1 | 22 33 25 | 0 25 1 | 2 19 15 | 4 16 51 | 6 24 54 |
| 28 | 20 40 10 | 22 42 10 | 0 28 39 | 2 23 2 | 4 20 54 | 6 29 3 |
| 29 | 20 44 18 | 22 45 55 | 0 32 17 | 2 26 50 | 4 24 58 | 6 33 11 |
| 30 | 20 48 25 | | 0 35 55 | 2 30 38 | 4 29 2 | 6 37 20 |
| 31 | 20 52 31 | | 0 39 33 | | 4 33 7 | |

## SUN

| Days. | July. | August. | September. | October. | November. | December. |
|---|---|---|---|---|---|---|
| | h m s | h m s | h m s | h m s | h m s | h m s |
| 1 | 6 41 28 | 8 46 14 | 10 42 14 | 12 30 19 | 14 26 39 | 16 30 36 |
| 2 | 6 45 36 | 8 50 6 | 10 45 52 | 12 33 57 | 14 30 35 | 16 34 56 |
| 3 | 6 49 44 | 8 53 58 | 10 49 29 | 12 37 35 | 14 34 32 | 16 39 16 |
| 4 | 6 53 51 | 8 57 50 | 10 53 6 | 12 41 13 | 14 38 30 | 16 43 37 |
| 5 | 6 57 58 | 9 1 41 | 10 56 43 | 12 44 52 | 14 42 28 | 16 47 59 |
| 6 | 7 2 5 | 9 5 31 | 11 0 20 | 12 48 31 | 14 46 27 | 16 52 21 |
| 7 | 7 6 11 | 9 9 21 | 11 3 56 | 12 52 11 | 14 50 27 | 16 56 44 |
| 8 | 7 10 18 | 9 13 10 | 11 7 33 | 12 55 51 | 14 54 28 | 17 1 7 |
| 9 | 7 14 23 | 9 16 59 | 11 11 9 | 12 59 31 | 14 58 30 | 17 5 31 |
| 10 | 7 18 29 | 9 20 47 | 11 14 45 | 13 3 12 | 15 2 33 | 17 9 55 |
| 11 | 7 22 34 | 9 24 34 | 11 18 21 | 13 6 53 | 15 6 36 | 17 14 20 |
| 12 | 7 26 38 | 9 28 21 | 11 21 56 | 13 10 35 | 15 10 41 | 17 18 44 |
| 13 | 7 30 42 | 9 32 7 | 11 25 32 | 13 14 17 | 15 14 46 | 17 23 10 |
| 14 | 7 34 46 | 9 35 53 | 11 29 8 | 13 18 0 | 15 18 52 | 17 27 35 |
| 15 | 7 38 49 | 9 39 38 | 11 32 43 | 13 21 44 | 15 22 59 | 17 32 1 |
| 16 | 7 42 51 | 9 43 23 | 11 36 19 | 13 25 28 | 15 27 6 | 17 36 27 |
| 17 | 7 46 53 | 9 47 7 | 11 39 54 | 13 29 12 | 15 31 15 | 17 40 53 |
| 18 | 7 50 55 | 9 50 51 | 11 43 29 | 13 32 57 | 15 35 24 | 17 45 19 |
| 19 | 7 54 55 | 9 54 34 | 11 47 5 | 13 36 43 | 15 39 34 | 17 49 45 |
| 20 | 7 58 56 | 9 58 16 | 11 50 40 | 13 40 29 | 15 43 45 | 17 54 12 |
| 21 | 8 2 56 | 10 1 58 | 11 54 16 | 13 44 16 | 15 47 57 | 17 58 38 |
| 22 | 8 6 55 | 10 5 40 | 11 57 51 | 13 48 3 | 15 52 9 | 18 3 5 |
| 23 | 8 10 53 | 10 9 21 | 12 1 27 | 13 51 52 | 15 56 22 | 18 7 31 |
| 24 | 8 14 51 | 10 13 2 | 12 5 3 | 13 55 41 | 16 0 36 | 18 11 59 |
| 25 | 8 18 49 | 10 16 42 | 12 8 39 | 13 59 30 | 16 4 51 | 18 16 25 |
| 26 | 8 22 45 | 10 20 22 | 12 12 15 | 14 3 21 | 16 9 7 | 18 20 51 |
| 27 | 8 26 42 | 10 24 2 | 12 15 51 | 14 7 12 | 16 13 23 | 18 25 17 |
| 28 | 8 30 37 | 10 27 41 | 12 19 28 | 14 11 4 | 16 17 40 | 18 29 44 |
| 29 | 8 34 32 | 10 31 20 | 12 23 4 | 14 14 56 | 16 21 58 | 18 34 10 |
| 30 | 8 38 26 | 10 34 58 | 12 26 42 | 14 18 50 | 16 26 17 | 18 38 35 |
| 31 | 8 42 20 | 10 38 36 | | 14 22 44 | | 18 43 1 |

This Table is adapted to Leap Year, particularly the year 1828, and is only intended to answer the purposes of information when no great degree of accuracy is required, and the Nautical Almanack not at hand.

In order to adapt it to common years, *one-fourth* of the difference between the given and preceding days is to be subtracted from the right ascension in the table for the first after Leap Year, *one-half* for the second after Leap Year, and *three-fourths* for the third ; and in the months of January and February, the right ascension is to be taken for the day following that given.

This Table may be employed in finding the apparent time by the altitude of a star, for finding the time of a star's transit when that is required, for obtaining the latitude by a meridian altitude, &c.

## TABLE II.

### Sun's Declination for every Day in the Year 1828.

| Days. | January. | February. | March. | April. | May. | June. |
|---|---|---|---|---|---|---|
| | South. | South. | South. | North. | North. | North. |
| | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| 1 | 23 4 22 | 17 17 44 | 7 28 10 | 4 38 49 | 15 9 11 | 22 5 43 |
| 2 | 22 59 29 | 17 0 43 | 7 5 18 | 5 1 52 | 15 27 8 | 22 13 57 |
| 3 | 22 54 8 | 16 43 23 | 6 42 21 | 5 24 50 | 15 44 50 | 22 21 7 |
| 4 | 22 48 20 | 16 25 46 | 6 19 18 | 5 47 43 | 16 2 17 | 22 28 14 |
| 5 | 22 42 5 | 16 7 53 | 5 56 9 | 6 10 29 | 16 19 27 | 22 34 57 |
| 6 | 22 35 23 | 15 49 42 | 5 32 56 | 6 33 9 | 16 36 22 | 22 41 17 |
| 7 | 22 28 14 | 15 31 15 | 5 9 38 | 6 55 43 | 16 53 0 | 22 47 12 |
| 8 | 22 20 38 | 15 12 32 | 4 46 15 | 7 18 10 | 17 9 22 | 22 52 44 |
| 9 | 22 12 36 | 14 53 34 | 4 22 50 | 7 40 30 | 17 25 26 | 22 57 52 |
| 10 | 22 4 8 | 14 34 21 | 3 59 21 | 8 2 42 | 17 41 13 | 23 2 36 |
| 11 | 21 55 14 | 14 14 53 | 3 35 48 | 8 24 46 | 17 56 43 | 23 6 55 |
| 12 | 21 45 54 | 13 55 10 | 3 12 13 | 8 46 42 | 18 11 54 | 23 10 50 |
| 13 | 21 36 9 | 13 35 14 | 2 48 36 | 9 8 29 | 18 26 48 | 23 14 20 |
| 14 | 21 25 59 | 13 15 5 | 2 24 57 | 9 30 6 | 18 41 22 | 23 17 26 |
| 15 | 21 15 24 | 12 54 43 | 2 1 16 | 9 51 35 | 18 55 38 | 23 20 7 |
| 16 | 21 4 24 | 12 34 8 | 1 37 35 | 10 12 51 | 19 9 35 | 23 22 24 |
| 17 | 20 53 0 | 12 13 21 | 1 13 52 | 10 34 2 | 19 23 12 | 23 24 16 |
| 18 | 20 41 13 | 11 52 22 | 0 50 10 | 10 55 1 | 19 36 29 | 23 25 43 |
| 19 | 20 29 2 | 11 31 13 | 0 26 27 | 11 15 48 | 19 49 27 | 23 26 45 |
| 20 | 20 16 27 | 11 9 52 | 0 2 45 S | 11 36 24 | 20 2 4 | 23 27 22 |
| 21 | 20 3 30 | 10 48 22 | 0 20 56 N | 11 56 49 | 20 14 20 | 23 27 35 |
| 22 | 19 50 11 | 10 26 41 | 0 44 36 | 12 17 1 | 20 26 16 | 23 27 22 |
| 23 | 19 36 29 | 10 4 52 | 1 8 14 | 12 37 2 | 20 37 51 | 23 26 45 |
| 24 | 19 22 26 | 9 42 52 | 1 31 50 | 12 56 50 | 20 49 5 | 23 25 44 |
| 25 | 19 8 1 | 9 20 44 | 1 55 24 | 13 16 25 | 20 59 57 | 23 24 17 |
| 26 | 18 53 16 | 8 58 28 | 2 18 56 | 13 35 48 | 21 10 28 | 23 22 26 |
| 27 | 18 38 10 | 8 36 5 | 2 42 24 | 13 54 56 | 21 20 36 | 23 20 10 |
| 28 | 18 22 43 | 8 13 33 | 3 5 49 | 14 13 51 | 21 30 23 | 23 17 30 |
| 29 | 18 6 57 | 7 50 55 | 3 29 10 | 14 32 32 | 21 39 47 | 23 14 25 |
| 30 | 17 50 52 | | 3 52 27 | 14 50 59 | 21 48 49 | 23 10 56 |
| 31 | 17 24 27 | | 4 15 40 | | 21 57 28 | |

| Days. | July. | August. | September | October. | November | December. |
|---|---|---|---|---|---|---|
| | North. | North. | North. | South. | South. | South. |
| | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| 1 | 23 7 2 | 17 59 28 | 8 13 9 | 3 16 33 | 14 31 39 | 21 52 8 |
| 2 | 23 2 44 | 17 44 9 | 7 51 16 | 3 39 51 | 14 50 44 | 22 1 9 |
| 3 | 22 58 2 | 17 28 33 | 7 29 15 | 4 3 7 | 15 9 35 | 22 9 44 |
| 4 | 22 52 56 | 17 12 39 | 7 7 6 | 4 26 20 | 15 28 11 | 22 17 53 |
| 5 | 22 47 26 | 16 56 29 | 6 44 51 | 4 49 30 | 15 46 32 | 22 25 37 |
| 6 | 22 41 32 | 16 40 2 | 6 22 28 | 5 12 36 | 16 4 37 | 22 32 54 |
| 7 | 22 35 14 | 16 23 19 | 5 59 59 | 5 35 39 | 16 22 26 | 22 39 45 |
| 8 | 22 28 33 | 16 6 21 | 5 37 25 | 5 58 37 | 16 39 59 | 22 46 9 |
| 9 | 22 21 29 | 15 49 6 | 5 14 44 | 6 21 31 | 16 57 14 | 22 52 6 |
| 10 | 22 14 1 | 15 31 36 | 4 51 59 | 6 44 19 | 17 14 12 | 22 57 36 |
| 11 | 22 6 11 | 15 13 52 | 4 29 8 | 7 7 2 | 17 30 52 | 23 2 38 |
| 12 | 21 57 58 | 14 55 53 | 4 6 12 | 7 29 40 | 17 47 14 | 23 7 13 |
| 13 | 21 49 22 | 14 37 39 | 3 43 13 | 7 52 11 | 18 3 18 | 23 11 21 |
| 14 | 21 40 23 | 14 19 11 | 3 20 9 | 8 14 35 | 18 19 2 | 23 15 0 |
| 15 | 21 31 3 | 14 0 30 | 2 57 2 | 8 36 52 | 18 34 27 | 23 18 12 |
| 16 | 21 21 20 | 13 41 35 | 2 33 51 | 8 59 2 | 18 49 32 | 23 20 56 |
| 17 | 21 11 16 | 13 22 27 | 2 10 38 | 9 21 4 | 19 4 17 | 23 23 12 |
| 18 | 21 0 50 | 13 3 7 | 1 47 22 | 9 42 58 | 19 18 41 | 23 24 59 |
| 19 | 20 50 3 | 12 43 35 | 1 24 4 | 10 4 43 | 19 32 44 | 23 26 19 |
| 20 | 20 38 55 | 12 23 50 | 1 0 43 | 10 26 19 | 19 46 27 | 23 27 10 |
| 21 | 20 27 26 | 12 3 54 | 0 37 22 | 10 47 46 | 19 59 47 | 23 27 33 |
| 22 | 20 15 36 | 11 43 46 | 0 13 59N | 11 9 3 | 20 12 46 | 23 27 27 |
| 23 | 20 3 27 | 11 23 28 | 0 9 25 S | 11 30 10 | 20 25 22 | 23 26 53 |
| 24 | 19 50 57 | 11 2 58 | 0 32 50 | 11 51 7 | 20 37 35 | 23 25 51 |
| 25 | 19 38 7 | 10 42 18 | 0 56 15 | 12 11 53 | 20 49 26 | 23 24 21 |
| 26 | 19 24 58 | 10 21 28 | 1 19 39 | 12 32 28 | 21 0 53 | 23 22 22 |
| 27 | 19 11 30 | 10 0 28 | 1 43 4 | 12 52 51 | 21 11 57 | 23 19 55 |
| 28 | 18 57 42 | 9 39 18 | 2 6 26 | 13 13 2 | 21 22 36 | 23 17 0 |
| 29 | 18 43 36 | 9 17 59 | 2 29 51 | 13 33 1 | 21 32 52 | 23 13 37 |
| 30 | 18 29 11 | 8 56 31 | 2 53 13 | 13 52 47 | 21 42 43 | 23 9 46 |
| 31 | 18 14 28 | 8 34 54 | | 14 12 20 | | 23 5 27 |

This Table, like the last, is for the year 1828, or Leap Year. The correction for any other year must be made as before.

**SUN'S** *Semidiameter, &c.—(Naut. Alm.)*

## TABLE,

### *Of Sun's Semidiameter, and of the time of his semidiameter passing the meridian.*

|  | Time of Sun's ½ diam passing Meridian. | Semi-diameter. |  | Time of Sun's ½ diam passing Meridian. | Semi-diameter. |
|---|---|---|---|---|---|
| *Jan.* | M. S. | M. S. | *July.* | M. S. | M. S. |
| 1 | 1. 10,8 | 16. 17,8 | 1 | 1. 8,5 | 15. 45,5 |
| 7 | 1. 10,5 | 16. 17,7 | 7 | 1. 8,3 | 15. 45,5 |
| 13 | 1. 10.1 | 16. 17,4 | 13 | 1. 8,0 | 15. 45,8 |
| 19 | 1. 9,5 | 16. 16.9 | 19 | 1. 7,5 | 15. 46,1 |
| 25 | 1. 8,9 | 16. 16,3 | 25 | 1. 7,0 | 15. 46,6 |
| *Feb.* |  |  | *Aug.* |  |  |
| 1 | 1. 8,1 | 16. 15,3 | 1 | 1. 6,5 | 15. 47,4 |
| 7 | 1. 7,4 | 16. 14,4 | 7 | 1. 6,0 | 15. 48,3 |
| 13 | 1. 6,7 | 16. 13,2 | 13 | 1. 5,5 | 15. 49,3 |
| 19 | 1. 6,1 | 16. 12,0 | 19 | 1. 5,0 | 15. 50,4 |
| 25 | 1. 5,5 | 16. 10,7 | 25 | 1. 4,6 | 15. 51,6 |
| *Mar.* |  |  | *Sept.* |  |  |
| 1 | 1. 5,2 | 16. 9,7 | 1 | 1. 4,2 | 15. 53,1 |
| 7 | 1. 4,8 | 16 8,2 | 7 | 1. 3,9 | 15. 54,6 |
| 13 | 1. 4,5 | 16. 6,6 | 13 | 1. 3,8 | 15. 56,1 |
| 19 | 1. 4,3 | 16. 4,9 | 19 | 1. 3,8 | 15. 57,7 |
| 25 | 1. 4,2 | 16. 3,3 | 25 | 1. 3,9 | 15. 59,3 |
| *April.* |  |  | *Oct.* |  |  |
| 1 | 1. 4,2 | 16. 1,3 | 1 | 1. 4,1 | 16. 1,0 |
| 7 | 1. 4,4 | 15. 59,7 | 7 | 1. 4,4 | 16. 2,6 |
| 13 | 1. 4,6 | 15. 58,1 | 13 | 1. 4,8 | 16. 4,3 |
| 19 | 1. 4,9 | 15. 56,5 | 19 | 1. 5,3 | 16. 5,9 |
| 25 | 1. 5,4 | 15. 54,9 | 25 | 1. 5,9 | 16. 7,5 |
| *May.* |  |  | *Nov.* |  |  |
| 1 | 1. 5,8 | 15. 53,5 | 1 | 1. 6,7 | 16. 9,3 |
| 7 | 1. 6,3 | 15. 52,1 | 7 | 1. 7,4 | 16. 10,8 |
| 13 | 1. 6,8 | 15. 50,9 | 13 | 1. 8,1 | 16. 12,1 |
| 19 | 1. 7,2 | 15. 49,7 | 19 | 1. 8,7 | 16. 13,3 |
| 25 | 1. 7,7 | 15. 48,7 | 25 | 1. 9,4 | 16. 14,5 |
| *June.* |  |  | *Dec.* |  |  |
| 1 | 1. 8,1 | 15. 47,6 | 1 | 1. 10,0 | 16. 15,4 |
| 7 | 1. 8,3 | 15. 46,9 | 7 | 1. 10,5 | 16. 16,2 |
| 13 | 1. 8,5 | 15. 46,3 | 13 | 1. 10,8 | 16. 16,9 |
| 19 | 1. 8,6 | 15. 45,9 | 19 | 1. 10,9 | 16. 17,4 |
| 25 | 1. 8,6 | 15. 45,6 | 25 | 1. 11,0 | 16. 17,7 |

**SUN'S** *parallax in altitude.*

| *Altitude.* | *Parallax.* | *Altitude.* | *Parallax.* |
|---|---|---|---|
| 0° | 9″ | 60° | 4″ |
| 10 | 9 | 65 | 4 |
| 20 | 8 | 70 | 3 |
| 30 | 8 | 75 | 2 |
| 40 | 7 | 80 | 2 |
| 50 | 6 | 85 | 1 |
| 55 | 5 | 90 | 0 |

**SURFACES** *of Solids.*

Let $y =$ ordinate of any curve, $z =$ length; then

$$\text{Surface} = \text{fl. } 2\,\pi\,y\,dz.$$

*Ex.* 1. Surface of cone $= 2\,\pi\,b \times \frac{s}{2}$, where $b = \frac{1}{2}$ base, and $s =$ slant side, $=$ circumference of base $\times \frac{1}{2}$ slant side.

2. Surface of sphere $= 4\,\pi\,r^2 =$ four times the area of one of its great circles.

3. Surface of paraboloid $= \frac{\pi.\,(4\,y^2 + a^2)^{\frac{3}{2}}}{6\,a} - \frac{\pi\,a^2}{6}$.

4. Surface of cycloid $= \frac{8\,\pi\,a^2}{3}$ ($a =$ diameter of generating circle.)

*Guldinus' property.*

Let M D E K (*see Fig. Art. Solid*) be any plane figure, revolving about an axis $x\,y$ in its own plane; then the area of the surface generated by the perimeter of this figure, is equal to the circumference described by the centre of gravity of the perimeter multiplied into the perimeter.

*Ex.* Let D M E K be a circle, then the solid will represent the ring of an anchor, and if $r =$ radius of circle, and $a =$ A O, the surface $= 2\,\pi\,a \times 2\,\pi\,r = 4\,\pi^2\,a\,r$.

SURVEYING.

I. *Surveying Land.*

1. The area of a triangle $=$ base $\times \frac{1}{2}$ perpendicular altitude: or $=$ the product of any two sides $\times$ natural sine of their included $\angle$; or when three sides A B, A C, B C are given, their half sum being S, area $=$ $\sqrt{\{S \times (S - AB) \times (S - AC) \times (S - BC)\}}$

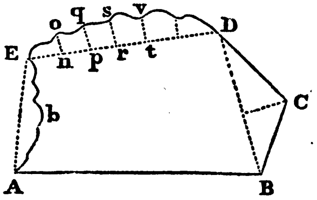
2. The area of a trapezium = base × ½ sum of the perpendiculars. And the area of a trapezoid = ½ sum of the parallel sides × perpendicular distance between them.

3. To find the area of any irregular polygon, divide it into trapeziums, or trapezoids, or triangles, and find their areas separately; and their sum is the area of the polygon.

4. To find the area of a long irregular figure E *q* D *p* bounded on one side by a curve.

Divide E D into any number of equal parts, and measure the perpendiculars, *n o*, *p q*, *r s*, *t v* &c. then the area is found nearly by adding together all the perpendiculars, dividing the sum by the number of perpendiculars increased by unity, and multiplying by the chord of the curve.

5. To find, by the foregoing rules, the content of the irregular field ABCDE, which will include most of the cases likely to occur in practice.



Find the area of the trapezium A B D E by Art. 3, the Δ B D C by Art. 1; and the curvilinear areas E *q* D, E *b* A by Art. 4; add the three first areas together and subtract the last, for the content of the field.

66 ft

Land is measured by a chain 22 yards long, and divided into 100 equal parts or links, each link being 7.92 inches: 10 square chains, or 100,000 square links, is one acre, viz.:—

625 square links is 1 perch.
25,000 square links or 40 perches, 1 rood.
100,000 square links or 4 roods, 1 acre.

The perch (which in statute measure is 16½ feet) varies by custom in different parts of England; and with it, consequently, varies the acre in proportion.

In Devonshire and part of Somersetshire, 15; in Cornwall, 18; in Lancashire and Yorkshire, 21; and in Cheshire and Staffordshire, 24 feet are accounted a perch.

Hence the following Table will give the number of square feet in a square perch, in the above-mentioned counties.

| | | | | |
|---|---|---|---|---|
| Statute perch .............. | 16,5 | × 16.5 | = 272.25 | square feet. |
| Devonshire perch ........ | 15 | × 15 | = 225 | do. |
| Cornwall perch ............ | 18 | × 18 | = 324 | do. |
| Lanc. and Yorks. perch | 21 | × 21 | = 441 | do. |
| Cheshire and Staff. perch | 24 | × 24 | = 576 | do. |

*Rules for reducing Statute Measure to Customary, and the contrary.*

1. To reduce statute measure to customary, multiply the number of perches statute measure, by the square feet in a square perch statute measure; divide the product by the square feet in a square perch customary measure, and the quotient will be the answer in square perches; which reduce to roods and acres, by dividing by 40 and 4.

2. To reduce customary measure to statute, multiply the number of perches, customary measure, by the square feet in a square perch customary measure; divide the product by the square feet in a square perch statute measure, and the quotient will be the answer in square perches; which reduce as before.

By these rules tables may be calculated to save the trouble of computing for particular cases; thus,

### TABLE I.

*To reduce Statute Measure to Customary of 21 feet to a perch.*

| *Stat. Acre.* | *Customary.* A. | R. | P. | *Stat. Rood.* | *Customary.* R. | P. |
|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 18,7 | 1 | 0 | 24,7 |
| 2 | 1 | 0 | 37,5 | 2 | 1 | 9,4 |
| 3 | 1 | 3 | 16,3 | 3 | 1 | 34,1 |
| 4 | 2 | 1 | 35,0 | *Stat. Perch* | | *Customary.* P. |
| 5 | 3 | 0 | 13,8 | | | |
| 6 | 3 | 2 | 34,6 | | | |
| 7 | 4 | 1 | 11,4 | 1 | | 0,6 |
| 8 | 4 | 3 | 30,1 | 5 | | 3,0 |
| 9 | 5 | 2 | 8,9 | 10 | | 6,1 |
| 10 | 6 | 0 | 27,7 | 15 | | 9,2 |
| 20 | 12 | 1 | 15,4 | 20 | | 12,3 |
| 30 | 18 | 2 | 3,1 | 25 | | 15,4 |
| 40 | 24 | 2 | 30,8 | 30 | | 18,5 |
| 50 | 30 | 3 | 18,5 | 35 | | 21,6 |

## TABLE II.

*To reduce Customary Measure of 21 feet to a perch, to Statute.*

| Cust. Acre. | Statute. | | | Cust. Rood. | Statute. | | |
|---|---|---|---|---|---|---|---|
| | A. | R. | P. | | A. | R. | P. |
| 1 | 1 | 2 | 19,17 | 1 | | 1 | 24,79 |
| 2 | 3 | 0 | 38,24 | 2 | | 3 | 9,58 |
| 3 | 4 | 3 | 17,44 | 3 | 1 | 0 | 34,39 |
| 4 | 6 | 1 | 36,64 | Cust. Perch | Statute. | | |
| 5 | 8 | 0 | 15,84 | | | R. | P. |
| 6 | 9 | 2 | 34,88 | | | | |
| 7 | 11 | 1 | 14,8 | 1 | | | 1,619 |
| 8 | 12 | 3 | 33,28 | 5 | | | 8,099 |
| 9 | 14 | 2 | 12,48 | 10 | | | 16,198 |
| 10 | 16 | 0 | 31,68 | 15 | | | 24,297 |
| 20 | 32 | 1 | 23,36 | 20 | | | 32,396 |
| 30 | 48 | 2 | 15,4 | 25 | | 1 | 0,495 |
| 40 | 64 | 3 | 36,88 | 30 | | 1 | 8,594 |
| 50 | 80 | 3 | 38,56 | 35 | | 1 | 16,693 |

*Ex.* 1. In 36A. 1R. 10P. statute how many acres, &c. customary measure of 21 feet to a perch?

Reduce to perches, which will be found 5810, ∴ 5810 × 272,25 = 1581772,50; this divided by 441 gives 3586,7 perches; divide by 40 and 4 and the result is 22A. 1R. 26,7P. The same answer may be had from Table I.

*Ex.* 2. Reduce 22A. 1R. 27P. customary, to statute measure.

Here the number of perches is 3587, which, multiplied by 441, and divided by 272.25, gives 36A. 1R. 10P. The same result may be obtained from Table II.

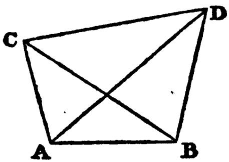
### II. *Surveying Trigonometrically.*

1. These large surveys have been undertaken principally for the accomplishment of one or other of these three objects, viz. (1) For finding the difference of longitude between two moderately distant and noted meridians, as the meridians of the observatories at Greenwich and Paris. (2) For the exact determination of the principal places in a country, with a view to give greater accuracy to maps. (3) For the measurement of a degree in various situations, in order to determine from thence the figure and magnitude of the earth.

These important objects can only be attained, by the greatest possible degree of accuracy in the instruments employed, the operations performed, and the computations required.

The following must only be considered a mere outline of the method pursued in surveying a country: the niceties necessary to be attended to, in order to render such survey available for scientific purposes, cannot be here described.

2. To carry on a measurement by a series of triangles.



Let a base line A B be measured,* and having fixed upon two objects C and D, observe the ∠'s B A C, B A D, A B C, A B D; then in the Δ A B C, the ∠'s B A C, A B C, being known, their supplement A C B is known, ∴ A C and C B may be found by Case 1. Plane Trigonometry. The relative bearings and distances therefore of A, B, C are thus determined. Again in the Δ A B D, the ∠'s D A B, D B A being known, A D B is known, and ∴ D B may be found. Lastly, in the Δ D B C, the sides B C, B D and the included ∠ C B D are known, ∴ the remaining ∠'s B C D, B D C may be found, and consequently also the side C D (see Case 2. Plane Trigonometry). The bearings ∴ and distances of B, C, D are also known. In the same way, by considering either A C, C D or D B as a new base, and fixing upon two other points; the measurement may be continued at pleasure.

In conducting geodetical operations, the following rules by Hutton should be observed, to diminish the probability of error.

(1) When one side only of a triangle is to be determined, the measured base should be nearly equal to the required side.

(2) When two sides of a triangle are to be determined, the triangle should, if possible, be equilateral.

(3) When the base cannot be equal to one or both the required sides, it should be as long as possible, and the two angles at the base equal, and not less than 20 or 30 degrees.

In the late survey of England the base first measured was upon Hounslow Heath. By continuing the measurement to Salisbury Plains, the distance of two objects was there found by calculation to be 36574,4 feet, and, by actual admeasurement, the distance was found to be 36574,3 feet, differing very little more than an inch from the computed distance.

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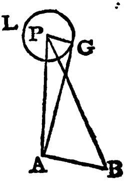
* To reduce a base on an elevated level to that at the surface of the sea, let $r =$ rad. of earth at the surface of the sea, $r + h$ the rad. referred to the level of the base measured, the altitude $h$ being determined by the barometer, B the length of the measured base at the altitude $h$, then the correction is $\frac{Bh}{r}$ nearly, which must be subtracted from the measured base, to give the true base at the level of the sea.

We shall close this necessarily imperfect article with the methods of finding the difference of longitudes and difference of latitudes of places upon the earth's surface, as practiced in the late government survey of this country.

*Difference of Longitude.*

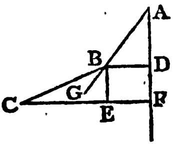


Let P be the pole, L G the circle described by the pole star, A and B the two places. Take by the instrument the $\angle$ G A B = $\angle$ contained by B and the pole star when at its greatest azimuth; then knowing P A, P G, we may find P A G the greatest azimuth, which added to G A B gives P A B; hence in the spherical $\Delta$ P A B, we have P A, P B and $\angle$ P A B to find $\angle$ A P B the difference of longitude. Hence if A be Greenwich, the longitude of B is known.

N.B. For four minutes either before or after the pole star's greatest elongation, it moves only about a second in azimuth; hence a good pocket watch gives the time of greatest azimuth with sufficient accuracy

*Difference of Latitude.*

In finding the difference of longitude above, the latitude was supposed known; this was found by geometrical admeasurement thus,



Let A, B, and C be three places, A F the meridian of A. Find the $\angle$ B A D = supplement of $\angle$ P A B in the former figure (and which may be found, if A's latitude is known, as was shewn in the last Article); and find by observation A B, B C, and $\angle$ A B C; then in the $\Delta$ A B D we can find A D in feet, and $\therefore$ (knowing the dimensions of the earth) in seconds, which gives the difference of latitudes of A and B. Again, in the $\Delta$ C B E, we have C B G the supplement of A B C and G B E (= B A D) and $\therefore$ C B E; hence B E may be found, and $\therefore$ A F, = difference of latitude of A and C; and thus we may proceed through any number of $\Delta$'s. If A be Greenwich, or any place whose latitude is accurately known, the latitude of the rest will be had.

In the same $\Delta$'s, we can also find B D, C F, or the perpendicular distance of each place from A's meridian.

The latitudes thus determined are more accurate than those deduced from astronomical observation; since the best instruments could not have given the zenith distances nearer than about one second, answering in these parts to 101 feet on the surface of the earth.

## TABLE

*Of the different lengths of a degree, as measured in various parts of the earth, the time of its measurement, the latitude of its middle point, &c. (Barlow.)*

| Date. | Latitude. | Extent in Eng. miles and dec. | Measurers. | Countries. |
|---|---|---|---|---|
| 1525 | 49° 20½′ N. | 68·763 | M. Fernel ....... | France. |
| 1620 | 52 4 N. | 66· 91 | Snellius........... | Holland. |
| 1635 | 53 15 N. | 69·545 | Norwood ........ | England. |
| 1644 | | 75·066 | Riccioli........... | Italy. |
| 1669 | 49 22 N. | 68·945 | Picard............. | France. |
| 1718 | | 69·119 | Cassini ........... | |
| 1737 | 66 20 N. | 69·403 | Maupertuis, &c. | Lapland. |
| | 49 22 N. | 69·121 | Cassini and La Caille.......... | France. |
| 1740 | 45 00 N. | 69·092 | | |
| 1744 | 0 0 | 68·751 | Juan and Ulloa | Peru. |
| | | 68·732 | Bouguer ........ | |
| | | 68·713 | Condamine ...... | |
| 1752 | 33 18½ S. | 69·076 | La Caille ........ | C. of GoodHope |
| 1755 | 43 0 N. | 68·998 | Boscovich ........ | Italy. |
| 1764 | 44 44 N. | 69·061 | Beccaria .......... | |
| 1766 | 47 40 N. | 69·142 | Leisganig ........ | Germany. |
| 1768 | 39 12 N. | 68·893 | Mason & Dixon | America. |
| 1802 | 51 29 54½ N. | 69.146 | Lt.-Col. Mudge | England. |
| 1803 | 66 20½ N. | 69·292 | Swanberg, &c... | Lapland. |
| | 12 32 N. | 68·743 | Lambton ......... | Misore. |
| 1808 | 44 52½ N. | 68·769 | Biot, Arago, &c. | France. |

SWEEPER.—*See Nightglass.*

SYNODICAL *revolution of the planets.*

1. If two planets revolve in circular orbits, to find the time from conjunction to conjunction.

Let P = periodic time of the earth, $p$ = that of the planet, suppose an inferior, $t$ = the time required; then

$$t = \frac{Pp}{P - p}.$$

For a superior planet,

$$t = \frac{Pp}{p - P}.$$

This will also give the time between two oppositions, or between any two similar situations.

*Cor.* Since $t = \frac{Pp}{P - p}$, $p = \frac{Pt}{t + P}$.

Therefore, from the earth's period (P) known, and the synodic ($s$) observed, we can determine the periodic time ($p$) of the planet.

For the synodical periods of the planets—*see Planets elements of.*

2. To find the same for three bodies.

Let $T$ = time between the conjunctions of the 1st and 2d found as above; $t$ = do. between the 2d and 3d.

Then if $m$ = least common multiple of $T$ and $t$, $m$ = time between two conjunctions of the three bodies.

# T

## TANGENTS, *method of drawing.*

1. Method of drawing a tangent to any curve, whose equation is given.

Let $x$ and $y$ = abscissa and ordinate; then

$$\text{Subtangent} = \frac{y\,dx}{dy}.$$

*Ex.* 1. In parabola, subtangent $= 2x$.

2. In circle and ellipse, subtan. $= \frac{2ax - x^2}{a - x}$.

3. In hyperbola, subtan. $= \frac{2ax + x^2}{a + x}$.

2. To find the equations to the tangent and normal.

Let $y' = ax' + b$ be the equation to the tangent; then $a = \frac{dy}{dx}$, also since the curve and line have a point in common, $y = ax + b$,

$$\text{and } y' - y = \frac{dy}{dx}(x' - x) \text{ which is the required equation.}$$

Again let $y''$ and $x''$ be the coordinates to the normal, then since it passes through a point whose coordinates are $x$ and $y$, and is perpendicular to a line whose equation is $y' - y = \frac{dy}{dx}(x' - x)$, the equation to the normal will be $y'' - y = -\frac{dx}{dy}(x'' - x)$. *(See Line, Art. 4.)*

*Ex.* In the parabola, equation to the tangent is $y' - y = \frac{2a}{y}(x' - x)$; and that to the normal $y'' - y = -\frac{y}{2a}(x'' - x)$.

For tangents to Spirals—*see Spiral.*

TAYLOR'S *Theorem.—(Higman.)*

If $x$ and $y$ be the coordinates to any point of a curve, and if, when $x$ becomes $x + h$, $y$ becomes $y'$; then will

$$y' = y + \frac{dy}{dx}h + \frac{d^2y}{dx^2}\frac{h^2}{1.2} + \frac{d^3y}{dx^3}\frac{h^3}{1.2.3} + \text{\&c.}$$

*Cor.* 1. If when $x$ becomes $x - h$, $y$ becomes $y_{,}$ then will

$$y_{,} = y - \frac{dy}{dx}h + \frac{d^2y}{dx^2}\frac{h^2}{1.2} - \frac{d^3y}{dx^3}\frac{h^3}{1.2.3} + \text{\&c.}$$

*Cor.* 2. If $h = dx$.

$$y' = y + dy + \frac{d^2y}{1.2} + \frac{d^3y}{1.2.3} + \text{\&c.}$$

*Cor.* 3. The above theorem may be expressed in general terms thus:—The variable of a function being supposed to consist of two parts $x$ and $h$, to develope the function in a series of powers of one of the parts $h$.

*Maclaurin's Theorem.*

To expand a function in a series of ascending integral and positive powers of the variable.

Let $u =$ any function of $x$, then $u = (u) + \left(\frac{du}{dx}\right)x + \left(\frac{d^2u}{dx^2}\right)\frac{x^2}{1.2} + \left(\frac{d^3u}{dx^3}\right)\frac{x^3}{1.2.3} + \text{\&c.}$ where $(u), \left(\frac{du}{dx}\right), \left(\frac{d^2u}{dx^2}\right)$, &c. denote the values of $u, \frac{du}{dx}, \frac{d^2u}{dx^2}$ &c. when $x = 0$.

This theorem is only a particular case of Taylor's, for take $x = 0$ in Taylor's series, and we have

$$f(h) = (u) + \left(\frac{du}{dx}\right)h + \left(\frac{d^2u}{dx^2}\right)\frac{h^2}{1.2} + \text{\&c.}$$

which is the same as the theorem above, if for $h$ we write $x$.

*Ex.* 1. To expand $(x+h)^m$.

Let $u = x^m$ and $u' = (x+h)^m$. By differentiation we have $\frac{du}{dx} = m x^{m-1}$; $\frac{d^2u}{dx^2} = m.(m-1)\,x^{m-2}$; $\frac{d^3u}{dx^3} = m.(m-1).(m-2)\,x^{m-3}$ &c.

Hence, by Taylor's theorem, $u' = x^m + m x^{m-1} h + m.\frac{m-1}{2} x^{m-2} h^2$ + &c.

*Ex.* 2. To expand $(a+x)^m$ by Maclaurin's theorem.

Let $u = (a+x)^m$; ∴ $\frac{du}{dx} = m.(a+x)^{m-1}$; $\frac{d^2u}{dx^2} = m.(m-1.)(a+x)^{m-2}$ &c. &c. Now let $x = 0$; then $(u) = a^m$, $\left(\frac{du}{dx}\right) = m a^{m-1}$; $\left(\frac{d^2u}{dx^2}\right) = m.(m-1.)\,a^{m-2}$ &c. ∴

$$u = (a+x)^m = a^m + m a^{m-1} x + m.\frac{m-1}{2} a^{m-2} x^2 + \&c.$$

*Ex.* 3. To expand $a^x$ in a series.

Let $u = a^x$, then $\frac{du}{dx} = ku$ ($k$ = h. l. $a$), $\frac{d^2u}{dx^2} = k^2 u$, &c. Now let $x = 0$, then $u = 1$; ∴, by Maclaurin's theorem,

$$a^x = 1 + \frac{kx}{1} + \frac{k^2x^2}{1.2} + \frac{k^3x^3}{1.2.3.} + \&c.$$

*Ex.* 4. To expand log. $(x+h)$.

Let $u = l.x$, and $u' = l(x+h)$; ∴

$$\frac{du}{dx} = \frac{m}{x},\ \frac{d^2u}{dx^2} = -\frac{m}{x^2},\ \frac{d^3u}{dx^3} = \frac{2m}{x^3}\ \&c.$$

∴ by Taylor's theorem,

$$u' = u + m\left(\frac{h}{x} - \frac{h^2}{2x^2} + \frac{h^3}{3x^3} - \frac{h^4}{4x^4} + \&c.\right)$$

*Cor.* If $x = 1$, we have

$$l(1+h) = m\left(\frac{h}{1} - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} + \&c.\right)$$

*Ex.* 5. Expand sin. $x$ in a series.

Let $u$ = sin. $x$. Take the successive differentials of sin. $x$, and find their value when $x = 0$, and we shall have by Maclaurin's theorem

$$\sin. x = x - \frac{x^3}{1.2.3} + \frac{x^5}{1.2.3.4.5} - \&c.$$

In like manner,

$$\cos. x = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \&c.$$

TELESCOPE, *theory of.—(Coddington, Wood.)*

1. *Astronomical Telescope.*

Let F, F′ represent the focal lengths of the object and eye glass; then the magnifying power is $\frac{F}{F'}$.

*Cor.* The linear magnitude of the greatest visible area is measured by the ∠, which the diameter of the eye glass subtends at the centre of the object glass, increased by the difference between the ∠'s which the diameter of the object glass subtends at the image, and at the eye glass.

2. *Galileo's telescope.*

The magnifying power as before $= \frac{F}{F'}$.

*Cor.* The linear magnitude of the field of view, when the eye is placed close to the concave lens, is measured by the angle which the diameter of the pupil subtends at the centre of the eye glass, increased by the difference between the ∠'s which the diameter of the object glass subtends at the pupil, and at the image.

3. *Herschel's and Newton's telescope.*

Let $f$ and F′ be the focal lengths of the speculum and eye glass; then the magnifying power $= \frac{f}{F'}$.

*Cor.* The field of view is nearly equal to the apparent magnitude of the eye glass seen from the speculum.

4. *The Gregorian and Cassegrain's telescope.*

Let $f, f'$, F, be the focal lengths of the great and small mirror, and the lens respectively, $l$ the distance of the mirrors; then the magnifying power of the Gregorian is nearly

$$\frac{(l - f')^2}{f' F}.$$

and of Cassegrain's is

$$\frac{(l + f')^2}{f' F}.$$

5. In refracting telescopes, if A be the linear aperture of the object glass, the density of rays in the picture upon the retina varies as

$$\frac{A^2 F'^2}{F^2}.$$

And in the Newtonian telescope, as

$$\frac{A^2 F'^2}{f^2}.$$

6. *To place a telescope in the meridian by the pole star.—(Wollaston.)*

Calculate the time of the meridian passage of the star correctly, and apply that to your chronometer. Then having the star in the field of your telescope (the instrument being first truly adjusted, and the adjusting screw for azimuth between your finger and thumb) and keeping it bisected, or covered by your meridian wire till the exact instant calculated, clamp the instrument there in azimuth, and you will find it very nearly in the meridian indeed.

Having thus placed the telescope *very nearly* in the meridian; we may adjust it accurately so, by either of the following formulæ :—

*Formula for correcting the error of a Meridian Telescope by the observation of any circumpolar star above and below the pole.*

If the western interval be greater than the eastern one, the telescope points to the east of that end of the true meridian which lies under the the elevated pole (be that N. or S.) and v. v.

The angle of this deviation may be investigated thus:—

To the log. of half the difference between the intervals in seconds (or the difference between either interval and 12*h*. sid. time.)

Add the log. tangent of the star's P D.

And the log. secant of the lat. of the station.

The sum (abating 20 from the Index) will give the log. of a number of seconds of sid. time; which converted into degrees, &c. will express the angular deviation of the instrument from the true meridian, to be applied as above.

This method depends not at all upon knowing truly the R A of the star; nor its P D with any very great accuracy: the Z D or alt. read off with the instrument, as it passes the meridian, will give the latter with fully sufficient precision.

Formula from which the above rule is deduced.—*(Maddy.)*

Deviation $\doteq \frac{1}{2} \, \frac{180^0 - (t - t')}{\cos. l. \ \tan. \delta}$ ; where $t$ and $t'$ are the two intervals, $\delta$ the star's declination, and $l$ the latitude of the place.

### *Formula for the correction of a Meridian Telescope by the observation of two stars differing considerably in polar distance.*

If the *southern* of two stars passes a meridian telescope *too soon* for the calculated difference of apparent R A between them (whether its passage be before or after the northern one, is immaterial) the telescope when turned down towards the *south* horizon will point to the *east* of the true meridian, and v. v. This holds universally, whether the latitude of the station be N. or S.

The angle of this deviation from the meridian may be found thus:—

The quantity of sidereal time, by which the observed difference of R A varies from the calculated difference between the stars, being reduced to seconds of time;

To the log. of that number of seconds; add
  the log. cosines of the declination of each star;
  the log. cosecant of the difference between them in declination;
and the log. secant of the lat. of the station:

The sum (abating 40 from the Index) will give the log. of a number of seconds of sidereal time; which reduced to degrees, &c. will express the angle made by the instrument and the true meridian.

Formula from which the above rule is deduced.—*(Maddy.)*

Let $T - T'$ be the difference of right ascensions of the two stars from the Tables.

$t - t'$ the difference of right ascension as observed by the telescope, $\delta$ and $\delta'$ the declinations, $l$ the latitude, then

$$\text{Deviation} = \left\{ T - T' - (t - t') \right\} \cdot \frac{\cos.\ \delta.\ \cos.\ \delta'}{\cos.\ l.\ \sin.\ (\delta - \delta')}.$$

7. *To find the field of view of a telescope.*

Direct the telescope to a star in the equator, or very near it, which will answer quite well enough for all usual purposes, and observe the number of seconds occupied in its passage across the field of view, and multiply this number by 4, to obtain *in degrees* a measure of the field.

It would evidently be inconsistent with the limits of this small work to enter into any explanation of the nature, use, and adjustment, of mathematical instruments; nevertheless as a telescope is in the hands of almost every one at all conversant in scientific pursuits, the following practical observations on this instrument, selected from the works of eminent practical astronomers, may not be unacceptable to the inexperienced observer.

*Proper size of telescopes.*

The smallest achromatic that can be used with success for astronomical purposes is the 3½ feet, aperture 2¾ inches.—*(Kitchiner.)*

*Magnifying powers of telescopes.*

For day purposes, a power of 90 or 100 is the maximum that can be generally used in this country, except on very fine days, and on objects uncommonly well lighted up. In telescopes of different apertures, the maximum power for day purposes is had by multiplying the diameter of the object speculum or glass in inches by 30. For astronomical purposes the rotatory motion of the earth prevents the application of a much higher power than 300 being used with any advantage: when a higher power than 300 is used, it requires uncommon dexterity both to find the object and manage the instrument. The following powers are proper for a fine achromatic. (1) A comet eye piece, made with two plano convexes not magnifying more than 12 or 15 times, which is also a delightful eye piece for viewing nebulæ and the milky way. (2) For a series of powers for *planetary* observations, multiply the diameter of the object glass in inches by 20, 30, 40, 50 and 60; this last is the maximum that can be used for the planets, and requires a very perfect telescope, and every circumstance to be favourable, to admit of its application with good effect. (3) A positive eye piece magnifying 500 times for close double stars; yet unless the telescope be an uncommonly fine one, a higher power than 200 only renders the object less distinct. (4) A circle of six single double convex lenses magnifying 50, 100, 150, 200, 300, and 400 times, but when the highest power is used, the distinct field of view is reduced to a very small diameter.—*(Kitchiner.)*

*Eye glasses for telescopes.*

In very delicate observations Herschel observes, no double eye glass should be used, as that occasions a too great waste of light. With the double eye glass he could not see the belts of Saturn, which he very plainly saw with the single one. Of single glasses he decidedly prefers concave to convex glasses, as they give a much more distinct image. Their very small field of view is a considerable imperfection, but in objects such as double stars, or the satellites of Saturn, and the Georgian, this inconvenience is not so material.—*(Phil. Trans.)*

*Best criterion of a good telescope.*

The most difficult object to define in the day time, and the best test of the distinctness and correctness of our instruments, is the dial plate of a watch, when the sun shines upon it, placed about 100 feet from the glass. In the night time a fixed star of the first magnitude is the best test, as the least defect in the figure or adjustment of the object glass is immediately seen by the star not appearing round, but surrounded by false lights and luminous accompaniments. For a test of the perfection of a telescope as to its *light and distinctness*, the pole star is as proper as any, as the small accompanying star is not visible except in a very perfect instrument. The examination of a bright object on a dark ground, as a card by daylight or Jupiter by night, with high magnifying powers, affords the severest test of the *perfect achromaticity* of a telescope, by the production of green and purple borders about their edges in the contrary case.—(*Kitchiner.—Mem. Astr. Soc.*)

*On the evenings and situations favourable or otherwise to astronomical observations.*

The rule upon which almost all the rest are founded is *that an uniform temperature* is necessary for the proper performance of a telescope. Upon this principle the following facts, the results of long experience, may be satisfactorily explained.

(1) A frost after mild weather, and a thaw after frost, will derange the telescope, till either the frost or mild weather are sufficiently settled. (2) No telescope just brought out of a warm room can act properly. (3) No delicate observation with high powers can be made when looking through a door, window, or slit, in the roof of on observatory; even a confined place in the open air is detrimental. (4) Windy weather is unfavourable. (5) Stars seen over the roof of a house, when very near, are not distinct, being disturbed probably by warm exhalations from the roof. (6) Dry air is unfavourable; but those evenings wherein the air is saturated with moisture, so as to drop down the tube of the telescope, are particularly favourable to distinct vision.

Upon the whole Dr Herschel observes that to use the highest magnifying powers to the greatest advantage, the air must be very clear, the moon absent, no twilight, no haziness, no violent wind, no sudden change of temperature; under all these circumstances a year that will afford 100 hours must be called a very productive one.—(*Herschel, Phil. Trans.*)

*Rules necessary to be observed for examining delicate objects with success.*

(1) If the telescope has been kept in a warm room, the cap of the object end should be taken off, the eye piece taken out, and the air suffered to pass through the tube for ten minutes, that it may acquire the temperature of the open air.—(*Kitchiner.*)

(2) The observer should in like manner be exposed in the open air for 15 or 20 minutes, and the eye carefully kept from all stimulating and

bright objects, so that the pupil may be in its most expanded state; it requiring at least 20 minutes before the eye can admit a view of very delicate objects (such as faint nebulæ); and the observation of a star, though only of the 2d or 3d magnitude, disorders the eye again, so as to require nearly the same time for the re-establishment of its tranquillity. —*(Herschel, Phil. Tran.)*

(3) We should never use a greater magnifying power than we absolutely want; the lower the power, the more beautiful and brilliant the object appears. In objects however that require great nicety to discern, such as the spheroidical shape of the planets, &c. it is proper in the first instance to use a considerable power, till the eye is accustomed to the phænomenon, after which the power may be gradually lowered.—*(Herschel, Phil. Trans.)*

(4) It may be proper to observe, in order to prevent disappointment, that in the prints usually given of Jupiter, Saturn, &c., the outlines and all the other features of the engraving are far more distinct than we can ever see them in the telescope in one view, it being the very intention of a copper-plate to collect together in one view all that has been successfully discovered by repeated and occasional perfect glimpses, and to represent it united to our conceptions. And this is the case with all drawings in books of Astronomy.—*(Hersch. Phil. Trans.)*

(5) In attempting to determine the apparent shape or magnitude of any planetary body or satellite, it is useful to compare it with some other known object of a similar kind. Thus to form an idea of the peculiar shape of Saturn, compare it with Jupiter several times in succession. To form some notion of the apparent magnitudes of Juno, Pallas, Ceres, and Vesta, compare them with each other, or with Jupiter's satellites.—*(Hersch. Phil. Trans.)*

(6) When we wish to discover very delicate and minute objects, which, with the finest instruments, are only to be seen under the most favourable circumstances, it is indispensable that we should be in a position of the greatest ease; no cramped or painful posture must distort the body or irritate the mind, the whole powers of which must be concentrated in the eye.—*(Kitchiner.)*

(7) In adjusting the telescope to close double stars, Dr. Herschel advises the observer previously to adjust the focus of his glass with the utmost delicacy on a star known to be single, of as nearly as possible the same altitude, magnitude, and colour, as the star which is to be examined, carefully observing whether it be round and well defined, or surrounded by little flitting appendages, as is the case when the object glass is not quite perfect.—*(Phil. Trans.)*

(8) To those who have not been long in the habit of observing double stars, it is necessary to mention, that when first seen they will appear nearer together than after a certain time; nor is it so soon as might be expected that we see them at their greatest distance. I have known it take up two or three months before the eye was sufficiently acquainted with the object to judge with the requisite precision.—*(Hersch. Phil. Trans.)*

(9) It is a singular fact, that a double star, where one of them is of the last degree of faintness, may be best seen by directing the eye to another part of the field. In this way a faint star in the neighbourhood of a large one will often become very conspicuous, though it will totally disappear, as if suddenly blotted out, when the eye is turned full upon it. The small companion of 23 (*h*) Ursæ Maj., is a remarkable instance of this; also ζ Persei, 7 Tauri, 43 Persei, ι Leporis. The lateral portions of the retina, less fatigued by strong lights, and less exhausted by perpetual attention, are probably more sensible to faint impressions than the central ones, which may serve to account for this phænomenon.—*(Hersch. jun., Phil. Trans.)*

### *Of the powers necessary for observing various celestial objects.*

*Comets* may be advantageously seen with a power of about 15.

The *sun*, *moon*, and *nebulæ*, with powers of from 45 to 60.

*Jupiter* and his moons from 80 to 130; but for estimating the brightness and apparent magnitude of the satellites, a lower power than 180 should not be attempted. *The belts of Jupiter* are scarcely discernable in a one foot achromatic, but may be seen with an 18 inch of $1\frac{3}{10}$ aperture, and power of 40; and are easily visible in a two feet, with an aperture of $1\frac{6}{10}$, with a power of from 30 to 60. *Note.* The 3d satellite is considerably larger than any of the rest; the 1st is a little larger than the 2d, and nearly of the size of the 4th.

*Saturn.* The best powers for general purposes are from 130 to 200. To view him with effect, he should not be more than two, or, in very fine nights, three hours from the meridian. The phænomena most worthy of observation in this planet, are the following: his belts; the singular compression at his poles; his double ring; the shadow of the ring upon the planet, and of the planet upon the ring; and his seven satellites. The ring *may* be seen in the 18 inch telescope with a power of 40; but for observing the division of the ring, its shadow upon the planet, his belts, and the compression at his poles, we should not have a less power than 200. As to his satellites, the visibility of these minute and extremely faint objects depends more upon the penetrating, than on the magnifying power, of our telescopes; and with a ten feet Newtonian, charged with a power of only 60, Sir W. Herschell saw all the five old satellites.

*Georgium Sidus.* The satellites of this planet were discovered by Herschel with a power of 157; but their faint scintillations were only perceived by interrupted glimpses, but magnifiers of 300, 460, 600, and 800, were most effective. It is in vain, however, to attempt any considerable power, unless in telescopes which have a prodigious quantity of light; and in Herschell's ten feet telescope, with none of its highest powers, could he possibly ascertain even the existence of the satellites.

*Ceres, Pallas, Juno, and Vesta.*—Herschel applied a distinct magnifying power of 500 or 600 and even higher to these asteroids, and yet no disc was discoverable, any more than in very small stars.

*Double stars.* In a fine telescope the powers employed should be from 200 to 400.

*List of a few double stars, which are proper objects for common telescopes.*

ζ Ursæ Majoris.
γ Delphini.
β. Cygni.
γ Arietis.
γ Andromedæ.
δ Orionis.
σ Orionis double.
11 Monocerotis appears double
ε Lyræ appears double.
ϑ Serpentis.
β Lyræ.
ι Delphini.
π Bootis.
γ Virginis.
ι Cassiopeæ.
μ Cygni.

*List of a few double stars, in which proximity or faintness renders one of them difficult to be seen, and which are proper objects for the finest telescopes.*

ζ Aquarii.
μ Draconis.
β Orionis.
α Piscium.
ξ Ursæ Majoris.
α Aurigæ.
δ Geminorum.
k Cygni.
ε Persei.
α Lyræ.
ρ Herculis.
α Geminorum.
ξ Libræ Seq.
π Bootis.
ε Lyræ.
11 Monocerotis.
κ Bootis.
β Scorpii.
Polaris.
γ Herculis.

40 Serpentis.
γ Leonis.
ξ Libræ Præ.
70 Ophiuchi.
α Herculis.
ζ Coronæ.
ξ Bootis.
44 Bootis.
δ Serpentis.
ε Bootis.
95 Herculis.
β Serpentis.
μ Herculis.
δ Herculis.

*Performance of different telescopes.*

Dr Kitchiner has seen the small star accompanying Polaris with a 2½ feet achromatic, aperture 1¾ inches; and the small star accompanying Rigel; but the telescope was exquisitely perfect.

ε Bootis, α Herculis, γ Andromedæ, β Cygni, ζ Aquarii, Pole Star, Castor, Rigel, may be seen with a fine 44 inch achromatic, of 2¾ aperture; but not one instrument in a hundred will shew them without a false light round the larger star.

With an exquisite achromatic of 46 inches focus and a treble object glass of 3⅜ inches aperture, Dr Kitchiner has seen the Pole star with the following powers, 40, 80, 150, 250, 350, 420, 700, and even with 1123 times the small star was still visible. This shews only how far magnifying power could be carried with this instrument, as it was with evident detriment to vision when higher than 80.

With a most perfect achromatic of 44 inches focus, aperture 2¾ inches made by Dollond, Mr Walker made the following observations. With a negative power of 180, he saw ε Bootis double; ι Bootis; η Coronæ Borealis. Three satellites of Saturn; the shadow of his ring on the planet; and a belt; *d* Serpentis; δ Herculis; the Pole star; ι Bootis, and *h* Draconis; powers 423 single eye glass, and 180, and 133 negative powers, — Rigel with 133, and the star in Monoceros' right foot treble with powers 133, 180, and 423.

The ordinary powers used by Messrs South and Herschel, (*see Phil. Trans.*) in forming their catalogue of double stars, was 179; though occasionally a lower power of 105, and a higher one of 273 were also used.

TEMPERATURE *of Atmosphere.—See Atmosphere.*

| THERMOMETER. | *Freezing point.* | *Boiling point.* |
|---|---|---|
| Fahrenheit's Thermometer | 32° | 212 |
| Reaumur's do. | 0 | 80 |
| Centrigrade do. | 0 | 100 |

To convert the degrees of Reaumur into those of Fahrenheit, and the contrary.

$$F = \frac{R \times 9}{4} + 32^\circ \text{ — and } R = \frac{(F - 32^\circ) \times 4}{9}.$$

To convert the centrigrade to Fahrenheit and the contrary.

$$F = \frac{C \times 9}{5} + 32^\circ \text{ — and } C = \frac{(F - 32^\circ) \times 5}{9}.$$

To convert the Centrigrade to Reaumur and the contrary.

$$R = \frac{C \times 4}{5} \text{ —— and } C = \frac{R \times 5}{4}.$$

THERMOMETRICAL *Barometer.—See Heat.*

TIDES.—*(Vince and Robison from Bernouilli.)*

1. If a fluid sphere at rest be attracted by a distant body S also at rest, it will put on the form of a spheroid; and if P and Q represent respectively the attraction of the spheroid at the extremities of the minor and major axes, $m$ be the addititious force of S upon P, and $n$ that upon the point E

Major axis : Minor :: $P + m : E - 2n$.

*Cor.* If the sphere were the earth, and S the sun or moon; then, upon the above supposition, the difference of the diameters or height of the tide, as caused by the sun, would = 2,033 feet; and the height, as caused by the moon, = 5,412 feet; ∴ in syzygy the height would be 7,445 feet.

2. The altitude of the high tide above the level of the water, if there had been no tide, is double of the depression of the low tide below.

3. Find (1) The elevation of the water at any point above the natural level of the undisturbed ocean. (2) The depression below the natural level at any point. (3) The falling of the water from the highest point, and (4) The rising of the water from the lowest point.

Put $\theta$ = angular distance of the point from the place of high water, or the hour $\angle$ from the time of high tide; $m$ = perpendicular height of high above low water; then the equations will stand thus:

(1) Elevation $= \frac{3 \cos.^2 \theta - 1}{3} \times m.$

(2) Depression $= \frac{3 \sin.^2 \theta - 2}{3} \times m.$

(3) Fall ......... $= m \times \sin.^2 \theta.$

(4) Rise ......... $= m \times \cos.^2 \theta.$

*Cor.* To find the distance of high tide from the point where the water is at the same height at which it would have been if there had been no tide, put $3\cos.^2\theta - 1 = 0$; $\therefore \cos.\theta = \frac{1}{\sqrt{3}} = \cos. 54^\circ. 44'$.

4. To find the elevation and depression as before, produced by the joint action of the sun and moon.

Let $m$ = perpendicular height of high above low water, as caused by the sun, $n$ = ditto arising from the moon, $\theta$ = hour angle from the time of high tide for the sun, $\theta'$ = ditto for the moon; then the elevation above the natural level is

$$\frac{3\cos.^2\theta - 1}{3} \times m + \frac{3\cos.^2\theta' - 1}{3} \times n;$$

and depression is $\frac{3\sin.^2\theta - 2}{3} \times m + \frac{3\sin.^2\theta' - 2}{3} \times n$.

*Cor.* 1. If the sun and moon be in syzygy, $\theta = \theta'$;

$\therefore$ elevation $= (m+n)\cos.^2\theta - \frac{m+n}{3}$;

and depression $= (m+n)\sin.^2\theta - \frac{2}{3}(m+n)$.

Hence at high water, elevation $= \frac{2}{3}(m+n)$, and at low water, depression $= \frac{1}{3}(m+n)$.

*Cor.* 2. If the moon is in quadrature, elevation at S $= \frac{2}{3}m - \frac{1}{3}n$, and depression at M $= \frac{1}{3}m - \frac{2}{3}n$; also the elevation at S above the *inscribed sphere* $= m - n$, and the elevation at M above the same $= n - m$. Hence since $n$ is greater than $m$ in the ratio (according to Bernouilli) of $2\frac{1}{3} : 1$, it is plain that when the moon is in quadrature, it is high water under the moon, and low water under the sun.

*Cor.* 3. Supposing the sun and moon to be in any other position, and it were required to find an intermediate point between them where there is high tide; in this case we must take the expression $\frac{3\cos.^2\theta - 1}{3} \times m + \frac{3\cos.^2\theta' - 1}{3} \times n$, and make the differential $= 0$, and we shall get $m : n :: \sin. 2\theta : \sin. 2\theta'$. Hence we have only to divide an arc $2(\theta + \theta)$ into two parts, so that the ratio of the sines may be given; and the half of each part will give $\theta$ and $\theta'$, and thus we get the point where the tide is highest.

*Cor.* 4. By computing by the last Cor. the $\angle$'s $\theta$ and $\theta'$ for every day from the new or full moon, we might get the time of the high tide when compared with the passage of the sun and moon over the meridian; and thus from these we might construct a table, shewing the theoretical times of high tide during the month.

Hitherto we have supposed the luminary to be in the equator: we come in the next place to consider the effect arising from the declination of the moon.

5. Let D = moon's declination, L = latitude of the place, $\theta$ = hour ∠ from high water; then the height of the water from the lowest point is

$$(\cos.\ D \times \cos.\ L \times \cos.\ \theta + \sin.\ D \times \sin.\ L)^2 \times m.$$

Hence we may consider the following cases:—

I. To find the interval from high to low tide, put $\cos.\ D \times \cos.\ L \times \cos.\ \theta + \sin.\ D \times \sin.\ L = 0$; $\therefore \cos.\ \theta = -\frac{\sin.\ D \times \sin.\ L}{\cos.\ D \times \cos.\ L}$.

II. When the latitude of the place = comp. of moon's declination, $\cos.\ \theta = -1$; $\therefore \theta = 180^\circ$, i.e. the interval between high and low tide = 12 hours, i.e there is only one high and one low tide in 24 hours.

III. When the distance of the place from the pole is less than the moon's declination, the expression in Art. 5 never can become = 0 within the limits of $\cos.\ \theta$; $\therefore$ there is only one high and one low tide in 24 lunar hours. And if we make $\cos.\ \theta = 1$, and $\cos.\ \theta = -1$, we have the difference of the altitudes of the two tides $= 4 \cos.\ D \times \cos.\ L \times \sin.\ D \times \sin.\ L \times m$.

IV. When D = L, make $\cos.\ \theta = 1$, and we have the greatest altitude $= m$; also $\cos.\ \theta = \frac{\sin^2 D}{\cos.^2 D}$ = interval from high to low water.

V. When the moon is in the equator, the altitude of the tide $= \cos.^2 L \times m$.

VI. The height of the tide, when the moon passes the meridian, $= (\cos.\ D \times \cos.\ L + \sin.\ D \times \sin.\ L)^2 \times m$; and when the moon is at the opposite meridian, the height is $(-\cos.\ D \times \cos.\ L + \sin.\ D \times \sin.\ L)^2 \times m$. Hence when the moon is in the equator, $\sin.\ D = 0$, and the height of both tides is equal. To a place on the north of the equator, when the moon has south declination, sin. D becomes negative, and the latter tides are the greatest; but when the moon has north declination, sin. D is positive, and the former is the greatest. Hence, to us in this case, the high tide is greater when the moon is above the horizon than when below. The difference of the two tides is always what is given in Case III.

VII. The height of the two tides, when the moon passes the meridian, being $(\cos. D \times \cos. L + \sin. D \times \sin. L)^2 \times m$, and $(-\cos. D \times \cos. L + \sin. D \times \sin. L)^2 \times m$, the mean height is $(\cos.^2 D \times \cos.^2 L + \sin.^2 D \times \sin.^2 L) \times m$. Hence the same north and south declination of the moon give the same mean altitude.

VIII. Under the equator the mean height $= \cos.^2 D \times m$.

The general phænomena of the tides agree very well with the conclusions deduced from the theory of gravity, indeed much more accurately than could have been expected, when we consider that the theory supposes the whole surface of the earth to be covered with deep waters; that there is no inertia of the waters; that the major axis of the spheroid is constantly directed to the moon; and that there is an equilibrium of all the parts; none of which suppositions are strictly founded in fact.

As a sequel to this Article we will subjoin a few of the principal phænomena of the tides, as deduced from actual observation.—*(Playfair.)*

The time from one high water to the next, is, at a mean, 12*h*. 25*m*. 24*s*. The instant of low water is not exactly in the middle of this interval; the tide in general taking 9 or 10 minutes more in ebbing than in flowing.

At new and full moon, or at the spring tides, the interval between the consecutive tides is the least, viz. 12*h*. 19*m*. 28*s*. At the quadratures, or neap tides, the interval is greatest, viz. 12*h*. 30*m*. 7*s*.

The gradual subsidence of the waters is such, that the diminution of heights are nearly as the squares of the times from high water.

The time of high water in the open sea is from 2 to 3 hours after the moon has been on the meridian, either above or under the horizon; but on the shores of large continents, and where there are shallows and obstructions, there are great irregularities in this respect; but for any given place the hour of high water is always nearly at the same distance from that of the moon's passage over the meridian.

The highest of the spring tides is not the tide that immediately follows the syzygy, but is in general the third, and in some cases the fourth.

At Brest, the spring tides rise to 19,317 feet; and those of the neap to 9,151. In the Pacific Ocean, the rise, in the first case, is 5 feet; in the second, 2 or 2,5. Indeed it may happen, that although the greatest elevation produced by the joint action of the sun and moon, in the open sea, does not exceed 8 or 9 feet, the tide in some singular situations may amount considerably higher. For instance, in the harbour of Annapolis-Royal, it sometimes rises 120 feet; the water accumulating to this astonishing height in consequence of its being stopped in the Bay of Fundy as in a hook.

The greater the rise of high water above the level of a fixed point, the greater is the depression of the corresponding low water relatively to the same point.

The height of the tide is affected by the vicinity of the moon to the earth, and increases, cæteris paribus, when the parallax and apparent diameter of the moon increase, but in a higher ratio.

The rise of the tide is affected by the declination of the luminaries; it is greatest, cæteris paribus, at the equinoxes, and least at the solstices.

When the moon is in the northern signs, the tide of the day, in all northern latitudes, is somewhat greater than the tide of the night: and the contrary when the moon is in the southern signs.

If the tides be considered relatively to the whole earth, and to the open sea, there is a meridian about 30° eastward of the moon, where it is always high water; on the west side of this circle, the tide is flowing; on the east, it is ebbing; and on the meridian, at right ∠'s to the same, it is every where low water.

In high latitudes, whether south or north, the rise and fall of the tide are inconsiderable. It is probable that at the poles there are no tides.

The tides, in narrow seas, and on shores far from the main body of the ocean, are not produced in those seas by the direct action of the luminaries, but are waves propagated from the great diurnal undulation, and moving with much less velocity. For instance, the high water transmitted from the tide in the Atlantic, reaches Ushant between three and four hours after the moon has passed the meridian. This wave then divides itself into three; one passing up the British Channel, another ranging along the west side of Ireland and Scotland, and the third entering the Irish Channel. The first of these flows through the channel at about 50 miles an hour, and reaches the Nore about 12 at night. The second moves more rapidly, so as to reach the North of Ireland by six, and the Orkneys by nine, and the Naze of Norway by 12; and in 12 hours more it reaches the Nore, where it meets the morning tide, that left the mouth of the channel only eight hours before. Thus these two tides travel round Britain in about 28 hours, in which time the primitive tide has gone round the whole circumference of the earth and nearly 45 degrees more.

## TABLE

***Of the time of High Water at the full and change of the Moon, at the principal ports and places on the coasts of Great Britain and Ireland.***

| Places. | Situation. | Time | Places. | Situation. | Time |
|---|---|---|---|---|---|
| | | H. M. | | | H. M. |
| Aberdeen ......... | Scotland | 12 45 | Cowes ............... | I. of Wight | 11 15 |
| Aberdovy ......... | Wales | 7 30 | Cromartie ......... | Scotland | 11 45 |
| Aberistwith ...... | Wales | 7 30 | Cuckold's Point | Ri. Thames | 2 15 |
| Achill Head ...... | Ireland | 6 0 | | | |
| Agnes (St) ........ | Scilly Isles | 4 40 | Dartmouth ...... | England | 6 10 |
| Air Point .......... | Isle of Man | 10 30 | Deal ............... | England | 11 15 |
| Aldborough ....... | England | 10 45 | Dee (River) ...... | Scotland | 12 45 |
| Alne River ....... | England | 2 45 | Dingle Bay ....... | Ireland | 3 20 |
| Amlwick Point | Anglesea | 10 30 | Dover Pier ...... | England | 11 16 |
| Arran Isle ......... | Scotland | 11 15 | Downs ............ | England | 11 15 |
| Arundel ............ | England | 9 20 | Dublin ............ | Ireland | 9 30 |
| | | | Dudgeon Lights | North Sea | 6 0 |
| Balta ................ | Shetland | 3 0 | Dunbar ............ | Scotland | 2 15 |
| Baltimore ......... | Ireland | 3 45 | Dundalk Bay ... | Ireland | 10 45 |
| Banff ................ | Scotland | 11 30 | Dundee ............ | Scotland | 2 15 |
| Bantry Bay ...... | Ireland | 3 15 | Dungarvon ...... | Ireland | 4 20 |
| Barmouth ......... | Wales | 8 0 | Dungeness ...... | England | 11 15 |
| Barnstaple Bar | England | 5 30 | | | |
| Beachy, on Shore | England | 9 45 | Eddystone ......... | Eng. Chan. | 5 15 |
| Beachy Offing ... | England | 11 0 | Exmouth Bar ... | England | 6 25 |
| Beaumaris ....... | Wales | 10 15 | | | |
| Berwick ......... | England | 2 15 | Falmouth ......... | England | 5 30 |
| Blakeney ......... | England | 6 0 | Flamboro' Head | England | 4 30 |
| Blyth .............. | England | 2 45 | Flats (Kentish) | England | 11 0 |
| Bolt Head ......... | England | 5 55 | Foreland (N) ... | England | 11 15 |
| Boston ............ | England | 7 15 | Foreland (S) ...... | England | 11 0 |
| Brassa Sound ... | Shetland | 10 0 | Fowey .............. | England | 5 30 |
| Bree Bank ...... | North Sea | 3 30 | | | |
| Bridgewater ...... | England | 6 45 | Galloper ......... | Ri, Thames | 12 45 |
| Bridlington ...... | England | 4 30 | Galway Bay ...... | Ireland | 4 30 |
| Bridport ......... | England | 6 45 | Galloway (Mull) | Scotland | 11 15 |
| Brighton ......... | England | 10 0 | Goodwyn ......... | Downs | 1 30 |
| Bristol ............ | England | 7 0 | Gravesend ...... | England | 1 30 |
| Burnt Island ... | Scotland | 2 30 | Gunfleet ........... | Ri. Thames | 12 0 |
| | | | | | |
| Caernarvon Bar | Wales | 9 0 | Harwich ......... | England | 11 30 |
| Cairston ............ | Orkney | 9 0 | Hastings ......... | England | 10 30 |
| Calf of Man ...... | St Geo. Cha | 10 30 | Helen's (St) ...... | England | 11 45 |
| Cantire (Mull) ... | Scotland | 9 0 | Holyhead Bay ... | Wales | 10 0 |
| Cardigan Bar ... | Wales | 7 0 | Hull ................. | England | 6 0 |
| Carlingford ...... | Ireland | 9 0 | Humber R. Ent. | England | 5 15 |
| Carmarthen ...... | Wales | 6 0 | | | |
| Chatham ......... | England | 1 0 | Ives (St) ............ | England | 4 30 |
| Chester Bar ...... | England | 10 30 | | | |
| Chichester Harb. | England | 11 30 | Kenmare River | Ireland | 3 30 |
| Clear Cape ........ | Ireland | 4 30 | Kentish Knock | Ri. Thames | 11 30 |
| Cornwall Cape ... | England | 4 25 | Kinsale ........... | Ireland | 5 0 |
| Cork Harb. Ent. | Ireland | 4 30 | | | |

| Places. | Situation. | Time | | Places. | Situation. | Time | |
|---|---|---|---|---|---|---|---|
| | | H. | M. | | | H. | M. |
| Land's End | England | 4 | 30 | Selsea Harbour | England | 11 | 15 |
| Leith Pier | Scotland | 2 | 20 | Shannon R. Ent. | Ireland | 3 | 45 |
| Lewis Islands | Scotland | 6 | 0 | Sheerness | England | 12 | 0 |
| Liverpool | England | 11 | 8 | Shields | England | 3 | 0 |
| London | England | 2 | 45 | Skerries | Ireland | 4 | 45 |
| Lyme Regis | England | 6 | 45 | Sligo | Ireland | 6 | 45 |
| | | | | Solebay | England | 10 | 30 |
| Margate Roads | England | 11 | 45 | Southampton | England | 11 | 45 |
| Milford Haven | England | 6 | 0 | Spithead | England | 9 | 30 |
| Montrose | Scotland | 1 | 30 | Sunderland | England | 3 | 0 |
| Mount's Bay | England | 4 | 55 | Swansea | Wales | 6 | 0 |
| | | | | Swin | Ri. Thames | 12 | 0 |
| Needles | England | 9 | 45 | | | | |
| Newcastle | England | 4 | 0 | Tay Bar | Scotland | 2 | 0 |
| Nore Light | Ri. Thames | 12 | 30 | Tees River | England | 3 | 30 |
| | | | | Tinmouth | England | 3 | 0 |
| Orfordness | England | 11 | 0 | Torbay | England | 6 | 10 |
| Orkney Isles | Scotland | 10 | 30 | Trallee Bay | Ireland | 3 | 45 |
| Pentland Frith | Scotland | 10 | 30 | Waterford Harb. | Ireland | 5 | 30 |
| Penzance | England | 4 | 30 | Wexford Harb. | Ireland | 7 | 30 |
| Plymouth Sound | England | 5 | 15 | Weymouth | England | 6 | 30 |
| Portland Race | England | 9 | 15 | Whitby | England | 3 | 45 |
| Portland Road | England | 6 | 15 | Wicklow | Ireland | 9 | 0 |
| Portsmouth Har. | England | 11 | 30 | Wisbeach | England | 7 | 30 |
| Ramsgate | England | 11 | 20 | Yarmouth Roads | England | 8 | 45 |
| Rye Harbour | England | 10 | 36 | Yarmouth Sands | England | 10 | 30 |
| | | | | Yorkshire Coast | England | 6 | 0 |
| Saltees | Ireland | 5 | 40 | Youghall | Ireland | 5 | 0 |
| Seaford | England | 10 | 16 | | | | |

*To find the time of high water on a given day at any place where the time of high water at full and change is known.*

Let the time of the moon's passing the meridian of the given place be found in the Nautical Almanack, and to this time apply the correction, from the following Table, corresponding to her meridian passage and semidiameter, and to the result add the time of high water at full and change at the given place, as given in the preceding Table, and the sum will be the time of high water on the given day. If this sum exceed 12*h* 24*m*, or 24*h* 49*m*, subtract those times from it, and the remainder will be nearly the time of high water on the afternoon of the given day.

***Corrections to be applied to the time of the moon's meridian passage in finding the time of high water.***

| Moon's Mer. Pass. | Moon's Semidiameter. | | | Moon's Mer. Pass. | Moon's Mer. Pass. | Moon's Semidiameter. | | | Moon's Mer. Pass. |
|---|---|---|---|---|---|---|---|---|---|
| | 14′ 30″ | 15′ 30″ | 16′ 30″ | | | 14′ 30″ | 15′ 30″ | 16′ 30″ | |
| h m | h m | h m | h m | h m | h m | h m | h m | h m | h m |
| 0 0 | — 0 4 | 0 0 | + 0 5 | 12 0 | 6 0 | — 0 55 | — 1 2 | — 1 12 | 18 0 |
| 0 30 | — 0 10 | — 0 8 | — 0 5 | 12 30 | 6 30 | — 0 46 | — 0 51 | — 0 58 | 18 30 |
| 1 0 | — 0 17 | — 0 16 | — 0 15 | 13 0 | 7 0 | — 0 32 | — 0 34 | — 0 37 | 19 0 |
| 1 30 | — 0 24 | — 0 25 | — 0 25 | 13 30 | 7 30 | — 0 17 | — 0 16 | — 0 14 | 19 30 |
| 2 0 | — 0 31 | — 0 34 | — 0 36 | 14 0 | 8 0 | — 0 1 | + 0 3 | + 0 9 | 20 0 |
| 2 30 | — 0 38 | — 0 41 | — 0 46 | 14 30 | 8 30 | + 0 8 | + 0 15 | + 0 24 | 20 30 |
| 3 0 | — 0 44 | — 0 49 | — 0 55 | 15 0 | 9 0 | + 0 14 | + 0 21 | + 0 32 | 21 0 |
| 3 30 | — 0 50 | — 0 56 | — 1 4 | 15 30 | 9 30 | + 0 16 | + 0 24 | + 0 36 | 21 30 |
| 4 0 | — 0 55 | — 1 2 | — 1 12 | 16 0 | 10 0 | + 0 15 | + 0 23 | + 0 34 | 22 0 |
| 4 30 | — 0 58 | — 1 6 | — 1 16 | 16 30 | 10 30 | + 0 12 | + 0 19 | + 0 29 | 22 30 |
| 5 0 | — 1 0 | — 1 8 | — 1 19 | 17 0 | 11 0 | + 0 7 | + 0 14 | + 0 23 | 23 0 |
| 5 30 | — 0 59 | — 1 7 | — 1 18 | 17 30 | 11 30 | + 0 2 | + 0 7 | + 0 15 | 23 30 |
| 6 0 | — 0 56 | — 1 2 | — 1 12 | 18 0 | 12 0 | — 0 4 | 0 0 | + 0 5 | 24 0 |

*Ex.* Required the time of high water at London, Sept. 2, 1823, the time of the moon's transit being 22*h.* 39*m.*, and her ½ diameter 16′. 26″, by the Naut. Alm.

| | *h.* | *m.* |
|---|---|---|
| Moon's transit | 22 | 39 |
| Correction from the above Table | + 0 | 29 |
| | 23 | 8 |
| High water at full and change by 1st Table | 2 | 45 |
| | 25 | 53 |
| Subtract | 24 | 49 |
| Time required | 1 | 4 |

**TIMBER** *measuring.*

The customary rule for the measurement of timber is erroneous; for, according to the common rule, a tree frequently contains one-fourth more timber than it is estimated at. The following formulæ give both the customary and true content.

Let L = the length of the tree in feet and decimals, and G the mean girth taken in inches; then

$$\frac{L\ G^2}{2304} = \text{cubic feet customary.}$$

$$\frac{L\ G^2}{1807} = \text{cubic feet true content.}$$

If G as well as L be in feet,

$$.08\ L\ G^2 = \text{cubic feet true content.}$$

Sometimes a certain allowance is made in girting a tree for the thickness of the bark, which is generally one inch to every foot in girt, or $\frac{1}{12}$ of the whole girt; in that case,

$$\frac{L\ G^2}{2742} = \text{cubic feet customary.}$$

$$\frac{L\ G^2}{2150} = \text{cubic feet true content.}$$

If the tree tapers regularly from one end to the other, take half the sum of the girts at the two ends for the mean girt. If the tree do not taper regularly, but is unequal, being thick in some places and small in others, it is usual to take several different dimensions, the sum of which divided by the number of them is accounted the mean girt. But when the tree is very irregular, it is best to divide it into several lengths, and to find the content of each separately. That part of a tree, or of the branches, whose ¼ girt is less than ½ a foot, is not accounted timber.

**TIMBER,** *on the strength and stress of.—See Elastic bodies, equilibrium of.*

**TIME,** *equation of.—See Equation of Time.*

**TIME,** *various tables relating to.—(Vince.)*

### TABLE I.

*For converting degrees, minutes, and seconds into sidereal time.*

| Deg. Min. | Hou. Min. Min. Sec. | Deg. Min. | Hou. Min. Min. Sec. | Sec. | Dec. of Sec. |
|---|---|---|---|---|---|
| 1 | 0. 4 | 30 | 2. 0 | 1 | ,067 |
| 2 | 0. 8 | 40 | 2. 40 | 2 | ,133 |
| 3 | 0. 12 | 50 | 3. 20 | 3 | ,2 |
| 4 | 0. 16 | 60 | 4. 0 | 4 | ,266 |
| 5 | 0. 20 | 70 | 4. 40 | 5 | ,333 |
| 6 | 0. 24 | 80 | 5. 20 | 6 | ,4 |
| 7 | 0. 28 | 90 | 6. 0 | 7 | ,466 |
| 8 | 0. 32 | 100 | 6. 40 | 8 | ,533 |
| 9 | 0. 36 | 200 | 13. 20 | 9 | ,6 |
| 10 | 0. 40 | 300 | 20. 0 | 10 | ,666 |
| 20 | 1. 20 | | | | |

*Ex.* **Reduce 74° 39′ 57″ into time.**

| | | | |
|---|---|---|---|
| 70° | 4*h* | 40*m* | 0*s* |
| 4° | 0 | 16 | 0 |
| 30′ | 0 | 2 | 0 |
| 9′ | 0 | 0 | 36 |
| 50″ | 0 | 0 | 3,333 |
| 7″ | 0 | 0 | 0,466 |
| Time required | 4 | 58 | 39,799 |

## TABLE II.

*For converting sidereal time into degrees, minutes, and seconds.*

| Hou. | Deg. | Min.<br>Sec. | Deg. Min.<br>Min. Sec. | Dec.<br>of Sec. | Sec. |
|---|---|---|---|---|---|
| 1 | 15 | 1 | 0. 15 | ,1 | 1,5 |
| 2 | 30 | 2 | 0. 30 | ,2 | 3,0 |
| 3 | 45 | 3 | 0. 45 | ,3 | 4,5 |
| 4 | 60 | 4 | 1. 0 | ,4 | 6,0 |
| 5 | 75 | 5 | 1. 15 | ,5 | 7,5 |
| 6 | 90 | 6 | 1. 30 | ,6 | 9,0 |
| 7 | 105 | 7 | 1. 45 | ,7 | 10,5 |
| 8 | 120 | 8 | 2. 0 | ,8 | 12,0 |
| 9 | 135 | 9 | 2. 15 | ,9 | 13,5 |
| 10 | 150 | 10 | 2. 30 | | |
| 11 | 165 | 20 | 5. 0 | | |
| 12 | 180 | 30 | 7. 30 | | |
| 16 | 240 | 40 | 10. 0 | | |
| 20 | 300 | 50 | 12. 30 | | |

The manner of applying this Table is evident from the last Example.

## TABLE III.

*Decimal parts of an Hour.*

| | | | |
|---|---|---|---|
| 1′ | ,01666 | 1″ | ,00028 |
| 2 | ,03333 | 2 | ,00056 |
| 3 | ,05 | 3 | ,00083 |
| 4 | ,06666 | 4 | ,00111 |
| 5 | ,08333 | 5 | ,00139 |
| 6 | ,1 | 6 | ,00167 |
| 7 | ,11666 | 7 | ,00194 |
| 8 | ,13333 | 8 | ,00222 |
| 9 | ,15 | 9 | ,00250 |
| 10 | ,16666 | 10 | ,00277 |
| 20 | ,33333 | 20 | ,00556 |
| 30 | ,5 | 30 | ,00833 |
| 40 | ,66666 | 40 | ,01111 |
| 50 | ,83333 | 50 | ,01388 |

## TABLE IV.

*Decimal parts of a Degree.*

| Min. | Dec. | Min. | Dec. | Sec. | Dec. | Sec. | Dec. |
|---|---|---|---|---|---|---|---|
| 1 | ,01667 | 31 | ,51667 | 1 | ,00028 | 31 | ,00861 |
| 2 | ,03333 | 32 | ,53333 | 2 | ,00056 | 32 | ,00889 |
| 3 | ,05000 | 33 | ,55000 | 3 | ,00083 | 33 | ,00917 |
| 4 | ,06667 | 34 | ,56667 | 4 | ,00111 | 34 | ,00944 |
| 5 | ,08333 | 35 | ,58333 | 5 | ,00138 | 35 | ,00972 |
| 6 | ,10000 | 36 | ,60000 | 6 | ,00167 | 36 | ,01000 |
| 7 | ,11667 | 37 | ,61667 | 7 | ,00194 | 37 | ,01028 |
| 8 | ,13333 | 38 | ,63333 | 8 | ,00222 | 38 | ,01056 |
| 9 | ,15000 | 39 | ,65000 | 9 | ,00250 | 39 | ,01083 |
| 10 | ,16667 | 40 | ,66667 | 10 | ,00278 | 40 | ,01111 |
| 11 | ,18333 | 41 | ,68333 | 11 | ,00306 | 41 | ,01139 |
| 12 | ,20000 | 42 | ,70000 | 12 | ,00333 | 42 | ,01167 |
| 13 | ,21667 | 43 | ,71667 | 13 | ,00361 | 43 | ,01194 |
| 14 | ,23333 | 44 | ,73333 | 14 | ,00389 | 44 | ,01222 |
| 15 | ,25000 | 45 | ,75000 | 15 | ,00417 | 45 | ,01250 |
| 16 | ,26667 | 46 | ,76667 | 16 | ,00444 | 46 | ,01278 |
| 17 | ,28333 | 47 | ,78333 | 17 | ,00472 | 47 | ,01306 |
| 18 | ,30000 | 48 | ,80000 | 18 | ,00500 | 48 | ,01333 |
| 19 | ,31667 | 49 | ,81667 | 19 | ,00528 | 49 | ,01361 |
| 20 | ,33333 | 50 | ,83333 | 20 | ,00556 | 50 | ,01389 |
| 21 | ,35000 | 51 | ,85000 | 21 | ,00583 | 51 | ,01417 |
| 22 | ,36667 | 52 | ,86667 | 22 | ,00611 | 52 | ,01444 |
| 23 | ,38333 | 53 | ,88333 | 23 | ,00639 | 53 | ,01472 |
| 24 | ,40000 | 54 | ,90000 | 24 | ,00667 | 54 | ,01500 |
| 25 | ,41667 | 55 | ,91667 | 25 | ,00694 | 55 | ,01528 |
| 26 | ,43333 | 56 | ,93333 | 26 | ,00722 | 56 | ,01556 |
| 27 | ,45000 | 57 | ,95000 | 27 | ,00750 | 57 | ,01583 |
| 28 | ,46667 | 58 | ,96667 | 28 | ,00778 | 58 | ,01611 |
| 29 | ,48333 | 59 | ,98333 | 29 | ,00806 | 59 | ,01639 |
| 30 | ,50000 | 60 | 1,00000 | 30 | ,00833 | 60 | ,01667 |

## TABLE V.

*Decimal Numbers for each Day in the Year.*

| | MONTHS. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug | Sept. | Oct. | Nov. | Dec. |
| 1 | 0.000 | 0.085 | 0.162 | 0.246 | 0.329 | 0.414 | 0.496 | 0.581 | 0.666 | 0.747 | 0.832 | 0.914 |
| 2 | 0.003 | 0.088 | 0.164 | 0.249 | 0.331 | 0.416 | 0.499 | 0.583 | 0.668 | 0.750 | 0.835 | 0.917 |
| 3 | 0.006 | 0.091 | 0.167 | 0.252 | 0.334 | 0.419 | 0.502 | 0.586 | 0.671 | 0.753 | 0.838 | 0.920 |
| 4 | 0.008 | 0.093 | 0.170 | 0.255 | 0.337 | 0.422 | 0.504 | 0.589 | 0.673 | 0.755 | 0.840 | 0.922 |
| 5 | 0.011 | 0.096 | 0.173 | 0.258 | 0.340 | 0.425 | 0.507 | 0.592 | 0.675 | 0.758 | 0.843 | 0.925 |
| 6 | 0.014 | 0.099 | 0.175 | 0.260 | 0.342 | 0.427 | 0.509 | 0.594 | 0.678 | 0.760 | 0.845 | 0.928 |
| 7 | 0.017 | 0.102 | 0.178 | 0.263 | 0.345 | 0.430 | 0.512 | 0.597 | 0.681 | 0.763 | 0.848 | 0.931 |
| 8 | 0.019 | 0.101 | 0.181 | 0.266 | 0.348 | 0.433 | 0.515 | 0.600 | 0.684 | 0.766 | 0.851 | 0.933 |
| 9 | 0.022 | 0.107 | 0.181 | 0.269 | 0.351 | 0.436 | 0.518 | 0.602 | 0.687 | 0.769 | 0.854 | 0.936 |
| 10 | 0.025 | 0.109 | 0.186 | 0.271 | 0.353 | 0.438 | 0.520 | 0.605 | 0.689 | 0.772 | 0.856 | 0.939 |
| 11 | 0.028 | 0.112 | 0.189 | 0.274 | 0.356 | 0.441 | 0.523 | 0.608 | 0.692 | 0.775 | 0.859 | 0.942 |
| 12 | 0.030 | 0.115 | 0.192 | 0.277 | 0.359 | 0.444 | 0.526 | 0.610 | 0.695 | 0.777 | 0.862 | 0.944 |
| 13 | 0.033 | 0.118 | 0.195 | 0.280 | 0.362 | 0.447 | 0.529 | 0.613 | 0.698 | 0.780 | 0.865 | 0.947 |
| 14 | 0.036 | 0.120 | 0.197 | 0.282 | 0.364 | 0.449 | 0.531 | 0.616 | 0.701 | 0.782 | 0.867 | 0.950 |
| 15 | 0.039 | 0.123 | 0.200 | 0.285 | 0.367 | 0.452 | 0.534 | 0.619 | 0.703 | 0.785 | 0.870 | 0.953 |
| 16 | 0.041 | 0.127 | 0.203 | 0.288 | 0.370 | 0.455 | 0.537 | 0.622 | 0.706 | 0.788 | 0.873 | 0.955 |
| 17 | 0.044 | 0.129 | 0.206 | 0.291 | 0.373 | 0.458 | 0.540 | 0.625 | 0.709 | 0.791 | 0.876 | 0.958 |
| 18 | 0.046 | 0.131 | 0.208 | 0.293 | 0.375 | 0.460 | 0.542 | 0.627 | 0.711 | 0.793 | 0.878 | 0.961 |
| 19 | 0.019 | 0.134 | 0.211 | 0.296 | 0.378 | 0.463 | 0.545 | 0.630 | 0.714 | 0.796 | 0.882 | 0.964 |
| 20 | 0.052 | 0.137 | 0.214 | 0.299 | 0.381 | 0.466 | 0.548 | 0.633 | 0.717 | 0.799 | 0.884 | 0.966 |
| 21 | 0.056 | 0.140 | 0.217 | 0.302 | 0.383 | 0.468 | 0.551 | 0.636 | 0.720 | 0.802 | 0.887 | 0.969 |
| 22 | 0.057 | 0.142 | 0.219 | 0.304 | 0.386 | 0.471 | 0.553 | 0.638 | 0.722 | 0.804 | 0.890 | 0.971 |
| 23 | 0.060 | 0.145 | 0.222 | 0.307 | 0.389 | 0.473 | 0.556 | 0.641 | 0.725 | 0.807 | 0.893 | 0.974 |
| 24 | 0.063 | 0.148 | 0.225 | 0.309 | 0.392 | 0.476 | 0.559 | 0.644 | 0.728 | 0.810 | 0.895 | 0.977 |
| 25 | 0.066 | 0.151 | 0.227 | 0.312 | 0.395 | 0.479 | 0.562 | 0.647 | 0.731 | 0.813 | 0.898 | 0.980 |
| 26 | 0.068 | 0.153 | 0.230 | 0.315 | 0.397 | 0.482 | 0.564 | 0.649 | 0.733 | 0.815 | 0.900 | 0.983 |
| 27 | 0.071 | 0.156 | 0.233 | 0.318 | 0.400 | 0.485 | 0.567 | 0.652 | 0.736 | 0.818 | 0.903 | 0.985 |
| 28 | 0.074 | 0.159 | 0.236 | 0.320 | 0.403 | 0.487 | 0.570 | 0.655 | 0.739 | 0.821 | 0.906 | 0.988 |
| 29 | 0.077 | 0.162 | 0.239 | 0.323 | 0.406 | 0.490 | 0.573 | 0.657 | 0.742 | 0.824 | 0.909 | 0.991 |
| 30 | 0.079 | | 0.241 | 0.326 | 0.408 | 0.493 | 0.575 | 0.660 | 0.744 | 0.826 | 0.912 | 0.994 |
| 31 | 0.082 | | 0.244 | | 0.411 | | 0.578 | 0.663 | | 0.829 | | 0.997 |

## TABLE VI.

*For reducing Sidereal to Mean Solar Time.*

| Hou. | Min. Sec. | Min. | Sec. | Sec. | Sec. |
|---|---|---|---|---|---|
| 1 | 0. 9, 83 | 1 | 0,16 | 1 | 0,00 |
| 2 | 0. 19, 66 | 2 | 0,33 | 2 | 0,01 |
| 3 | 0. 29, 49 | 3 | 0,49 | 3 | 0,01 |
| 4 | 0. 39, 32 | 4 | 0,66 | 4 | 0,01 |
| 5 | 0. 49, 15 | 5 | 0,82 | 5 | 0,01 |
| 6 | 0. 58, 98 | 6 | 0,98 | 6 | 0,02 |
| 7 | 1. 8, 81 | 7 | 1,15 | 7 | 0,02 |
| 8 | 1. 18, 64 | 8 | 1,31 | 8 | 0,02 |
| 9 | 1. 28, 47 | 9 | 1,47 | 9 | 0,02 |
| 10 | 1. 38, 30 | 10 | 1,64 | 10 | 0,03 |
| 11 | 1. 48, 13 | 11 | 1,80 | 11 | 0,03 |
| 12 | 1. 57, 96 | 12 | 1,97 | 12 | 0,03 |
| 13 | 2. 7, 78 | 13 | 2,13 | 13 | 0,04 |
| 14 | 2. 17, 61 | 14 | 2,29 | 14 | 0,04 |
| 15 | 2. 27, 44 | 15 | 2,46 | 15 | 0,04 |
| 16 | 2. 37, 27 | 16 | 2,62 | 16 | 0,04 |
| 17 | 2. 47, 10 | 17 | 2,78 | 17 | 0,05 |
| 18 | 2. 56, 93 | 18 | 2,95 | 18 | 0,05 |
| 19 | 3. 6, 76 | 19 | 3,11 | 19 | 0,05 |
| 20 | 3. 16, 59 | 20 | 3,28 | 20 | 0,05 |
| 21 | 3. 26, 42 | 30 | 4,91 | 30 | 0,08 |
| 22 | 3. 36, 25 | 40 | 6,55 | 40 | 0,11 |
| 23 | 3. 46, 08 | 50 | 8,19 | 50 | 0,14 |
| 24 | 3. 55, 91 | 60 | 9,83 | 60 | 0,16 |

**Rule.** Subtract the numbers found in the table corresponding to the given sidereal time from that time, and it reduces it to mean solar time.

*Ex.* Reduce 17*h*. 19*m*. 23*s*. sidereal time into mean solar time.

| | | | |
|---|---|---|---|
| 17*h* | | 2*m* | 47,10*s* |
| 19*m* | | 0 | 3,11 |
| 20*s* | | 0 | 0,05 |
| 3*s* | | 0 | 0,01 |
| | | 2 | 50,27 |
| | 17*h* | 19 | 23 |
| Mean solar time | 17 | 16 | 32,73 |

TABLE VII.—*For converting Mean Solar into Sidereal Time.*

| Hou. | Min. Sec. | Min. | Sec. | Sec. | Sec. |
|---|---|---|---|---|---|
| 1 | 0. 9, 86 | 1 | 0,16 | 1 | 0,00 |
| 2 | 0. 19, 71 | 2 | 0,33 | 2 | 0,01 |
| 3 | 0, 29, 57 | 3 | 0,49 | 3 | 0,01 |
| 4 | 0. 39, 43 | 4 | 0,66 | 4 | 0,01 |
| 5 | 0. 49, 28 | 5 | 0,82 | 5 | 0,01 |
| 6 | 0. 59, 14 | 6 | 0,99 | 6 | 0,02 |
| 7 | 1. 8, 99 | 7 | 1,15 | 7 | 0,02 |
| 8 | 1. 18, 85 | 8 | 1,31 | 8 | 0,02 |
| 9 | 1. 28, 71 | 9 | 1,48 | 9 | 0,02 |
| 10 | 1. 38, 56 | 10 | 1,64 | 10 | 0,03 |
| 11 | 1. 48, 42 | 11 | 1,82 | 11 | 0,03 |
| 12 | 1. 58, 28 | 12 | 1,97 | 12 | 0,03 |
| 13 | 2. 8, 13 | 13 | 2,14 | 13 | 0,04 |
| 14 | 2. 17, 99 | 14 | 2,30 | 14 | 0,04 |
| 15 | 2. 27, 85 | 15 | 2,46 | 15 | 0,04 |
| 16 | 2. 37, 70 | 16 | 2,63 | 16 | 0,04 |
| 17 | 2. 47, 56 | 17 | 2,79 | 17 | 0,05 |
| 18 | 2. 57, 42 | 18 | 2,96 | 18 | 0,05 |
| 19 | 3. 7, 27 | 19 | 3,12 | 19 | 0,05 |
| 20 | 3. 17, 13 | 20 | 3,28 | 20 | 0,05 |
| 21 | 3. 26, 98 | 30 | 4,93 | 30 | 0,08 |
| 22 | 3. 36, 84 | 40 | 6,57 | 40 | 0,11 |
| 23 | 3. 46, 70 | 50 | 8,21 | 50 | 0,14 |
| 24 | 3. 56, 55 | 60 | 9,86 | 60 | 0,16 |

RULE. Add the acceleration or the numbers found in the table corresponding to the given mean solar time, to that time, and it reduces it to sidereal time.

The application of this rule is evident, from the last example.

*Time, sidereal and mean solar.*

Given the hour in mean solar time, to find the sidereal time.

RULE. To the given mean solar time apply the equation of time at the preceding noon from the Naut. Alm., but with a contrary sign, which gives the time since the sun's centre was on the meridian; reduce this time so corrected to sidereal time, by adding the acceleration from Tab. VII.; to which add the sun's R. A. at preceding noon from the Naut. Alm.

Or thus at short—

Sid. time $=$ mean solar time $\mp$ equation of time at prec. noon $+$ acceleration for that hour $+$ sun's R. A. at prec. noon.

Hence conversely,

Mean sol. time $=$ sid. time $-$ sun's R. A. at prec. noon $-$ acceleration for the hour so deduced by Tab. VI. + equation of time at prec. noon.

This last rule also gives the time of a star's passage over the meridian in mean solar time, the star's R. A. being substituted for sid. time.

*Ex.* Given mean solar time 5*h.* 19*m.* 17,4*s.*, Nov. 8, 1827; to find the corresponding sidereal time,

| | *d.* | *m.* | *s.* |
|---|---|---|---|
| | 5 | 19 | 17,4 |
| Equation of time | + | 16 | 6,9 |
| | 5 | 35 | 24,3 |
| | | | 55,1 |
| | 5 | 36 | 19,4 |

| By Tab. VII. | *s.* |
|---|---|
| 5*h* | 49,28 |
| 30*m* | 4,93 |
| 5*m* | 0,82 |
| 20*s* | 0,05 |
| 4*s* | 0,01 |
| | 55,09 |

| | | | |
|---|---|---|---|
| | 5 | 36 | 19,4 |
| R. A. Sun. N. Alm. | 14 | 51 | 25,8 |
| Sid. Time | 20 | 27 | 45,2 |

When the longitude is different from that of Greenwich, a proportional correction must be made for the difference.

If the Naut. Alm. is not at hand, sidereal time may be found very nearly by the following Table, merely adding the sun's *mean* R. A. in the table to the time of day where you are.—(*Woodhouse.*)

*Sun's mean R. A.*

| | Hours. | Days. | M. | S. |
|---|---|---|---|---|
| Jan. 6 | 19 | 1 | 3 | 56 |
| 21 | 20 | 2 | 7 | 53 |
| Feb. 5 | 21 | 3 | 11 | 49 |
| 20 | 22 | 4 | 15 | 46 |
| Mar. 7 | 23 | 5 | 19 | 43 |
| 22 | 0 | 6 | 23 | 39 |
| Apr. 7 | 1 | 7 | 27 | 36 |
| 22 | 2 | 8 | 31 | 32 |
| May 7 | 3 | 9 | 35 | 29 |
| 22 | 4 | 10 | 39 | 28 |
| June 7 | 5 | 11 | 43 | 22 |
| 22 | 6 | 12 | 47 | 18 |
| July 7 | 7 | 13 | 51 | 15 |
| 22 | 8 | 14 | 55 | 12 |
| Aug. 7 | 9 | 15 | 59 | 8 |
| 22 | 10 | | | |
| Sept. 6 | 11 | | | |
| 21 | 12 | | | |
| Oct. 6 | 13 | | | |
| 21 | 14 | | | |
| Nov. 6 | 15 | | | |
| 21 | 16 | | | |
| Dec. 6 | 17 | | | |
| 21 | 18 | | | |

*Ex.* Given as before; to find the sidereal time.

| | *h.* | *m.* | *s.* |
|---|---|---|---|
| Nov. 6th | 15 | 0 | 0 |
| 2d | | 7 | 53 |
| | 15 | 7 | 53 |
| Given time | 5 | 19 | 17,4 |
| Sid. time | 20 | 27 | 10,4 |

Or more accurately by adding the acceleration 55,1*s*, as found in the last example, to the given time, we should have sid. time = 20. 28. 5,5.

From this same Table and the Table of R. A. of the principal stars (*see Stars, catalogue of*), may also be found the time of a star's transit over the meridian in mean solar time nearly without the aid of the Naut. Alm.; the rule being,

Star's R. A. — sun's mean R. A. — acceleration = mean solar time at the time of the star's transit.

*To find the time of the moon or a planet's passing the meridian.—(Woodhouse.)*

Let the increment of sun's R. A. in 24*h*. be $a$; do. of a planet or the moon be A: let also the difference between the R. A. of the heavenly body and that of the sun at the *preceding noon*, expressed in sidereal time, be $t$; then time of a planet's transit =

$$t - \frac{a - A}{24} t + \left(\frac{a - A}{24}\right)^2 t.$$

Or when the planet is retrograde, time =

$$t - \frac{a + A}{24} t + \left(\frac{a + A}{24}\right)^2 t.$$

In the case of the moon, the time =

$$t + \frac{A - a}{24} t + \left(\frac{A - a}{24}\right)^2 t.$$

And in the case of a star, the time =

$$t - \frac{a\,t}{24} + \left(\frac{a}{24}\right)^2 t.$$

*Time error in, corresponding to any small given error or variation in the declination, latitude, or altitude.—(Woodhouse.)*

(1) *Declination.*

Let $t$ be the exact time from noon, $\delta$ = change of declination, $\iota$ = variation in the time, then

$$\iota = \delta\,(\text{tan. declination} \times \text{cot. } t - \text{tan. lat.} \times \text{cosec. } t)$$

This formula is used in finding the time from equal altitudes of the sun, when there is a change of declination, in the interval between the two observations, which there is always, except at the solstices.

(2) *Given the error in latitude to find the error in time.*

Let $\lambda$ = error in latitude, $\iota$ = do. in time, then

$$\iota = \lambda\,(\text{tan. dec.} \times \text{cosec. } t - \text{tan. lat.} \times \text{cot. } t.)$$

This formula is useful at sea; for between the observation which determines the latitude from the sun's meridian altitude, and the observation of the altitude, the observer, if on board a ship, may have changed his place, and if so may have probably changed his latitude.

(3) *Given the error in altitude to find the error in time.*

Let $\alpha$ be the error in altitude, then

$$t = \frac{\alpha}{\text{sin. azim.} \times \text{cos. lat.}}.$$

Hence for a given error in altitude $t$ is the least when the body is on the prime vertical, the altitude ∴ should be taken near the east or west points.

*Time of sun's passing the meridian, or the horizontal or vertical wire of a telescope.—(Vince.)*

(1) Let $d''$ = diameter of the sun estimated in seconds of a great circle; then the time of passing the meridian is

$$\frac{d'' \times \text{sec. declin.}}{15''}.$$

The same will do for the moon if $d''$ = its diameter.

(2). The time of passing an horizontal wire is

$$\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}.$$

The same expression must also give the time which the sun takes in rising.

If $d'' = 1980''$ the horizontal refraction, we have the time that refraction accelerates the rising of the sun =

$$132'' \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}.$$

(3) The time in which the sun would pass the vertical wire of a telescope is

$$\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. } \theta \times \text{cos. dec.}},$$ where $\theta = \angle$ formed by the circles of altitude and declination.

**TORSION,** *elasticity of.—See Elastic Bodies, equilibrium of.*

**TRADE-WIND.**—*See Wind.*

**TRANSITS** *of Mercury and Venus.—(Vince.)*

Let P = the periodic time of the earth, $p$ that of Venus or Mercury. Now that a transit may happen again at the same node, the earth must perform a certain number of complete revolutions in the same time that the planet performs a certain number, for then they must come into conjunction again at the same point of the earth's orbit, or nearly in the same position in respect to the node. Let the earth perform $x$ revolutions whilst the planet performs $y$ revolutions; then will $Px = py$, $\therefore \frac{x}{y} = \frac{p}{P}$. Now $P = 365{,}256$ and for *Mercury*, $p = 87{,}968$, therefore, $\frac{x}{y} = \frac{p}{P} = \frac{87{,}968}{365{,}256} =$ (by resolving it into its continued fractions) $\frac{1}{4}, \frac{6}{25}, \frac{7}{29}, \frac{13}{54}, \frac{33}{137}, \frac{46}{191}$, &c. That is, 1, 6, 7, 13, 33, 46, &c. revolutions of the earth are nearly equal to 4, 25, 29, 54, 137, 191, &c. revolutions of Mercury, approaching nearer to a state of equality, the further you go. The first period, or that of one year, is not sufficiently exact; the period of six years will sometimes bring on a return of the transit at the *same* node; that of seven years more frequently; that of 13 years still more frequently, and so on. Now there was a transit of Mercury at its descending node, in May, 1786; hence by continually adding 6, 7, 13, 33, 46, &c. to it, you get all the years when the transit may be *expected* to happen at that node. In 1789 there was a transit at the ascending node, and therefore by adding the same numbers to that year you will get the years in which the transits may be expected to happen at that node. The next transits at the descending node will happen in 1799, 1832, 1845, 1878, 1891; and at the ascending node, in 1802, 1815, 1822, 1835, 1848, 1861, 1868, 1881, 1894. For *Venus*, $p = 224{,}7$; hence $\frac{x}{y} = \frac{p}{P} = \frac{224{,}7}{365{,}256} = \frac{8}{13}, \frac{235}{382}, \frac{713}{1159}$, &c. Therefore the periods are 8, 235, 713, &c. years. The transits at the same node will therefore sometimes return at 8 years, but oftener in 235, and still oftener in 713, &c. Now in 1769 a transit happened at the descending node in June, and the next transits at the same node will be in 2004, 2012, 2247, 2255, 2490, 2498, 2733, 2741, and 2984. In 1639 a transit happened at the ascending node in November, and the next transits at the same node will be in 1874, 1882, 2117, 2125, 2360, 2368, 2603, 2611, 2846, and 2854. These transits are found to happen, by continually adding the periods, and finding the years when they may be *expected*, and then computing, for each time, the shortest geocentric distance of Venus from the sun's centre at the time of conjunction; and if it be less than the semidiameter of the sun, there will be a transit.

**TRANSIT** *of a star and planet over the Meridian.—See Time.*

**TRANSIT** *instrument, to bring it into the Meridian.—See Telescope.*

**TRAPEZIUM**, *area of.—See Surveying.*

**TRIANGLE**, *plane and spherical area of.—See Surveying and Trigonometry.*

## TRIGONOMETRY.—(*Woodhouse, Barlow.*)

### I. Plane Trigonometry.

*Solution of the cases of right angled triangles.*

Let $a$ be the base, $b$ the perpendicular, $c$ the hypothenuse, and A, B, C the angles opposite.

| *Given.* | *Sought* | *Solution.* | *Given.* | *Sought.* | *Solution.* |
|---|---|---|---|---|---|
| $c$, B | $a$ | $a = c. \cos. B$ | $a$, $c$ | $b$ | $b = \sqrt{c^2 - a^2}$ |
| | $b$ | $b = c. \sin. B$ | | B | $\cos. B = \frac{a}{c}$ |
| | A | $A = \frac{\pi}{2} - B$ | | | |
| $a$, B | $b$ | $b = a. \tan. B$ | $a$, $b$ | $c$ | $c = \sqrt{a^2 + b^2}$ |
| | $c$ | $c = a. \sec. B$ | | B | $\tan. B = \frac{b}{a}$ |
| $b$, B | $a$ | $a = b. \cot. B$ | | | |
| | $c$ | $c = b. \operatorname{cosec}. B$ | | | |

*Solution of the cases of oblique angled triangles.*

Let $a$, $b$, $c$ be the sides of the Δ; A, B, C the ∠'s opposite to them.

*Case* 1.

Given two sides and an ∠ opposite to one of them; or two ∠'s and a side; to find the rest.

*Solution.*—The sides are proportional to the sines of the opposite ∠'s.

*Note.* When two sides and an ∠ opposite to one of them are given, the case is sometimes ambiguous, viz. when the side adjacent is greater than the side opposite to the given ∠, and that ∠ is acute. But in practical cases there will be found some circumstance or other to remove the ambiguity.

*Case* 2.

Given two sides $a$ and $b$, and included ∠ C.

Solution 1st. $\text{Tan.} \frac{A - B}{2} = \frac{a - b}{a + b} \cdot \text{Tan.} \frac{A + B}{2}$. Hence A + B and A − B are known, and consequently A and B.

Solution 2d. Let $a$ be greater than $b$. Find in the tables an ∠ $\theta$ such that $\tan. \theta = r. \frac{a}{b}$; then $r. \tan. \frac{A - B}{2} = \tan. \frac{A + B}{2}. \tan. (\theta - 45^{\circ})$.

The latter method is the most concise in those cases in which the logs. of $a$ and $b$ are given.

*Case 3d.*

Given $a, b, c$ to find A, B, C.

Solution 1st. Let $S = \frac{a + b + c}{2}$; then $\left(\sin. \frac{A}{2}\right)^2 = r^2 \times \frac{(S - b).\ (S - c)}{c\,b}$.

Solution 2d. $\left(\text{Cos.} \frac{A}{2}\right)^2 = r^2 \times \frac{S.\ (S - a)}{b\,c}$.

Solution 3d. $\left(\text{Tan.} \frac{A}{2}\right)^2 = r^2 \times \frac{(S - b).\ (S - c)}{S.\ (S - a)}$.

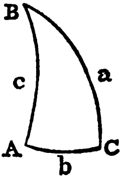
If the ∠ sought be less than 90° use 1st method.

If ——————— greater than 90° use 2d method.

The third method may be used in all cases, except when the ∠ sought is nearly 180°. When the ∠ sought is very small, and great accuracy is required, a peculiar computation is necessary.

## II. Spherical Trigonometry.

*Solution of the six cases of right angled spherical triangles.*



| *Given.* | *Soug.* | *Values of the terms required.* | *Cases where the terms required are less than* $90^{\circ}$. |
|---|---|---|---|
| $c$, $b$ | B | Sin. $B = \frac{\sin. b}{\sin. c}$ | If $b$ be less than $90^{\circ}$ |
| | A | Cos. $A = \frac{\tan. b}{\tan. c}$ | If $c$ and $b$ are of same affection |
| | $a$ | Cos $a = \frac{\cos\ c}{\cos. b}$ | Do. |
| $b$, B | $c$ | Sin. $c = \frac{\sin\ b}{\sin. B}$ | Ambiguous |
| | $a$ | Sin. $a = \frac{\tan. b}{\tan. B}$ | Do. |
| | A | Sin. $A = \frac{\cos. B}{\cos. b}$ | Do. |
| $b$, A | $c$ | Tan. $c = \frac{\tan\ b}{\cos. A}$ | If $b$ and A are of same affection |
| | B | Cos. $B = \cos. b \times \sin. A$ | If $b$ be less $90^{\circ}$. |
| | $a$ | Tan. $a = \sin. b \times \tan. A$ | If A be less $90^{\circ}$. |
| $c$, A | $b$ | Tan. $b = \tan. c \times \cos. A$ | If $c$ and A be of same affection |
| | $a$ | Sin. $a = \sin. c \times \sin. A$ | If A be acute |
| | B | Tan. $B = \frac{\cot. A}{\cos. c}$ | If $c$ and A be of same affection |
| $a$, $b$ | $c$ | Cos. $c = \cos. a \times \cos. b$ | If $a$ and $b$ are of same affection |
| | A | Tan. $A = \frac{\tan. a}{\sin. b}$ | If $a$ be less than $90^{\circ}$. |
| A, B | $c$ | Cos. $c = \cot. A \times \cot. B$ | If A and B are of same affection |
| | $a$ | Cos, $a = \frac{\cos. A}{\sin. B}$ | If A be acute |

If the Δ, instead of being right angled, is a quadrantal Δ, the surest, and perhaps the most expeditious method is to take the supplemental or polar Δ, and solve it by the above table, taking the supplements of the given sides for the ∠'s of the polar Δ, and the supplements of the given ∠'s for the sides.

## *Solution of the six cases of oblique ∠d spherical Δs.*

### *Case* 1.

Given the three sides $a$, $b$, $c$ to find A.

Solution 1st. $\left(\text{Cos.}\,\frac{\text{A}}{2}\right)^2 = r^2 \times \frac{\text{sin. S. sin. (S} - a)}{\text{sin. } b\text{. sin. } c}$.

Solution 2d. $\left(\text{Sin.}\,\frac{\text{A}}{2}\right)^2 = r^2 \times \frac{\text{sin. (S} - b)\text{. sin. (S} - c)}{\text{sin. } b\text{. sin. } c}$.

Solution 3d. $\left(\text{Tan.}\,\frac{\text{A}}{2}\right)^2 = r \times \frac{\text{sin. (S} - b)\text{. sin. (S} - c)}{\text{sin. S. sin. (S} - a)}$.

Sometimes one of these methods may be more convenient than another, see corresponding case in Plane Trigonometry.

### *Case* 2.

Given A, B, C to find $a$, &c.

Solution 1st. Let $\text{S}' = \frac{\text{A} + \text{B} + \text{C}}{2}$, then

$$\left(\text{Sin.}\,\frac{a}{2}\right)^2 = r^2 \times \frac{-\text{cos. S}'\text{. cos. (S}' - \text{A)}}{\text{sin. B. sin. C}}.$$

Solution 2d. $\left(\text{Cos.}\,\frac{a}{2}\right)^2 = r^2 \times \frac{\text{cos. (S}' - \text{B.) cos. (S}' - \text{C)}}{\text{sin. B. sin. C}}$.

Solution 3d. $\left(\text{Tan.}\,\frac{a}{2}\right)^2 = r^2 \times \frac{-\text{cos. S}'\text{. cos. (S}' - \text{A)}}{\text{cos. (S}' - \text{B.) cos. (S}' - \text{C)}}$.

S′ is greater than 90° and less than 270°, ∴ — cos. S′ is positive, and whole quantity is positive.

### *Case* 3.

Given $a$, $b$ and included ∠ C. Required A and B.

$$\text{Solution 1st.}\quad \text{Tan.}\,\frac{\text{A}+\text{B}}{2} = \frac{\text{cos.}\,\frac{a-b}{2}}{\text{cos.}\,\frac{a+b}{2}}\text{. Cot.}\,\frac{\text{C}}{2}$$

$$\text{and Tan.}\,\frac{\text{A}-\text{B}}{2} = \frac{\text{sin.}\,\frac{a-b}{2}}{\text{sin.}\,\frac{a+b}{2}}\text{. Cot.}\,\frac{\text{C}}{2}$$

from whence A + B and A — B, and consequently A and B may be found, as also $c$.

Solution 2d. But if $c$ be required alone, then it may be thus determined independently of A and B.

Assume $(\tan.\ \theta)^2 = \dfrac{\sin.\ a.\ \sin.\ b.\ \text{v.}\ \sin.\ C}{\text{v.}\ \sin.\ (a-b)}$; then $2\left(\sin.\ \dfrac{c}{2}\right)^2$

$\dfrac{\text{v.}\ \sin.\ (a-b.)\ (\sec.\ \theta)^2}{r}$.

*Case* 4.

Given A, B, and included side $c$. Required $a$, $b$ and C.

$$\text{Solution 1st.}\quad \text{Tan.}\ \frac{a+b}{2} = \frac{\cos.\ \dfrac{A-B}{2}}{\cos.\ \dfrac{A+B}{2}} \times \tan.\ \frac{c}{2}.$$

$$\text{and Tan.}\ \frac{a-b}{2} = \frac{\sin.\ \dfrac{A-B}{2}}{\sin.\ \dfrac{A+B}{2}} \times \tan.\ \frac{c}{2}.$$

From whence $a+b$ and $a-b$ and ∴ $a$ and $b$ may be found.

Solution 2d. Or C may be determined independently of $a$ and $b$ thus

Assume $(\tan.\ \theta)^2 = \dfrac{\text{ver.}\ \sin.\ c.\ \sin.\ A.\ \sin.\ B}{\left(\cos.\ \dfrac{A+B}{2}\right)^2}$, then $\left(\sin.\ \dfrac{C}{2}\right)^2 =$

$\dfrac{\left(\cos.\ \dfrac{A+B}{2}\right)^2.\ (\sec.\ \theta)^2}{r^2}$.

*Case* 5.

Given $a$, $b$ and A opposite to $a$; to find the rest.

To find B, $\sin.\ B = \dfrac{\sin.\ A.\ \sin.\ b}{\sin.\ a}$.

To find C, $\cot.\ \dfrac{C}{2} = \tan.\ \tfrac{1}{2}\,(A+B).\ \dfrac{\cos.\ \frac{1}{2}\,(a+b)}{\cos.\ \frac{1}{2}\,(a-b)}$.

To find $c$, $\sin.\ c = \sin.\ a.\ \dfrac{\sin.\ C}{\sin.\ A}$.

*Case* 6.

Given A, B and $a$ opposite to A.

$\text{Sin.}\ b = \sin.\ a.\ \dfrac{\sin.\ B}{\sin.\ A}$; then C and $c$ as in the last case.

**To find the area of a spherical Δ.**

Let A, B, C be the three angles, then

Area $= A + B + C - 180^\circ$. or, if $r$ = radius of the sphere, area $= r \times (A + B + C - 180^\circ)$.

### III. TRIGONOMETRICAL FORMULÆ.

1. If $s$ = sin. and $c$ = cos. of an arc A; the arcs, of which $s$ is the sine, are comprehended within the two formulæ.

$$2 n \pi + A, \text{ and } (2 n + 1) \pi - A, \text{ where } \pi = 180^\circ.$$

Do., of which $-s$ is the sine, are

$$(2 n + 1) \pi + A, \text{ and } (2 n + 2) \pi - A.$$

Do., of which $c$ is the cosine, are

$$2 n \pi + A \text{ and } (2 n + 2) \pi - A.$$

Do., of which $-c$ is cosine, are

$$(2 n + 1) \pi - A \text{ and } (2 n + 1) \pi + A$$

in all which cases $n$ may be 0, 1, 2, 3, &c.

2. $$\text{Sin.} \left(\frac{\pi}{2} + A\right) = \text{sin.} \left(\frac{\pi}{2} - A\right).$$

3. $$\text{Cos. } A. = \text{sin.} \left(\frac{\pi}{2} - A\right) = \text{sin.} \left(\frac{\pi}{2} + A\right).$$

4. $$\text{Sin. } A. = \text{cos.} \left(\frac{\pi}{2} - A\right) = -\text{cos.} \left(\frac{\pi}{2} + A\right).$$

$$= \frac{\text{cos. } A}{\text{cot. } A} = \sqrt{1 - \text{cos.}^2 A} = \frac{1}{\text{cosec. } A}$$

$$= \text{cos. } A. \text{ tan. } A = \frac{1}{\sqrt{1 + \text{cot.}^2 A}}$$

$$= \frac{\text{tan. } A}{\sqrt{1 + \text{tan.}^2 A}} = 2 \text{ sin. } \tfrac{1}{2} A. \; 2 \text{ cos. } \tfrac{1}{2} A.$$

$$= \sqrt{\frac{1 - \text{cos. } 2 A}{2}} = \frac{2 \text{ tan. } \frac{1}{2} A}{1 + \text{tan.}^2 \frac{1}{2} A}$$

$$= \frac{2}{\text{cot. } \frac{1}{2} A + \text{tan. } \frac{1}{2} A} = \frac{1}{\text{cot. } A + \text{tan. } \frac{1}{2} A}.$$

5. $$\text{Cos. } A = \frac{\text{sin. } A}{\text{tan. } A} = \text{sin. } A. \text{ cot. } A. = \frac{1}{\text{sec. } A}$$

$$= \sqrt{1 - \sin.^2 A} = \frac{1}{\sqrt{1 + \tan.^2 A}}.$$

$$= \frac{\cot. A}{\sqrt{1 + \cot.^2 A}} = \cos.^2 \tfrac{1}{2} A - \sin.^2 \tfrac{1}{2} A.$$

$$= 1 - 2 \sin.^2 \tfrac{1}{2} A = 2 \cos.^2 \tfrac{1}{2} A - 1$$

$$= \sqrt{\frac{1 + \cos. 2 A}{2}} = \frac{1 - \tan.^2 \frac{1}{2} A}{1 + \tan.^2 \frac{1}{2} A}$$

$$= \frac{\cot. \frac{1}{2} A - \tan. \frac{1}{2} A}{\cot. \frac{1}{2} A + \tan. \frac{1}{2} A} = \frac{1}{1 + \tan. A . \tan. \frac{1}{2} A}.$$

6. $$\text{Tan. } A = \frac{\sin. A}{\cos. A} = \frac{1}{\cot. A}$$

$$= \sqrt{\frac{1}{\cos.^2 A} - 1} = \frac{\sin. A}{\sqrt{1 - \sin.^2 A}}$$

$$= \frac{\sqrt{1 - \cos.^2 A}}{\cos. A} = \frac{2 \tan. \frac{1}{2} A}{1 - \tan.^2 \frac{1}{2} A}.$$

$$= \frac{2 \cot. \frac{1}{2} A}{\cot.^2 \frac{1}{2} A - 1} = \frac{2}{\cot. \frac{1}{2} A - \tan \frac{1}{2} A}$$

$$= \cot. A - 2 \cot. 2 A = \frac{1 - \cos. 2 A}{\sin. 2 A}$$

$$= \frac{\sin. 2 A}{1 + \cos. 2 A} = \sqrt{\frac{1 - \cos. 2 A}{1 + \cos. 2 A}}.$$

*Formulæ relating to two arcs.*

1. $\text{Sin. } (A + B) = \sin. A . \cos. B + \cos. A . \sin. B.$
2. $\text{Sin. } (A - B) = \sin. A . \cos. B - \cos. A . \sin. B.$
3. $\text{Cos. } (A + B) = \cos. A . \cos. B - \sin. A . \sin. B.$
4. $\text{Cos. } (A - B) = \cos. A . \cos. B + \sin. A . \sin. B.$
5. $\text{Tan. } (A + B) = \dfrac{\tan. A + \tan. B}{1 - \tan. A . \tan. B}.$
6. $\text{Tan. } (A - B) = \dfrac{\tan. A - \tan. B}{1 + \tan. A . \tan. B}.$
7. $\dfrac{\text{Sin. } (A + B)}{\text{Sin. } (A - B)} = \dfrac{\tan. A + \tan. B}{\tan. A - \tan. B} = \dfrac{\cot. B + \cot. A}{\cot. B - \cot. A}.$

8. $$\frac{\text{Cos. }(A+B)}{\text{Cos. }(A-B)} = \frac{\text{cot. }B - \text{tan. }A}{\text{cot. }B + \text{tan. }A} = \frac{\text{cot. }A - \text{tan. }B}{\text{cot. }A + \text{tan. }B}.$$

9. $$\frac{\text{Sin. }A + \text{sin. }B}{\text{Sin. }A - \text{sin. }B} = \frac{\text{tan. }\frac{1}{2}(A+B)}{\text{tan. }\frac{1}{2}(A-B)}.$$

10. $$\frac{\text{Cos. }B + \text{cos. }A}{\text{Cos. }B - \text{cos. }A} = \frac{\text{cot. }\frac{1}{2}(A+B)}{\text{tan. }\frac{1}{2}(A-B)}.$$

11. $\text{Sin. }A \;.\; \text{cos. }B = \frac{1}{2}\text{ sin. }(A+B) + \frac{1}{2}\text{ sin. }(A-B).$

12. $\text{Cos. }A \;.\; \text{sin. }B = \frac{1}{2}\text{ sin. }(A+B) - \frac{1}{2}\text{ sin. }(A-B).$

13. $\text{Sin. }A \;.\; \text{sin. }B = \frac{1}{2}\text{ cos. }(A-B) - \frac{1}{2}\text{ cos. }(A+B).$

14. $\text{Cos. }A \;.\; \text{cos. }B = \frac{1}{2}\text{ cos. }(A+B) + \frac{1}{2}\text{ cos. }(A-B).$

15. $\text{Sin. }A + \text{sin. }B = 2\text{ sin. }\frac{1}{2}(A+B)\text{ cos. }\frac{1}{2}(A-B).$

16. $\text{Cos. }A + \text{cos. }B = 2\text{ cos. }\frac{1}{2}(A+B)\text{ cos. }\frac{1}{2}(A-B).$

17. $$\text{Tan. }A + \text{tan. }B = \frac{\text{sin. }(A+B)}{\text{cos. }A \,.\, \text{cos. }B}.$$

18. $$\text{Cot. }A + \text{cot. }B = \frac{\text{sin. }(A+B)}{\text{sin. }A \,.\, \text{sin. }B}.$$

19. $\text{Sin. }A - \text{sin. }B = 2\text{ sin. }\frac{1}{2}(A-B) \,.\, \text{cos. }\frac{1}{2}(A+B).$

20. $\text{Cos. }B - \text{cos. }A = 2\text{ sin. }\frac{1}{2}(A-B) \,.\, \text{sin. }\frac{1}{2}(A+B).$

21. $$\text{Tan. }A - \text{tan. }B = \frac{\text{sin. }(A-B)}{\text{cos. }A \,.\, \text{cos. }B}.$$

22. $$\text{Cot. }B - \text{cot. }A = \frac{\text{sin. }(A-B)}{\text{sin. }A \,.\, \text{sin. }B}.$$

23. $$\left.\begin{array}{l}\text{Sin.}^2 A - \text{sin.}^2 B = \\ \text{Cos.}^2 B - \text{cos.}^2 A =\end{array}\right\} \text{sin. }(A-B) \,.\, \text{sin. }(A+B).$$

24. $\text{Cos.}^2 A - \text{sin.}^2 B = \text{cos. }(A-B) \,.\, \text{cos. }(A+B).$

25. $$\text{Tan.}^2 A - \text{tan.}^2 B = \frac{\text{sin. }(A-B) \,.\, \text{sin. }(A+B)}{\text{cos.}^2 A \,.\, \text{cos.}^2 B}.$$

26. $$\text{Cot.}^2 B - \text{cot.}^2 A = \frac{\text{sin. }(A-B) \,.\, \text{sin. }(A+B)}{\text{sin.}^2 A \,.\, \text{sin.}^2 B}.$$

27. $\text{Sin. }B = \text{sin. }(A+B) \,.\, \text{cos. }A - \text{sin. }A \,.\, \text{cos. }(A+B).$

28. $\text{Cos. }B = \text{sin. }(A+B)\text{ sin. }A + \text{cos. }A \,.\, \text{sin. }(A+B).$

*Note.*—To express the formulæ to rad. *r*, multiply each term by that power of *r* that will make each term of the same dimensions as that term which has the highest dimensions.

*Expressions for the sines and cosines of multiple arcs.*

1. $\text{Cos.}\ (n+1)\,A = 2\cos.\ n\,A.\ \cos.\ A - \cos.\ (n-1)\,A.$

2. $2\,\text{Cos.}\ m\,A = (2\cos.\ A)^{m} - m\,(2\cos.\ A)^{m-2} + m.\frac{m-3}{2}\,(2\cos.\ A)^{m-4} - \frac{m.\,(m-4).\,(m-5)}{2.\,3}\,(2\cos.\ A)^{m-6} + \&c.$

3. $\text{Sin.}\ (n+1)\,A = 2\sin.\ n\,A.\ \cos.\ A - \sin.\ (n-1)\,A.$

4. $\text{Sin.}\ m\,A = m\sin.\ A - \frac{m.\,(m^2-1)}{2.\,3}\,(\sin.\ A)^3 + \frac{m.\,(m^2-1.)\,(m^2-9)}{2.\,3.\,4.\,5}\,(\sin.\ A)^5\ \&c.\ (m,\ \text{odd.})$

5. $\text{Sin.}\ m\,A = \cos.\ A\left(m.\ \sin.\ A - \frac{m.\,(m^2-4)}{2.\,3}\,(\sin.\ A)^3 + \frac{m.\,(m^2-4)\,(m^2-16)}{2.\,3.\,4.\,5}\,(\sin.\ A)^5\ \&c.\right)\ (m\ \text{even.})$

6. Let $2\cos.\ A = x + \frac{1}{x}$ then $2\cos.\ n\,A = x^n + \frac{1}{x^n}$ ($n$ any No.)

7. $(\text{Cos.}\ A + \sqrt{-1}\ \sin.\ A)^m = \cos.\ m\,A + \sqrt{-1}\ \sin.\ m\,A.$
   and $(\cos.\ A - \sqrt{-1}\ \sin.\ A)^m = \cos.\ m\,A - \sqrt{-1}\ \sin.\ m\,A.$
   whence we have in another form

8. $\text{Cos.}\ m\,A = (\cos.\ A)^m - \frac{m.\,(m-1)}{2}\,(\cos.\ A)^{m-2}\,(\sin.\ A)^2 + \frac{m.\,(m-1)\,(m-2)\,(m-3)}{2.\,3.\,4}\,(\cos.\ A)^{m-4}\,(\sin.\ A)^4 - \&c.$

and $\sin.\ m\,A = m\,(\cos.\ A)^{m-1}\sin.\ A - \frac{m.\,(m-1)\,(m-2)}{2.\,3}(\cos.\ A)^{m-3}\,(\sin.\ A)^3\ \&c.$

9. Also if $e$ = No. whose hyp. log. = 1 we have in terms of the impossible quantity $\sqrt{-1}$

$$\text{Cos.}\ n\,A = \frac{e^{n\,A\,\sqrt{-1}} + e^{-n\,A\,\sqrt{-1}}}{2},\ \&\sin.\ n\,A = \frac{e^{n\,A\,\sqrt{-1}} - e^{-n\,A\,\sqrt{-1}}}{2\sqrt{-1}}$$

*Expressions for the powers of the sine and cosine of an arc.*

1. $2^{n-1}\,(\cos.\ A)^n = \cos.\ n\,A + n.\ \cos.\ (n-2)\,A + n.\ \frac{n-1}{2}\ \cos.\ (n-4)\,A + \&c.$

*Note.* If $n$ be even the last term must always be $\frac{1.3.5.7\ldots n-1.}{1.2.3\ldots\frac{n}{2}}\,2^{\frac{n}{2}-1}$

*Exs.* $2\,(\cos.\,A)^2 = \cos.\,2\,A + 1.$

$2^2\,(\cos.\,A)^3 = \cos.\,3\,A + 3\cos.\,A.$

$2^3\,(\cos.\,A)^4 = \cos.\,4\,A + 4\cos.\,2\,A + 3.$

$2^4\,(\cos.\,A)^5 = \cos.\,5\,A + 5\cos.\,3\,A + 10\cos.\,A.$

$2^5\,(\cos.\,A)^6 = \cos.\,6\,A + 6\cos.\,4\,A + 15\cos.\,2\,A + 10.$

&c. &c.

2. $2^{n-1}\,(\sin.\,A)^n = \pm\cos.\,n\,A \mp \cos.\,(n-2).\,A \pm n.\frac{n-1}{2}\cos.$ $(n-4)\,A$ &c. where the upper sign must be used when $n$ is 4, 8, 12, &c. and the lower when $n$ is 2, 6, 10, &c., and in both cases the last term is as before.

3. $2^{n-1}\,(\sin.\,A)^n = \pm\sin.\,n\,A \mp \sin.\,(n-2)\,A \pm n.\frac{n-1}{2}\sin.$ $(n-4)\,A$, &c., where the upper sign must be used when $n$ is 1, 5, 9, &c., and the lower when $n$ is 3, 7, 11, &c.

*Exs.* $2\,(\sin.\,A)^2 = -\cos.\,2\,A + 1.$

$2^2\,(\sin.\,A)^3 = -\sin.\,3\,A + 3\sin.\,A.$

$2^3\,(\sin.\,A)^4 = \cos.\,4\,A - 4\cos.\,2\,A + 3.$

$2^4\,(\sin.\,A)^5 = \sin.\,5\,A - 5\sin.\,3\,A + 10\sin.\,A.$

$2^5\,(\sin.\,A)^6 = -\cos.\,6\,A + 6\cos.\,4\,A - 15\cos.\,2\,A + 10.$

*Series for the sine and cosine in terms of the arc.*

1. $\text{Sin.}\,x = x - \frac{x^3}{2.3} + \frac{x^5}{2.3.4.5} + \&c.$

2. $\text{Cos.}\,x = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \&c.$

*Value of the sine in some of the most simple cases.*

$\text{Sin.}\,0^\circ = 0.$

$\text{Sin.}\,9^\circ = \frac{1}{4}\sqrt{3+\sqrt{5}} - \frac{1}{4}\sqrt{5-\sqrt{5}}.$

$\text{Sin.}\,15^\circ = \frac{1}{2}\sqrt{1+\frac{1}{2}} - \frac{1}{2}\sqrt{1-\frac{1}{2}}.$

$\text{Sin.}\,18^\circ = \frac{1}{4}.\left(\sqrt{5}-1\right).$

$\text{Sin.}\,27^\circ = \frac{1}{4}\sqrt{5+\sqrt{5}} - \frac{1}{4}\sqrt{3-\sqrt{5}}.$

$\text{Sin.}\,30^\circ = \frac{1}{2}.$

$$\text{Sin. } 36^\circ = \tfrac{1}{4}\sqrt{10 - 2\sqrt{5}}.$$

$$\text{Sin. } 45^\circ = \frac{1}{\sqrt{2}}.$$

$$\text{Sin. } 54^\circ = \tfrac{1}{4}\left(\sqrt{5} + 1\right).$$

$$\text{Sin. } 60^\circ = \frac{\sqrt{3}}{2}.$$

$$\text{Sin. } 63^\circ = \tfrac{1}{4}\sqrt{5 + \sqrt{5}} + \tfrac{1}{4}\sqrt{3 - \sqrt{5}}.$$

$$\text{Sin. } 72^\circ = \tfrac{1}{4}\sqrt{10 + 2\sqrt{5}}.$$

$$\text{Sin. } 81^\circ = \tfrac{1}{4}\sqrt{3 + \sqrt{5}} + \tfrac{1}{4}\sqrt{5 - \sqrt{5}}.$$

$$\text{Sin. } 90^\circ = 1.$$

TWILIGHT.—*See Refraction.*

# U, V.

VARIATION *and dip of the Magnetic Needle.*

TABLE I.

*Shewing the variation of the Needle in various parts of the earth, from Professor Hansteen, of Christiania.*

| *Authority.* | *Date.* | *Variation.* | *Latitude.* | *Longitude* | *Place.* |
|---|---|---|---|---|---|
| Luchtemacher | 1649 | 1° 30′ E | 55 41′ | 12 35 E | Copenhagen. |
| Bartholin ...... | 1672 | 3 35 W | | | |
| Lous, sen. ...... | 1730 | 10 37 | | | |
| Lous, jun. ...... | 1765 | 15 5 | | | |
| Do. ............... | 1779 | 17 5 | | | |
| Bugge ............ | 1784 | 18 0 | | | |
| Wleugel ........ | 1817 | 18 5 | | | |
| Elvius ......... | 1718 | 5 37 W | 59 20 | 18 4 E | Stockholm. |
| Wilcke .......... | 1763 | 11 48 | | | |
| Do. ............... | 1771 | 13 4 | | | |
| Do. ............... | 1786 | 15 34 | | | |
| Cronstrand .... | 1817 | 15 36 | | | |
| Holm ............ | 1761 | 13 50 W | 63 26 | 10 22 E | Drontheim. |
| Berlin ............ | 1779 | 18 0 | | | |
| Vibe .............. | 1786 | 19 0 | | | |

| *Authority.* | *Date.* | *Variation.* | *Latitude.* | *Longitude* | *Place.* |
|---|---|---|---|---|---|
| | | ° ′ | ° ′ | | |
| Holm | 1761 | 15 15 W | 59 55 | 10 42 E | Christiania. |
| Hansteen | 1817 | 20 3 | | | |
| Billings | 1735 | 1 5 W | 52 17 | 104 11 E | Irkutsk. |
| Schubert | 1805 | 0 32 E | | | |
| Mayer | 1726 | 3 15 W | 59 56 | 30 19 E | Petersburgh. |
| Krafft | 1774 | 4 50 | | | |
| Henry | 1805 | 11 0 | | | |
| Do. | 1812 | 7 16 | | | |
| Cook | 1779 | 6 19 E | 53 1 | 158 48 | Kamtschatka. |
| Krusenstern | 1805 | 5 20 | | | |
| Kirch | 1717 | 10 42 W | 52 32 | 13 21 E | Berlin. |
| Do. | 1751 | 14 16 | | | |
| Bernouilli | 1770 | 16 9 | | | |
| Schulze | 1785 | 18 3 | | | |
| Bode | 1805 | 18 2 | | | |
| V. Swinden | 1797 | 19 40 W | 46 12 | 6 9 E | Geneva. |
| | 1804 | 21 13 | | | |
| Bellarmatus | 1541 | 7 0 E | 48 50 | 2 20 E | Paris. |
| Picard | 1666 | 0 0 | | | |
| Cassini | 1687 | 5 12 W | | | |
| La Hire | 1707 | 10 10 | | | |
| Maraldi | 1720 | 13 0 | | | |
| Do. | 1740 | 15 30 | | | |
| Do. | 1760 | 18 30 | | | |
| Le Monnier | 1780 | 20 35 | | | |
| Cotte | 1800 | 22 12 | | | |
| Bouvard | 1814 | 22 34 | | | |
| Kendrick | 1745 | 18 0 W | 53 21 | 353 41 E | Dublin. |
| Harding | 1791 | 27 23 | | | |
| Burrows | 1580 | 11 15 E | 51 31 | 00 00 | London. |
| Gunter | 1622 | 5 56¼ | | | |
| Gellibrand | 1634 | 4 6 | | | |
| Bond | 1657 | 0 0 W | | | |
| Gellibrand | 1665 | 1 22½ | | | |
| Halley | 1672 | 2 30 | | | |
| Do. | 1692 | 6 0 | | | |
| Graham | 1723 | 14 17 | | | |
| Do. | 1745 | 17 0 | | | |
| Do. | 1745 | 17 0 | | | |
| Do. | 1746 | 17 10 | | | |
| Do. | Ma. 21 | 17 10 | | | |
| Do. | Ap. 22 | 17 15 | | | |
| Do. | May 4 | 17 18 | | | |
| Do. | 14 | 17 20 | | | |
| Do. | 16 | 17 15 | | | |
| Do. | De. 18 | 17 25 | | | |
| Do. | 1747 | 17 30 | | | |
| Do. | 1747 | 17 40 | | | |
| Do. | 1748 | 17 40 | | | |
| Heberden | 1773 | 21 9 | | | |
| Cavendish | 1774 | 21 16 | | | |

| *Authority.* | *Date.* | *Variation.* | *Latitude.* | *Longitude* | *Place.* |
|---|---|---|---|---|---|
| | | ° ′ | ° ′ | | |
| Cavendish ...... | 1775 | 21 43 | 51 31 | 00 00 | London. |
| Gilpin ........... | 1786 | 23 17 | | | |
| Do. .............. | 1787 | 23 19 | | | |
| Do. .............. | 1788 | 23 32 | | | |
| Do. .............. | 1789 | 23 19 | | | |
| Do. .............. | 1790 | 23 39 | | | |
| Do. .............. | 1791 | 23 36 | | | |
| Do. .............. | 1792 | 23 36 | | | |
| Do. .............. | 1793 | 23 49 | | | |
| Do. .............. | 1794 | 23 56 | | | |
| Do. .............. | 1795 | 23 57 | | | |
| Do. .............. | 1796 | 24 0 | | | |
| Do. .............. | 1797 | 24 1 | | | |
| Do. .............. | 1798 | 24 0·6 | | | |
| Do. .............. | 1799 | 24 1·8 | | | |
| Do. .............. | 1800 | 24 3·6 | | | |
| Do. .............. | 1801 | 24 4·2 | | | |
| Do. .............. | 1802 | 24 6·7 | | | |
| Do. .............. | 1803 | 24 8·8 | | | |
| Do. .............. | 1804 | 24 8·4 | | | |
| Do. .............. | 1805 | 24 8·8 | | | |
| Do. .............. | 1809 | 24 11·0 | | | |
| Do. .............. | 1814 | 24 16·7 | | | |
| Do. .............. | Jul. | 24 17·9 | | | |
| Do. .............. | Au. | 24 21·2 | | | |
| Do. .............. | Sep. | 24 20·5 | | | |
| Martinius ...... | 1638 | 7 39 E | 38 42 | 350 51 | Lisbon. |
| Do. .............. | 1668 | 0 50 W | | | |
| Ross .............. | 1762 | 17 32 | | | |
| Lowenorn ...... | 1782 | 19 51 | | | |
| Auzout .......... | 1670 | 2 15 W | 41 54 | 12 28 | Rome. |
| | 1788 | 17 12 | | | |
| Mathews ....... | 1721 | 5 12 W | 19 0 | 71 45 | Bombay. |
| Yeates ........... | 1817 | 0 0 | | | |
| Fontenay ...... | 1690 | 2 25 W | 22 13 | 113 35 | Canton. |
| Yeates ............ | 1817 | 0 0 | | | |
| Mathews ....... | 1722 | 2 52 W | 13 15 | 79 57 | Madras. |
| Yeates ............ | 1817 | 1 0 E | | | |
| Wallis ............ | 1766 | 14 10 W | 32 36 | 342 57 | Madeira. |
| Mudge ............ | 1820 | 19 59 | | | |
| Fleurieu ...... | 1769 | 15 43 W | 28 27 | 343 45 | Teneriffe. |
| Bligh ............. | 1788 | 20 1 | | | |
| Krusenstern ... | 1803 | 16 1 | | | |
| Keeling ......... | 1609 | 21 0 W | 20 10 | 57 28 | Isle of France. |
| Yeates ............ | 1817 | 15 0 | | | |
| Daunton ........ | 1614 | 1 45 W | 33 55 | 18 24 | Table Bay. |
| Caille ............ | 1752 | 19 0 | | | |
| Bonsoe ........... | 1804 | 25 4 | | | |
| Davis ............ | 1610 | 7 13 E | 15 55 | 354 12 | St Helena. |
| Halley ............ | 1677 | 0 40 | | | |
| Wallis ............ | 1768 | 12 47 W | | | |
| Krusenstern ... | 1806 | 17 18 | | | |

| *Authority.* | *Date.* | *Variation.* | *Latitude.* | *Longitude* | *Place.* |
|---|---|---|---|---|---|
| Mathews ........ | 1723 | 12 20 W | 12 47 | 56 0 | Socotra Isl. |
| Yeates ............ | 1817 | 6 0 | | | |
| Mathews ........ | 1726 | 4 31 E | 17 6 | 283 14¼ | P.R., Jamaica |
| Yeates ............ | 1817 | 6 0 | | | |
| Vancouver ..... | 1795 | 14 49 E | 33 0 | 287 46 | Valparaiso. |
| Basil Hall ..... | 1821 | 14 43 | | | |
| Cook .............. | 1774 | 4 30 E | 27 6 | 250 14 | Easter Island. |
| La Perouse .... | 1786 | 3 10 | | | |
| Cook .............. | 1779 | 8 6 E | 19 28 | 204 0 | Owhyee. |
| Broughton ...... | 1796 | 8 15 | | | |
| Tasman .......... | 1643 | 7 15 E | 21 9 | 184 55 | Tongataboo. |
| Cook .............. | 1777 | 9 44 | | | |
| Oxley ............ | 1817 | 7 47 E | 33 40 | 148 21 | New Holland. |

## TABLE II.

*Shewing the dip of the Needle in various parts of the earth.—(Hansteen.)*

| *Authority.* | *Date.* | *Dip.* | *Latitude.* | *Longitude* | *Place.* |
|---|---|---|---|---|---|
| | | ° ′ | | | |
| Lous .............. | 1773 | 71 45 N | ——— | ——— | Copenhagen. |
| Wlengal ....... | 1813 | 71 26 | | | |
| Schubert ....... | 1805 | 67 0 N | ——— | ——— | Irkutsk. |
| Euler ............ | 1755 | 73 30 N | ——— | ——— | Petersburgh. |
| Kraft ............ | 1802 | 76 42 | | | |
| | 1755 | 71 45 N | ——— | ——— | Berlin. |
| Euler ............ | 1769 | 72 45 | | | |
| Humboldt ...... | 1805 | 69 53 | | | |
| Richer .......... | 1671 | 75 0 N | ——— | ——— | Paris. |
| La Caille ....... | 1754 | 72 15 | | | |
| Cassini .......... | 1791 | 70 52 | | | |
| Humboldt ...... | 1806 | 69 12 | | | |
| Conn. de tems. | 1814 | 68 36 | | | |
| Norman ......... | 1576 | 71 50 N | ——— | ——— | London. |
| Gilbert ......... | 1600 | 72 0 N | | | |
| Ridley .......... | 1613 | 72 30 N | | | |
| Bond ............. | 1676 | 73 30 N | | | |
| Whiston ........ | 1720 | 73 45<br>75 10 | | | |
| Graham ......... | 1723 | 74 42<br>74 42 | | | |
| Nairne ........... | 1772 | 72 19 | | | |
| Cavendish ...... | 1775 | 72 31 | | | |
| Gilpin ............ | 1786 | 72 8·1 | | | |
| Do. ............... | 1787 | 72 2·5 | | | |
| Do. ............... | 1788 | 72 4·0 | | | |
| Do. ............... | 1789 | 71 54·8 | | | |
| Do. ............... | 1790 | 71 53·7 | | | |
| Do. ............... | 1791 | 71 23·7 | | | |

| *Authority.* | *Date.* | *Dip.* | *Latitude.* | *Longitude* | *Place.* |
|---|---|---|---|---|---|
| | | ° ′ | | | |
| Gilpin | 1795 | 71 11·4 | — | — | London. |
| Do. | 1797 | 70 59·4 | | | |
| Do. | 1798 | 70 55·4 | | | |
| Do. | 1799 | 70 52·2 | | | |
| Do. | 1801 | 70 35·6 | | | |
| Do. | 1803 | 70 32·0 | | | |
| Do. | 1805 | 70 21·0 | | | |
| Sabine | 1821 | 70 3·2 | | | |
| Kircher | 1640 | 65 40 N | — | — | Rome. |
| Humboldt | 1806 | 63 48 | | | |
| Abercrombie | 1775 | 5 15 N | — | — | Madras. |
| Mudge | 1820 | 63 47 N | — | — | Madeira. |
| La Caille | 1751 | 43 0 S | — | — | Good Hope. |
| Bayley | 1775 | 45 19 | | | |
| La Caille | 1754 | 9 0 S | — | — | St. Helena. |
| Cook | 1775 | 11 25 | | | |
| Panton | 1776 | 4 37 N | — | — | Socotra. |
| Vancouver | 1795 | 44 15 S | — | — | Valparaiso. |
| Basil Hall | 1821 | 38 46 | | | |
| Cook | 1777 | 40 51 S | — | — | Owhyee. |
| Vancouver | 1793 | 41 24 | | | |
| Cook | 1777 | 39 1 S | — | — | Tongataboo. |
| Cook | 1776 | 61 52 N | — | — | Teneriffe. |
| Mudge | 1820 | 58 22 | | | |
| Basil Hall | 1821 | 12 11¼ N | 2 13 S | 280 15 | Guayaquil. |
| Humboldt | 1805 | 61 35 N | 40 50 N | 14 16 | Naples. |
| Do. | 1799 | 13 22 N | 0 13 S | 281 15 | Quito. |
| Do. | 1805 | 64 45 N | 44 25 N | 8 58 | Genoa. |
| Do. | 1805 | 67 10 N | — | — | Lucern. |

The following recent observations on the dip and variation were selected by Mr. Barlow, as being entitled to the greatest credit:—

| *Place.* | *Date.* | *Lat.* | *Long.* | *Dip.* | *Variation* | *Authority.* |
|---|---|---|---|---|---|---|
| Tristan da Acunha | 1821 | 37 0 | 12 10 | 37 53 S | 12 0 W | Marryat. |
| Trinidad | 1820 | 20 30 | 29 0 | 10 27 S | 5 0 W | Do. |
| St. Jago | 1820 | 14 51 | 23 32 | 48 0 N | 15 55 W | Mean of Do. and Mudge's observations |
| Teneriffe | Do. | 28 28 | 16 16 | 58 22 N | 20 47 W | |
| Madeira | Do. | 32 38 | 17 51 | 63 47 N | 23 7 W | |
| Madrid | 1798 | 10 25 | 3 29 | 67 41 N | 19 59 W | Humboldt. |
| Paris | 1814 | 48 50 | 2 20 | 68 36 N | 22 34 W | Bouvard. |
| London | 1818 | 51 31 | 0 0 | 70 34 N | 24 30 W | Kater, the dip. |
| Berlin | 1806 | 52 32 | 13 21 | 69 53 N | 18 2 W | Humboldt |
| Copenhagen | 1812 | 55 41 | 12 35 | 71 26 N | 18 22 W | Wleugel. |
| Davis' Strait | 1820 | 64 0 | 61 50 | 83 43 N | 60 20 W | Parry. |
| Regent's Inlet | Do. | 72 45 | 80 41 | 88 26 N | 118 16 W | Do. |
| Baffin's Bay | Do. | 73 0 | 61 30 | 84 30 N | 82 2 W | Do. |
| Possession Bay | Do. | 73 31 | 77 22 | 86 4 N | 108 46 W | Do. |
| Melville Island | Do. | 74 47 | 110 48 | 88 43 N | 127 46 E | Do. |

**To find the area of a spherical Δ.**

Let A, B, C be the three angles, then

Area = A + B + C — 180°. or, if r = radius of the sphere, area = r × (A + B + C — 180°).

### III. Trigonometrical Formulæ.

1. If $s$ = sin. and $c$ = cos. of an arc A ; the arcs, of which $s$ is the sine, are comprehended within the two formulæ.

$$2 n \pi + A, \text{ and } (2 n + 1) \pi - A, \text{ where } \pi = 180^\circ.$$

Do., of which $-s$ is the sine, are

$$(2 n + 1) \pi + A, \text{ and } (2 n + 2) \pi - A.$$

Do., of which $c$ is the cosine, are

$$2 n \pi + A \text{ and } (2 n + 2) \pi - A.$$

Do., of which $-c$ is cosine, are

$$(2 n + 1) \pi - A \text{ and } (2 n + 1) \pi + A$$

in all which cases $n$ may be 0, 1, 2, 3, &c.

2. $\text{Sin.} \left(\frac{\pi}{2} + A\right) = \text{sin.} \left(\frac{\pi}{2} - A\right).$

3. $\text{Cos. A.} = \text{sin.} \left(\frac{\pi}{2} - A\right) = \text{sin.} \left(\frac{\pi}{2} + A\right).$

4. $\text{Sin. A.} = \text{cos.} \left(\frac{\pi}{2} - A\right) = - \text{cos.} \left(\frac{\pi}{2} + A\right).$

$$= \frac{\text{cos. A}}{\text{cot. A}} = \sqrt{1 - \text{cos.}^2 A} = \frac{1}{\text{cosec. A}}$$

$$= \text{cos. A. tan. A} = \frac{1}{\sqrt{1 + \text{cot.}^2 A}}$$

$$= \frac{\text{tan. A}}{\sqrt{1 + \text{tan.}^2 A}} = 2 \text{ sin. } \tfrac{1}{2} A. \; 2 \text{ cos. } \tfrac{1}{2} A.$$

$$= \sqrt{\frac{1 - \text{cos. } 2 A}{2}} = \frac{2 \text{ tan. } \frac{1}{2} A}{1 + \text{tan.}^2 \frac{1}{2} A}$$

$$= \frac{2}{\text{cot. } \frac{1}{2} A + \text{tan. } \frac{1}{2} A} = \frac{1}{\text{cot. } A + \text{tan. } \frac{1}{2} A}.$$

5. $\text{Cos. A} = \frac{\text{sin. A}}{\text{tan. A}} = \text{sin. A. cot. A.} = \frac{1}{\text{sec. A}}$

**VENUS.**—*See Planets, elements of.*

**VENUS,** *transit of.—See Transit.*

**VENUS,** *phases of.—(Vince.)*

In the case of Mercury, Venus, and Mars, if $\theta =$ exterior ∠ of elongation, i. e. = supplement of the ∠, which the earth and sun subtend at the planet, the visible enlightened part : the whole disc :: ver. sin. $\theta$ : diameter.

Hence Mercury and Venus will have the same phases, from their inferior to their superior conjunction, as the moon has from the new to the full; and the same from the superior to the inferior conjunction, as the moon has from the full to the new. Mars will appear gibbous in quadratures, as the ∠ $\theta$ will then differ considerably from two right ∠s; and consequently the versed sine from the diameter. For Jupiter, Saturn, and the Georgian, the ∠ $\theta$ never differs enough from two right ∠s to make them appear gibbous, so that they always appear to shine with a full face. In the case of the moon, the ∠ $\theta$ very nearly equals the ∠ of elongation; ∴ the visible enlightened part of the moon varies very nearly as the ver. sin. of its elongation.

Venus is brightest between its inferior conjunction and its greatest elongation; and its elongation at that time from the sun $= 39^\circ.\ 44'$. Also at that time the visible enlightened part : whole disc :: 0,53 : 2. Venus therefore appears a little more than one-fourth illuminated, and answers to the appearance of the moon when five days old. This situation happens about 36 days before and after its inferior conjunction.

Mercury is brightest between its greatest elongation and superior conjunction; the elongation of Mercury at this time $= 22^\circ.\ 18\frac{3}{4}'$.

**VERNIER.**

As instruments are now usually constructed, the following is a general rule for finding the value of each division on any vernier.

Find the value of each of the divisions or sub-divisions of the limb to which the vernier is applied. Divide the number of minutes or seconds thus found by the number of divisions on the vernier, and the quotient will give the value of the vernier division. Thus suppose each sub-division of the limb to be $5'$ or $300''$, and that the vernier has 20 divisions, then $\frac{300}{20} = 15'' =$ value of the vernier.

**VESTA.**—This planet was discovered by Dr. Olbers, of Bremen, March 29, 1807. For its elements, &c., *see Planets, elements of.*

8. $\frac{\text{Cos.}\ (A+B)}{\text{Cos.}\ (A-B)} = \frac{\text{cot.}\ B - \text{tan.}\ A}{\text{cot.}\ B + \text{tan.}\ A} = \frac{\text{cot.}\ A - \text{tan.}\ B}{\text{cot.}\ A + \text{tan.}\ B}.$

9. $\frac{\text{Sin.}\ A + \text{sin.}\ B}{\text{Sin.}\ A - \text{sin.}\ B} = \frac{\text{tan.}\ \frac{1}{2}\ (A+B)}{\text{tan.}\ \frac{1}{2}\ (A-B)}.$

10. $\frac{\text{Cos.}\ B + \text{cos.}\ A}{\text{Cos.}\ B - \text{cos.}\ A} = \frac{\text{cot.}\ \frac{1}{2}\ (A+B)}{\text{tan.}\ \frac{1}{2}\ (A-B)}.$

11. $\text{Sin.}\ A\ .\ \text{cos.}\ B = \frac{1}{2}\ \text{sin.}\ (A+B) + \frac{1}{2}\ \text{sin.}\ (A-B).$

12. $\text{Cos.}\ A\ .\ \text{sin.}\ B = \frac{1}{2}\ \text{sin.}\ (A+B) - \frac{1}{2}\ \text{sin.}\ (A-B).$

13. $\text{Sin.}\ A\ .\ \text{sin.}\ B = \frac{1}{2}\ \text{cos.}\ (A-B) - \frac{1}{2}\ \text{cos.}\ (A+B).$

14. $\text{Cos.}\ A\ .\ \text{cos.}\ B = \frac{1}{2}\ \text{cos.}\ (A+B) + \frac{1}{2}\ \text{cos.}\ (A-B).$

15. $\text{Sin.}\ A + \text{sin.}\ B = 2\ \text{sin.}\ \frac{1}{2}\ (A+B)\ \text{cos.}\ \frac{1}{2}\ (A-B).$

16. $\text{Cos.}\ A + \text{cos.}\ B = 2\ \text{cos.}\ \frac{1}{2}\ (A+B)\ \text{cos.}\ \frac{1}{2}\ (A-B).$

17. $\text{Tan.}\ A + \text{tan.}\ B = \frac{\text{sin.}\ (A+B)}{\text{cos.}\ A\ .\ \text{cos.} B}.$

18. $\text{Cot.}\ A + \text{cot.}\ B = \frac{\text{sin.}\ (A+B)}{\text{sin.} A\ .\ \text{sin.}\ B}.$

19. $\text{Sin.}\ A - \text{sin.}\ B = 2\ \text{sin.}\ \frac{1}{2}\ (A-B)\ .\ \text{cos.}\ \frac{1}{2}\ (A+B).$

20. $\text{Cos.}\ B - \text{cos.}\ A = 2\ \text{sin.}\ \frac{1}{2}\ (A-B)\ .\ \text{sin.}\ \frac{1}{2}\ (A+B).$

21. $\text{Tan.}\ A - \text{tan.}\ B = \frac{\text{sin.}\ (A-B)}{\text{cos.} A\ .\ \text{cos.} B}.$

22. $\text{Cot.}\ B - \text{cot.}\ A = \frac{\text{sin.}\ (A-B)}{\text{sin.} A\ .\ \text{sin.}\ B}.$

23. $\left\{ \begin{array}{l} \text{Sin.}^2\ A - \text{sin.}^2\ B = \\ \text{Cos.}^2 B - \text{cos.}^2\ A = \end{array} \right\} \text{sin.}\ (A-B)\ .\ \text{sin.}\ (A+B).$

24. $\text{Cos.}^2\ A - \text{sin.}^2\ B = \text{cos.}\ (A-B)\ .\ \text{cos.}\ (A+B).$

25. $\text{Tan.}^2\ A - \text{tan.}^2\ B = \frac{\text{sin.}\ (A-B)\ .\ \text{sin.}\ (A+B)}{\text{cos.}^2\ A\ .\ \text{cos.}^2 B}.$

26. $\text{Cot.}^2\ B - \text{cot.}^2\ A = \frac{\text{sin.}\ (A-B)\ .\ \text{sin.}\ (A+B)}{\text{sin.}^2\ A\ .\ \text{sin.}^2 B}.$

27. $\text{Sin.}\ B = \text{sin.}\ (A+B)\ .\ \text{cos.}\ A - \text{sin.}\ A\ .\ \text{cos.}\ (A+B).$

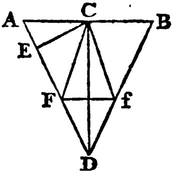
28. $\text{Cos.}\ B = \text{sin.}\ (A+B)\ \text{sin.}\ A + \text{cos.}\ A\ .\ \text{sin.}\ (A+B).$

*Note.*—To express the formulæ to rad. $r$, multiply each term by that power of $r$ that will make each term of the same dimensions as that term which has the highest dimensions.

*Cor.* 1. If the wedge be isosceles, $A\,D = D\,B$, and if resistances act at equal angles,

$$P : R + R' :: \frac{A\,B}{\text{sin. } L\,P\,M} : \frac{2\,A\,D}{\text{sin. } F\,R\,E}.$$

*Cor.* 2. If power act at right ∠s to the back, $P : R + R' :: \frac{\frac{1}{2} A\,B}{r} : \frac{A\,D}{\text{s. } F\,R\,E} :: \frac{\text{sin. } \frac{1}{2} \angle \text{ of wedge}}{r} : \frac{r}{\text{s. } F\,R\,E} ::$ sin. ½ ∠ of wedge × sin. $F\,R\,E : (\text{rad.})^2$.



*Cor.* 3. If the resistances in the last Corollary act perpendicularly on the sides of the wedge, $P : R + R' :: A\,C : A\,D$. If the directions of the resistances be perpendicular to the back, $P : R + R' :: A\,C^2 : A\,D^2$. And lastly, if they act parallel to the back, $P : R + R' :: C\,E : A\,D$.

*Cor.* 4. In the demonstration of the proposition, it has been supposed that the sides of the wedge are perfectly smooth; if, on account of the friction, the resistances C F, C *f* are wholly effective, we have

$$P : R + R' :: \text{sin. } C\,F\,f, \text{ or } F\,C\,A : \text{rad.}$$

The power applied to the wedge is usually percussion, and almost the only instance in which it is used for the purpose of equilibrium is in the construction of *arches*, built of truncated wedges.

WEIGHTS *and Measures, Tables of.*

## WEIGHTS.

### *TROY WEIGHT.*

| Grains. | | | |
|---|---|---|---|
| 24 | 1 Pennyweight. | | |
| 480 | 20 | 1 Ounce. | |
| 5760 | 240 | 12 | 1 Pound. |

By this weight, gold, silver, jewels, and precious stones are weighed. It is also used for ascertaining the strength of spirits, for experiments in Nat. Philosophy, and for comparing the different weights with each other.

Standard gold consists of 22 parts of fine gold, and 2 parts of alloy; and standard silver contains 37 parts of fine silver, and 3 of alloy.

The standard price of gold is £3. 17*s*. 10½*d*. per ounce, or £46. 14*s*. 6*d*. per pound, a pound being coined into 44½ guineas. A pound of standard silver is now coined into 66 shillings, instead of 62 shillings, as formerly.

By the Act of Parliament passed in June, 1824, all the weights remain as they were, the Act only declaring that the Imperial standard Pound Troy shall be the unit or only standard measure of weight from which all other weights shall be derived and computed; that this Troy pound is equal to the weight of 22.815 cubic inches of distilled water weighed in air at the temperature of 62° of Fahrenheit's thermometer, the barometer being at 30 inches; and that there being 5760 grains in a Troy pound, there will be 7000 such grains in a pound avoirdupoise.

### *APOTHECARIES WEIGHT.*

| Grains. | | | | |
|---|---|---|---|---|
| 20 | 1 Scruple. | | | |
| 60 | 3 | 1 Dram. | | |
| 480 | 24 | 8 | 1 Ounce. | |
| 5760 | 288 | 96 | 12 | 1 Pound. |

### *AVOIRDUPOIS WEIGHT.*

10 grs = 1 dram

| Drams. | | | | | |
|---|---|---|---|---|---|
| 16 | 1 Ounce. | | | | |
| 256 | 16 | 1 Pound. | | | |
| 7168 | 448 | 28 | 1 Quarter. | | |
| 28672 | 1792 | 112 | 4 | 1 Cwt. | |
| 573440 | 35840 | 2240 | 80 | 20 | 1 Ton. |

Avoirdupois weight is used for all coarse and heavy goods, such as butcher's meat, groceries, bread, cheese, butter, tea, &c., and all metals, except gold and silver.

The statute stone is 14lb., but it varies in different places; in London 8lb. make a stone of butcher's meat.

An avoirdupois pound : pound Troy :: 175 : 144 or :: 11 : 9 nearly; and an avoirdupois pound = 1lb. 2oz. 11dwts. 16 gr. Troy; and a Troy ounce = 1oz. 1,55dr. avoirdupois.

### *WOOL WEIGHT.*

| Pounds. | | | | | | |
|---|---|---|---|---|---|---|
| 7 | 1 Clove. | | | | | |
| 14 | 2 | 1 Stone. | | | | |
| 28 | 4 | 2 | 1 Tod. | | | |
| 182 | 26 | 13 | 6½ | 1 Wey. | | |
| 364 | 52 | 26 | 13 | 2 | 1 Sack. | |
| 4368 | 624 | 312 | 156 | 24 | 12 | 1 Last. |

In the northern counties woolstaplers allow 30lb. to the tod, and 8 tods to the pack.

### *HAY AND STRAW.*

| | |
|---|---|
| 36lb. of straw | make 1 truss. |
| 56lb. of old hay | |
| 60lb. of new hay | |
| 36 trusses | 1 load. |

| *BREAD.* | *lbs.* | *oz.* | *dr.* |
|---|---|---|---|
| A Peck Loaf weighs | 17 | 6 | 0 |
| A Half Peck | 8 | 11 | 0 |
| Quartern | 4 | 5 | 8 |

## MEASURES.

### *CLOTH MEASURE.*

| **Inch.** | | | | |
|---|---|---|---|---|
| 2¼ | 1 Nail. | | | |
| 9 | 4 | 1 Quarter of a Yard. | | |
| 36 | 16 | 4 | 1 Yard. | |
| 27 | 12 | 3 | 1 Flemish Ell. | |
| 45 | 20 | 5 | 1 English Ell. | |

### *LONG MEASURE.*

| **Barley corns.** | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 1 Inch. | | | | | |
| 36 | 12 | 1 Foot. | | | | |
| 108 | 36 | 3 | 1 Yard. | | | |
| 594 | 198 | 16½ | 5½ | 1 Pole. | | |
| 23760 | 7920 | 660 | 220 | 40 | 1 Furlong. | |
| 190080 | 63360 | 5280 | 1760 | 320 | 8 | 1 Mile. |

Also,

| | |
|---|---|
| 4 Inches | 1 Hand. |
| 1½ Feet | 1 Cubit. |
| 6 Feet | 1 Fathom. |
| 3 Miles | 1 League. |
| 60 Geographical Miles | 1 Degree. of latitude |
| 69½ English Miles | 1 Degree nearly. |

By the late Act of Parliament it is declared, that the Imperial standard yard (which is the same as the old yard) shall be the unit or only standard measure of extension, wherefrom all other measures of extension whatsoever, whether the same be lineal, superficial, or solid, shall be derived and computed; and that the Imperial standard yard, when compared with a pendulum vibrating seconds of mean time in the latitude of London, in a vacuum at the level of the sea, is in the proportion of 36 inches to 39.1393 inches.

*Note.*—The following standards, accurately measured, give these results:—

| | Inches. |
|---|---|
| General Lambton's scale, used in India | 35.99934 |
| Sir G. Shuckburgh's scale | 35.99998 |
| General Roy's scale | 36.00088 |
| Royal Society's standard | 36.00135 |
| Ramsden's bar | 36.00249 |

## *SQUARE OR LAND MEASURE.*

| Feet. | | | | |
|---|---|---|---|---|
| 9 | 1 Yard. | | | |
| 272¼ | 30¼ | 1 Pole. | | |
| 10890 | 1210 | 40 | 1 Rood. | |
| 43560 | 4840 | 160 | 4 | 1 Acre. |

For further observations on this measure—*see Surveying*.

## *WINE MEASURE.*

| Pints. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 1 Quart. | | | | | | |
| 8 | 4 | 1 Gallon. | | | | | |
| 336 | 168 | 42 | 1 Tierce. | | | | |
| 504 | 252 | 63 | 1½ | 1 Hogshead. | | | |
| 672 | 336 | 84 | 2 | 1⅓ | 1 Punch. | | |
| 1008 | 504 | 126 | 3 | 2 | 1½ | 1 Pipe. | |
| 2016 | 1008 | 252 | 6 | 4 | 3 | 2 | 1 Tun. |

This measure is used for wines, brandies, rum, honey, oil, vinegar, &c. A cask of rum, which contains from 95 to 110 gallons, is usually called a puncheon; a foreign pipe of wine varies from 110 to 140 gallons.

## *ALE AND BEER MEASURE.*

| Quarts. | | | | | | |
|---|---|---|---|---|---|---|
| 4 | 1 Gallon. | | | | | |
| 36 | 9 | 1 Firkin. | | | | |
| 72 | 18 | 2 | 1 Kilderkin. | | | |
| 144 | 36 | 4 | 2 | 1 Barrel. | | |
| 216 | 54 | 6 | 3 | 1½ | 1 Hogshead. | |
| 432 | 108 | 12 | 6 | 3 | 2 | 1 Butt. |

By the late Act the old Wine and Ale Gallons are abolished, and the Imperial standard gallon substituted in their place. This is declared to contain ten pounds avoirdupoise weight of distilled water weighed in air at the temperature of 62° of Fahrenheit, the barometer being at 30 inches. From this standard gallon all other measures of capacity, as well for wine, ale, beer, spirits, &c., as for dry goods not measured by

Coals

heap measure, shall be derived and computed. Two of these gallons make a peck, and 8 such gallons make a bushel, and 8 such bushels a quarter of corn, or other dry goods not measured by heaped measure.

The above bushel of 8 Imperial gallons is also to be used for coals, culm, fish, potatoes, fruit, and all other goods commonly sold by heaped measure, which goods are to be heaped up in the form of a cone of at least six inches in height, the base of the cone being 18½ inches diameter.

| | | |
|---|---|---|
| The Imperial gallon contains | 277.274 | cubic inches. |
| The old wine gallon | 231 | do. |
| The old corn | 268.8 | do. |
| The old ale | 282 | do. |

## TABLE OF FACTORS,

For converting old measures into new, and the contrary.

| | By decimals. | | | By vulgar fractions nearly. | | |
|---|---|---|---|---|---|---|
| | Corn Measure. | Wine Measure. | Ale Measure. | Corn Measure. | Wine Measure. | Ale Measure. |
| To convert old measures to new. | .96943 | .83311 | 1.01704 | $\frac{31}{32}$ | $\frac{5}{6}$ | $\frac{60}{59}$ |
| To convert new measures to old. | 1.03153 | 1.20032 | .98324 | $\frac{32}{31}$ | $\frac{6}{5}$ | $\frac{59}{60}$ |

N.B. For reducing the prices, these numbers must all be reversed.

*Ex.* Reduce 63 gallons wine measure to the equivalent number in Imperial measure.

$$63 \times .83311 \text{ or } 63 \times \frac{5}{6} = 52\tfrac{1}{2} \text{ Imperial gallons nearly.}$$

## *DRY OR CORN MEASURE.*

Pints.
8 ---- 1 Gallon.
16 ---- 2 ---- 1 Peck.
64 ---- 8 ---- 4 ---- 1 Bushel.
256 ---- 32 ---- 16 ---- 4 ---- 1 Coomb.
512 ---- 64 ---- 32 ---- 8 ---- 2 ---- 1 Quarter.
2560 ---- 320 ---- 160 ---- 40 ---- 10 ---- 5 ---- 1 Wey.
5120 ---- 640 ---- 320 ---- 80 ---- 20 ---- 10 ---- 2 ---- 1 Last.

Also,

2 bushels make 1 boll.
3 bushels ——— 1 sack.
36 bushels ——— 1 chaldron of coals at London.
68 bushels ——— 1 do. Newcastle.

The London chaldron weighs 28¼ cwt.; and the Newcastle chaldron 53 cwt.

A bushel, water measure, is 5 pecks. 8 chaldrons a keel.

### *MEASURE ITINERARY.*

| | Yds. | | Yds. |
|---|---|---|---|
| Mile of Russia | 1100 | Mile of Poland | 4400 |
| of Italy | 1467 | of Spain | 5028 |
| of England | 1760 | of Germany | 5867 |
| of Scotland & Ireland | 2200 | of Sweden | 7333 |
| Old league of France | 2200 | of Denmark | 7333 |
| Small league do. | 2933 | of Hungary | 8800 |
| Mean league do. | 3667 | | |
| Great league do. | 4400 | | |

### *TABLE OF MISCELLANEOUS ARTICLES.*

| | | |
|---|---|---|
| 24 | Sheets of Paper make | 1 quire. |
| 20 | Quires | 1 ream. |
| 21½ | Quires | 1 printer's ream. |
| 2 | Reams | 1 bundle. |
| 10 | Reams | 1 bale. |
| 60 | Skins | 1 roll of parchment. |
| 12 | Dozen of any thing | 1 gross. |
| 12 | Gross | 1 great gross. |
| 6 | Score | 1 great hundred. |
| 500 | Bricks | 1 load. |
| 384 | Bricks | 1 cubic yard. |
| 40 | Solid feet of hewn timber | 1 load. |
| 50 | Solid feet of unhewn timber | 1 load. |
| 100 | Acres | 1 hide of land. |
| 20 | Stone of flour | 1 sack. |
| 56 | Pounds of butter | 1 firkin. |
| 64 | Pounds of soap | 1 firkin. |

| | | |
|---|---|---|
| 19½ | Cwt. of lead | 1 fother. |
| 84 | Pounds of tea | 1 chest. |
| 168 | Pounds of rice | 1 bag. |
| 112 | Pounds of raisins | 1 barrel. |

## *FRENCH WEIGHTS & MEASURES.*

### *A few of the principal old French Measures.*

| | |
|---|---|
| A point | ,0148025 English inches. |
| A line | ,088815 |
| An inch | 1,06578 |
| A foot | 12,78936 |
| A toise | 6,394665 English feet. |

According to Gen. Roy, an English fathom : a French toise :: 1000 : 1065,75.

### *New, or Metre, System.*

In the new system the metre is the ten millionth part of the quadrant of the meridian = 3.281 English feet. The Are is the square decametre, and the litre the cubic decimetre.

### *Lineal Measure.*

| | |
|---|---|
| Millimetre | 0.03937 English inches. |
| Centimetre | 0.39371 |
| Decimetre | 3.93710 |
| Metre | 39.37100 |
| Decametre | 393.71000 |
| Hecatometre | 3937.10000 |
| Chiliometre | 39371.00000 |
| Myriometre | 393710.00000 |

### *Superficial Measure.*

| | |
|---|---|
| Are | 119.6046 Eng. square yards. |
| Decare | 1196.0460 |
| Hecatare | 11960.4600 |

### *Measure of capacity.*

| | |
|---|---|
| Millilitre | .06103 Eng. cubic inches |
| Centilitre | .61028 |
| Decilitre | 6.10280 |
| Litre | 61.02800 |

| | |
|---|---|
| Decalitre | 610.28000 |
| Hecatolitre | 6102.80000 |
| Chiliolitre | 61028.00000 |
| Myriolitre | 610280.00000 |

*New French Weights.*

| | English Troy grains. |
|---|---|
| Milligramme | .01544 |
| Centigramme | .15445 |
| Decigramme | 1.54457 |
| Gramme | 15.44579 |
| Decagramme | 154.45793 |
| Hecatogramme | 1544.57938 |
| Chiliogramme | 15445.79386 |
| Myriogramme | 154457.93860 |

WHEEL *and Axle.*

1. There is an equilibrium upon the wheel and axle, when the power is to the weight, as the radius of the axle to the radius of the wheel.

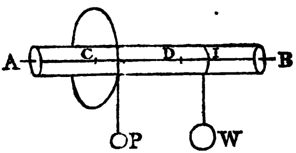
*Cor.* 1. Hence if the diameter of the wheel increases in the same proportion as the power decreases, the force with which the wheel is turned remains unaltered. This principle is introduced in the construction of the fuzee of a watch, and of the mainspring on the tumbler of gun locks.

*Cor.* 2. If 2 R be the thickness of the ropes by which the power and weight act, there will be an equilibrium when $P : W :: \text{rad. of axle} + R : \text{rad. of wheel} + R$. Hence the ratio of the power to the weight is greater in this case than the former.

*Cor.* 3. In a combination of wheels and axles, where the circumference of the first axle is applied to the circumference of the second wheel, by means of a string or by tooth and pinion, and the 2d axle to the 3d wheel, &c. there is an equilibrium, when P : W :: the product of the radii of all the axles : the product of the radii of all the wheels; or :: the product of all the teeth in the pinions : the product of all the teeth in the wheels.

If the wheels be all equal to each other, and the axles equal to each other; or if each wheel be to the axle on which it is fixed in a constant ratio, as for instance that of $r$ to $s$, and if $n$ be the number of wheels or axles, $P : W :: r^n : s^n$.

2. Required the pressure upon the pivots A and B of the wheel and axle when in equilibrio.



Let A = weight of the axle, $w$ = weight of the wheel, and let D be the centre of gravity of P and W, then the pressure on A from the axle = ½ A; do. from the wheel $= \frac{w \times BC}{AB}$; do. from P and W $= (P + W) \times \frac{BD}{AB}$; ∴ the whole pressure on A $= \frac{1}{2} A + \frac{w \times BC + (P + W) \times BD}{AB}$. In like manner the pressure on B $= \frac{1}{2} A + \frac{w \times AC + (P + W) \times AD}{AB}$.

WHEEL *undershot and overshot.—(Playfair.)*

1. An undershot wheel (i. e. one carried round by the impulse of a stream flowing *under* it) produces the greatest effect, or does the most work, when the wheel moves with one-third of the velocity of the water.

For if V and $v$ be the velocities of the water and floatboard, the effect in a given time will vary as $(V - v)^2 \times v$, which is a maximum when $v = \frac{1}{3} V$.

If $h$ = height due to velocity V, and $a^2$ = the section of the stream, the effect when a maximum $= \frac{8 a^2 h}{27}$, i. e. $\frac{8}{27}$ of the water expended.

Mr Smeaton found in *practice*, that when the effect is a maximum, $v$ equalled from ⅓ to ½ V; and that its effect instead of $\frac{8}{27}$, was about ⅓ of the water expended. He also found that the expence of water being the same, the effect is as the square of the velocity, and when the section of the water is the same, the effect is as the cube of the velocity.

2. In overshot wheels (i. e. when the wheel receives the water into buckets at or near the highest point) if A be the quantity of water issuing in a second, and $h$ the height due to the velocity of the circumference of the wheel, and $r$ the radius, the effect of the machine is proportional to $A. (2r - h.)$

*Cor.* Hence the effect will be the greater the less $h$ is, or the less the velocity of the wheel; this however in practice is found to be subject to some limitations. Mr Smeaton was led from experiment to conclude that overshot wheels do most work, when their circumferences move at the rate of 3 feet in a second, but this determination is also to be understood with some latitude.

3. In an overshot wheel, the machine will be in its greatest perfection; when the diameter of the wheel is ⅝ of the height of the water above the lowest point of the wheel.

4. The power of the overshot wheel is greater, cæteris paribus, than that of the undershot, nearly in the ratio of 13 to 5.

## WIND.

Winds may be divided into *constant*, or those which always blow in the same direction; *periodical*, or those which blow half a year in one direction, and half a year in the contrary direction, which last are called *monsoons;* and *variable*, which are subject to no rules.

### I. *Constant or Trade Winds.*

The trade wind at the Equator blows constantly from the east: from the Equator to the northern tropic, or even as far as the parallel 25° or 30°, it declines towards the N.E., and the more so the further you recede from the Equator: and from the Equator to the southern tropic, or to the parallel 25° or 30°, it has a S.E. direction. The line however that separates the opposite trade winds is not precisely the Equator, but the second or third parallel north. To a certain extent also they follow the course of the sun, reaching a little further into the southern ½ sphere, and contracting their limits in the north, when the sun is on the south side of the Equator; and making a reverse change when he declines to the north. In a zone of variable breadth in the middle of this tract, calms and rains prevail, caused probably by the mingling and ascending of the opposite aerial currents. The phænomenon of the trade winds may be thus explained. The air towards the poles being denser than that at the Equator, will continually rush towards the Equator; but as the velocity of the different parts of the earth's surface, from its rotation, increases as you approach the Equator; the air which is rushing from the north will not continue upon the same meridian, but it will be left behind; that is, in respect to the earth's surface, it will have a motion from the east; and these two motions combined produce a N.E. wind on the north side of the Equator. And in like manner there must be a S.E. wind on the south side. The air which is thus continually moving from the Poles to the Equator, being rarified when it comes there, ascends to the top of the atmosphere, and then returns back to the Poles.

II. *Periodical Winds, or Monsoons.*

Such would probably be the regular course of the trade winds supposing the parts between and near the tropics were open sea. But high lands change or interrupt their regular course. For instance, in the Indian Ocean the trade wind is curiously modified by the lands which surround it on the north, east, and west. There, the southern trade wind blows regularly as it ought to do from the E. and S.E., from 10° S. latitude to the tropic; but in the space from 10° S. latitude to the Equator, N.W. winds blow during our winter (from October to April); and S.E. in the other six months, while in the *whole* space north of the Equator S.W. winds blow during summer, and N.E. during winter. These winds are called monsoons. It was observed above, that the regular trade wind blows in the Indian Ocean from 10° S. latitude to the tropic, but there is an exception to this in all that part of the Indian Ocean which lies between Madagascar and Cape Comorin; for there, between the months of April and October, the wind blows from the S.W., and in the contrary direction from October to April. But of both the constant and periodical winds it may be observed, that they blow only at sea; at land the wind is always variable.

*Particulars of the Trade Winds, from Robertson.—(Young's Natural Philosophy.)*

1. For 30° on each side of the Equator, there is almost constantly an easterly wind in the Atlantic and Pacific Oceans: it is called the trade wind: near the Equator it is due east, further off it blows towards the Equator, and is N.E. or S.E.

2. Beyond 30° latitude, the wind is more uncertain.

3. The monsoons are, perhaps erroneously, deduced from a superior current in a contrary direction.

4. In the Atlantic, between 10° and 28° N. latitude, about 300 miles from the coast of Africa, there is a constant N.E. wind.

5. On the American side of the Caribbee Islands the N.E. wind becomes nearly E.

6. The trade winds extend 3° or 4° further N. and S. on the W. than on the E. side of the Atlantic.

7. Within 4° of the Equator, the wind is always S.E.: it is more E. towards America, and more S. towards Africa. On the coast of Brazil, when the sun is far northwards, the S.E. becomes more S., and the N.E. more E., and the reverse when the sun is far southwards.

8. On the coast of Guinea, for 1500 miles, from Sierra Leone to St.

Thomas, the wind is always S. or S.W. probably from an inclination of the trade wind towards the land.

9. Between lat. 4° and 10°, and between the longitudes of Cape Verd and the Cape Verd Islands, there is a track of sea very liable to storms of thunder and lightning. It is called the rains. Probably there are opposite winds that meet here.

10. In the Indian Ocean, between 10° and 20° S. latitude, the wind is regularly S.E. From June to November, these winds reach to within 2° of the Equator: but from December to May the wind is N.W. between lat. 3° and 10° near Madagascar, and from 2° to 12° near Sumatra.

11. Between Sumatra and Africa, from 3° S. latitude to the coasts on the N. the monsoons blow N.E. from September to April, and S.W. from March to October: the wind is steadier, and the weather fairer, in the former half year.

12. Between Madagascar and Africa, and thence northwards to the Equator, from April to October there is a S.S.W. wind, which further N. becomes W.S.W.

13. East of Sumatra, and as far as Japan, the monsoons are N. and S. but not quite so certain as in the Arabian gulf.

14. From New Guinea to Sumatra and Java, the monsoons are more N.W. and S.E. being on the south of the Equator; they begin a month or six weeks later than in the Chinese seas.

15. The changes of these winds are attended by calms and storms.

III. *Winds variable.*

In the temperate zones the direction of the winds is by no means so regular as between the tropics. In the north temperate zone, however, they blow most frequently from the S.W., in the south temperate zone, from the N.W.; but changing frequently to all points of the compass, and in the north temperate zones blowing, particularly during the spring, from the north-east.

From an average of 10 years of the register kept by order of the Royal Society, it appears that at London the winds blow in the following order:—

| *Winds.* | *Days.* | *Winds.* | *Days.* |
|---|---|---|---|
| South-west | 112 | South-east | 32 |
| North-east | 58 | East | 26 |
| North-west | 50 | South | 18 |
| West | 53 | North | 16 |

It appears from the same register, that the S.W. wind blows at an average more frequently than any other wind during every month of the year, and that it blows longest in July and August; that the N.E. blows most constantly during January, March, April, May, and June; and most seldom during February, July, September, and December; and that the N.W. wind blows oftener from November to March, and more seldom during September and October than any other months.—*(Phil. Trans.* 66, 2.*)*

The following Table of the winds at Lancaster, has been drawn up from a register kept for seven years at that place:—

| | *Days.* | | *Days.* |
|---|---|---|---|
| S.W. | 92 | S.E. | 35 |
| N.E. | 67 | N. | 30 |
| S. | 51 | N.W. | 26 |
| W. | 47 | E. | 17 |

The following Table is an abstract of nine years observation made at Dumfries, by Mr Copland:—

| | *Days.* | | *Days.* |
|---|---|---|---|
| S. | 28⅓ | N. | 36⅓ |
| W. | 69 | N.W. | 25½ |
| E. | 68 | S.E. | 18⅓ |
| S.W. | 50⅓ | N.E. | 14⅓ |

The following Table exhibits a view of the number of days during which the westerly and easterly winds blow in a year at different parts of the island. Under the term westerly are included the N.W., W., S.W., and S.; the term easterly is taken in the same latitude:—

| *Years of observation* | *Places.* | *Westerly.* | *Easterly.* |
|---|---|---|---|
| 10 | London | 233 | 132 |
| 7 | Lancaster | 216 | 149 |
| 51 | Liverpool | 190 | 175 |
| 9 | Dumfries | 227.5 | 137.5 |
| 10 | Branxholm, 54 miles S.W. of Berwick | 232 | 133 |
| 7 | Cambuslany, near Glasgow | 214 | 151 |
| 8 | Hawkhill, near Edinburgh | 229.5 | 135.5 |
| | Mean | 220.3 | 144.7 |

IV. *Wind, velocity of.*

The following Table, drawn up by Mr Smeaton, will give the reader a pretty precise idea of the velocity of the wind in different circumstances.—(*Phil. Trans.* 1757.)

| *Miles per Hour.* | *Feet per Second.* | *Perpendicular force on one square foot in avoirdupois pounds and parts.* | |
|---|---|---|---|
| 1 | 1.47 | .005 | Hardly perceptible. |
| 2 | 2.93 | .020 | Just perceptible. |
| 3 | 4.4 | .044 | |
| 4 | 5.87 | .079 | Gently pleasant. |
| 5 | 7.33 | .123 | |
| 10 | 14.67 | .492 | Pleasant, brisk. |
| 15 | 22.00 | 1.107 | |
| 20 | 29.34 | 1.968 | Very brisk. |
| 25 | 36.67 | 3.075 | |
| 30 | 44.01 | 4.429 | High wind. |
| 35 | 51.34 | 6.027 | |
| 40 | 58.68 | 7.873 | Very high wind. |
| 45 | 66.01 | 9.963 | |
| 50 | 73.35 | 12.300 | Storm or tempest. |
| 60 | 88.02 | 17.715 | Great storm. |
| 80 | 117.36 | 31.490 | Hurricane. |
| 100 | 146.7 | 49.200 | Hurricane that tears up trees and carries buildings before it. |

WINDMILL.—(*Playfair.*)

1. The impulse of a stream of air, striking with a velocity of $v$ feet per second, on a plane whose area in feet $= a^2$, inclined at an angle $\theta$ to the direction of the stream, is in avoirdupois pounds,

$$\frac{v^2 a^2 \sin.^2 \theta}{440}.$$

2. The sails of windmills are so constructed as to have different inclinations to the plane of their motions at different distances from the axis; greatest nearer the centre, and least at their extremities. This is done in order to make the momentum of the wind nearly the same as all different distances from the centre of motion.

3. Supposing the sail of a windmill to be a plane, inclined to the axis at an angle $\theta$, the effect of the wind to turn the sail in a plane, at right angles to its axis, will be the greatest when $\cos. \theta \times \sin.^2 \theta$ is a maximum, or when $\cos. \theta = \frac{1}{3}$.

This gives $\theta = 54^\circ\ 44'$, and therefore the inclination of the sail to the plane of its motion, or what is called the angle of *weather*, is $35^\circ\ 16'$. This is true only when the sail is at rest or just beginning to move.

When the sail is in motion, and of course near the extremities of the sail, when it moves faster, the angle of weather must be less.

Maclaurin makes the weather to vary from 26° 34′, at the point of the sail nearest the centre, to 9° at its extremity. Mr Smeaton, however, by experiment has found the following angles to answer as well as any. The radius is supposed to be divided into six parts, and ⅙th reckoning from the centre is called 1, the extremity being denoted 6.

| *No.* | *Angle with the axis.* | *Angle with the plane of motion.* |
|---|---|---|
| 1 | 72° | 18 |
| 2 | 71 | 19 |
| 3 | 72 | 18 middle. |
| 4 | 74 | 16 |
| 5 | 77½ | 12½ |
| 6 | 83 | 7 extremity. |

4. From Smeaton's experiments it appears, that a windmill works to the greatest advantage, when it is so constructed that the velocity of the sails is to their velocity when they go round without any load, as a number between 6 and 7 is to 10; and also that the load, when the mill works in this manner, is to the load that would just keep it from moving, nearly as 8,5 to 10.

5. With different velocities of wind the load that gives the maximum *effect* varies nearly as the square of the velocity, and the *effect* itself as the cube.

WIRE, *time of sun's passing.—See Time.*

# Y

YEAR, *length of.—See Earth elements of, and Calendar.*

## TABLE I.

Containing the Logarithms of all numbers from 1 to 1000; and of all the even numbers from 1000 to 10,000. The Logarithms of the odd numbers from 1000 to 10,000 may be had by simple subtraction.

| No. | Log. | No. | Log. | No. | Log. | No. | Log. | No. | Log. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.000000 | 21 | 1.322219 | 41 | 1.612784 | 61 | 1.785330 | 81 | 1.908485 |
| 2 | 0.301030 | 22 | 1.342423 | 42 | 1.623249 | 62 | 1.792392 | 82 | 1.913814 |
| 3 | 0.477121 | 23 | 1.361728 | 43 | 1.633468 | 63 | 1.799341 | 83 | 1.919078 |
| 4 | 0.602060 | 24 | 1.380211 | 44 | 1.643453 | 64 | 1.806180 | 84 | 1.924279 |
| 5 | 0.698970 | 25 | 1.397940 | 45 | 1.653213 | 65 | 1.812913 | 85 | 1.929419 |
| 6 | 0.778151 | 26 | 1.414973 | 46 | 1.662758 | 66 | 1.819544 | 86 | 1.934498 |
| 7 | 0.845098 | 27 | 1.431364 | 47 | 1.672098 | 67 | 1.826075 | 87 | 1.939519 |
| 8 | 0.903090 | 28 | 1.447158 | 48 | 1.681241 | 68 | 1.832509 | 88 | 1.944483 |
| 9 | 0.954243 | 29 | 1.462398 | 49 | 1.690196 | 69 | 1.838849 | 89 | 1.949390 |
| 10 | 1.000000 | 30 | 1.477121 | 50 | 1.698970 | 70 | 1.845098 | 90 | 1.954243 |
| 11 | 1.041393 | 31 | 1.491362 | 51 | 1.707570 | 71 | 1.851258 | 91 | 1.959041 |
| 12 | 1.079181 | 32 | 1.505150 | 52 | 1.716003 | 72 | 1.857332 | 92 | 1.963788 |
| 13 | 1.113943 | 33 | 1.518514 | 53 | 1.724276 | 73 | 1.863323 | 93 | 1.968483 |
| 14 | 1.146128 | 34 | 1.531479 | 54 | 1.732394 | 74 | 1.869232 | 94 | 1.973128 |
| 15 | 1.176091 | 35 | 1.544068 | 55 | 1.740363 | 75 | 1.875061 | 95 | 1.977724 |
| 16 | 1.204120 | 36 | 1.556303 | 56 | 1.748188 | 76 | 1.880814 | 96 | 1.982271 |
| 17 | 1.230449 | 37 | 1.568202 | 57 | 1.755875 | 77 | 1.886491 | 97 | 1.986772 |
| 18 | 1.255273 | 38 | 1.579784 | 58 | 1.763428 | 78 | 1.892095 | 98 | 1.991226 |
| 19 | 1.278754 | 39 | 1.591065 | 59 | 1.770852 | 79 | 1.897627 | 99 | 1.995635 |
| 20 | 1.301030 | 40 | 1.602060 | 60 | 1.778151 | 80 | 1.903090 | 100 | 2.000000 |

| N. | 0 | 2 | 4 | 6 | 8 | N. | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 000000 | 000868 | 001734 | 002598 | 003461 | 130 | 113943 | 114611 | 115278 | 115943 | 116608 |
| 1 | 4321 | 5181 | 6038 | 6894 | 7748 | 1 | 7271 | 7934 | 8595 | 9256 | 9915 |
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| 6 | 2308 | 2405 | 2502 | 2599 | 2696 | 6 | 6611 | 6705 | 6799 | 6892 | 6986 |
| 7 | 2792 | 2889 | 2986 | 3083 | 3180 | 7 | 7080 | 7173 | 7267 | 7361 | 7454 |
| 8 | 3276 | 3373 | 3470 | 3566 | 3663 | 8 | 7548 | 7642 | 7735 | 7829 | 7922 |
| 9 | 3760 | 3856 | 3953 | 4049 | 4146 | 9 | 8016 | 8109 | 8203 | 8296 | 8390 |
| 900 | 954243 | 954339 | 954435 | 954532 | 954628 | 930 | 968483 | 968576 | 968670 | 968763 | 968856 |
| 1 | 4725 | 4821 | 4918 | 5014 | 5110 | 1 | 8950 | 9043 | 9136 | 9229 | 9323 |
| 2 | 5207 | 5303 | 5399 | 5495 | 5592 | 2 | 9416 | 9509 | 9602 | 9695 | 9789 |
| 3 | 5688 | 5784 | 5880 | 5976 | 6072 | 3 | 9882 | 9975 | 970068 | 970161 | 970254 |
| 4 | 6168 | 6265 | 6361 | 6457 | 6553 | 4 | 970347 | 970440 | 0533 | 0626 | 0719 |
| 5 | 6649 | 6745 | 6840 | 6936 | 7032 | 5 | 0812 | 0904 | 0997 | 1090 | 1183 |
| 6 | 7128 | 7224 | 7320 | 7416 | 7512 | 6 | 1276 | 1369 | 1461 | 1554 | 1647 |
| 7 | 7607 | 7703 | 7799 | 7894 | 7990 | 7 | 1740 | 1832 | 1925 | 2018 | 2110 |
| 8 | 8086 | 8181 | 8277 | 8373 | 8468 | 8 | 2203 | 2295 | 2388 | 2481 | 2573 |
| 9 | 8564 | 8659 | 8755 | 8850 | 8946 | 9 | 2666 | 2758 | 2851 | 2943 | 3035 |

| N. | 0 | 2 | 4 | 6 | 8 | N. | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 940 | 973128 | 973220 | 973313 | 973405 | 973497 | 970 | 986772 | 986861 | 986951 | 987040 | 967130 |
| 1 | 3590 | 3682 | 3774 | 3866 | 3959 | 1 | 7219 | 7309 | 7398 | 7488 | 7577 |
| 2 | 4051 | 4143 | 4235 | 4327 | 4420 | 2 | 7666 | 7756 | 7845 | 7934 | 8024 |
| 3 | 4512 | 4604 | 4696 | 4788 | 4880 | 3 | 8113 | 8202 | 8291 | 8381 | 8470 |
| 4 | 4972 | 5064 | 5156 | 5248 | 5340 | 4 | 8559 | 8648 | 8737 | 8826 | 8916 |
| 5 | 5432 | 5524 | 5616 | 5707 | 5799 | 5 | 9005 | 9094 | 9183 | 9272 | 9361 |
| 6 | 5891 | 5983 | 6075 | 6167 | 6258 | 6 | 9450 | 9539 | 9628 | 9717 | 9806 |
| 7 | 6350 | 6442 | 6533 | 6625 | 6717 | 7 | 9895 | 9983 | 990072 | 990161 | 990250 |
| 8 | 6808 | 6900 | 6992 | 7083 | 7175 | 8 | 990339 | 990428 | 0516 | 0605 | 0694 |
| 9 | 7266 | 7358 | 7449 | 7541 | 7632 | 9 | 0783 | 0871 | 0960 | 1049 | 1137 |
| 950 | 977724 | 977815 | 977906 | 977998 | 978089 | 980 | 991226 | 991315 | 991403 | 991492 | 991580 |
| 1 | 8181 | 8272 | 8363 | 8454 | 8546 | 1 | 1669 | 1758 | 1846 | 1935 | 2023 |
| 2 | 8637 | 8728 | 8819 | 8911 | 9002 | 2 | 2111 | 2200 | 2288 | 2377 | 2465 |
| 3 | 9093 | 9184 | 9275 | 9366 | 9457 | 3 | 2554 | 2642 | 2730 | 2819 | 2907 |
| 4 | 9548 | 9639 | 9730 | 9821 | 9912 | 4 | 2995 | 3083 | 3172 | 3260 | 3348 |
| 5 | 980003 | 980094 | 980185 | 980276 | 980367 | 5 | 3436 | 3524 | 3613 | 3701 | 3789 |
| 6 | 0458 | 0549 | 0640 | 0730 | 0821 | 6 | 3877 | 3965 | 4053 | 4141 | 4229 |
| 7 | 0912 | 1003 | 1093 | 1184 | 1275 | 7 | 4317 | 4405 | 4493 | 4581 | 4669 |
| 8 | 1366 | 1456 | 1547 | 1637 | 1728 | 8 | 4757 | 4845 | 4933 | 5021 | 5108 |
| 9 | 1819 | 1909 | 2000 | 2090 | 2181 | 9 | 5196 | 5284 | 5372 | 5460 | 5547 |
| 960 | 982271 | 982362 | 982452 | 982543 | 982633 | 990 | 995635 | 995723 | 995811 | 995898 | 995986 |
| 1 | 2723 | 2814 | 2904 | 2994 | 3085 | 1 | 6074 | 6161 | 6249 | 6337 | 6424 |
| 2 | 3175 | 3265 | 3356 | 3446 | 3536 | 2 | 6512 | 6599 | 6687 | 6774 | 6862 |
| 3 | 3626 | 3716 | 3807 | 3897 | 3987 | 3 | 6949 | 7037 | 7124 | 7212 | 7299 |
| 4 | 4077 | 4167 | 4257 | 4347 | 4437 | 4 | 7386 | 7474 | 7561 | 7648 | 7736 |
| 5 | 4527 | 4617 | 4707 | 4797 | 4887 | 5 | 7823 | 7910 | 7998 | 8085 | 8172 |
| 6 | 4977 | 5067 | 5157 | 5247 | 5337 | 6 | 8259 | 8347 | 8434 | 8521 | 8608 |
| 7 | 5426 | 5516 | 5606 | 5696 | 5786 | 7 | 8695 | 8782 | 8869 | 8956 | 9043 |
| 8 | 5875 | 5965 | 6055 | 6144 | 6234 | 8 | 9131 | 9218 | 9305 | 9392 | 9479 |
| 9 | 6324 | 6413 | 6503 | 6593 | 6682 | 9 | 9565 | 9652 | 9739 | 9826 | 9913 |

# TABLE II.

## LOGARITHMIC

## SINES, COSINES, TANGENTS, AND COTANGENTS,

*TO EVERY EVEN MINUTE OF THE QUADRANT.*

*Note.*—From this Table may also be found the Logarithmic Secants and Cosecants; the logarithm of the secant of any arc being = 20 — log. cosine; and the logarithmic cosecant = 20 — log. sine.

| ′ | 0 Deg. |  |  |  | 1 Deg. |  |  |  | 2 Deg. |  |  |  | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. |  |
| 0 | 0.000000 | 10.000000 | 0.000000 | Infinite. | 8.241855 | 9.999934 | 8.241921 | 11.758079 | 8.542819 | 9.999735 | 8.543084 | 11.456916 | 60 |
| 2 | 6.764756 | 00 | 6.764756 | 13.235244 | 256094 | 29 | 256165 | 743835 | 549095 | 26 | 550268 | 449732 | 58 |
| 4 | 7.065786 | 00 | 7.065786 | 12.934214 | 269881 | 25 | 269956 | 730044 | 557054 | 17 | 557336 | 442664 | 56 |
| 6 | 241877 | 9.999999 | 241878 | 758122 | 283243 | 20 | 283323 | 716677 | 563999 | 08 | 564291 | 435709 | 54 |
| 8 | 366816 | 99 | 366817 | 633183 | 296207 | 15 | 296292 | 703708 | 570836 | 9.999699 | 571137 | 428863 | 52 |
| 10 | 463725 | 98 | 463727 | 536273 | 308794 | 10 | 308884 | 691116 | 577566 | 89 | 577877 | 422123 | 50 |
| 12 | 542906 | 97 | 542909 | 457091 | 321027 | 05 | 321122 | 678878 | 584193 | 80 | 584514 | 415486 | 48 |
| 14 | 609853 | 96 | 609857 | 390143 | 332924 | 9.999899 | 333025 | 666975 | 590721 | 70 | 591051 | 408949 | 46 |
| 16 | 667845 | 95 | 667849 | 332151 | 344504 | 94 | 344610 | 655390 | 597152 | 60 | 597492 | 402508 | 44 |
| 18 | 718997 | 94 | 719003 | 280997 | 355783 | 88 | 355895 | 644105 | 603489 | 50 | 603839 | 396161 | 42 |
| 20 | 764754 | 93 | 764761 | 235239 | 366777 | 82 | 366895 | 633105 | 609734 | 40 | 610094 | 389906 | 40 |
| 22 | 7.806146 | 9.999991 | 7.806155 | 12.193845 | 8.377199 | 9.999876 | 8.377622 | 11.622378 | 8.615891 | 9.999629 | 8.616262 | 11.383738 | 38 |
| 24 | 843934 | 89 | 843944 | 156056 | 387962 | 70 | 388092 | 611908 | 621962 | 19 | 622343 | 377657 | 36 |
| 26 | 878695 | 88 | 878708 | 121292 | 398179 | 64 | 398315 | 601085 | 627948 | 08 | 628340 | 371660 | 34 |
| 28 | 910879 | 86 | 910894 | 089106 | 408161 | 58 | 408304 | 591696 | 633854 | 9.999597 | 634256 | 365744 | 32 |
| 30 | 940842 | 83 | 940858 | 059142 | 417919 | 51 | 418068 | 581932 | 639680 | 86 | 640093 | 359907 | 30 |
| 32 | 968870 | 81 | 968889 | 031111 | 427462 | 44 | 427618 | 572382 | 645428 | 75 | 645853 | 354147 | 28 |
| 34 | 995198 | 79 | 995219 | 001781 | 436800 | 38 | 436962 | 563038 | 651102 | 64 | 651537 | 348463 | 26 |
| 36 | 8.020021 | 76 | 8.020045 | 11.979955 | 445941 | 31 | 446110 | 553890 | 656702 | 53 | 657149 | 342851 | 24 |
| 38 | 043501 | 73 | 043527 | 956473 | 454893 | 23 | 455070 | 544930 | 662230 | 41 | 662689 | 337311 | 22 |
| 40 | 065776 | 71 | 065806 | 934194 | 463665 | 16 | 463840 | 536151 | 667689 | 29 | 668160 | 331840 | 20 |
| 42 | 8.086965 | 9.999968 | 8.086997 | 11.913003 | 8.472263 | 9.999809 | 8.472454 | 11.527546 | 8.673080 | 9.999518 | 8.673563 | 11.326437 | 18 |
| 44 | 107167 | 64 | 107202 | 892797 | 480693 | 01 | 480892 | 519108 | 678405 | 06 | 678900 | 321100 | 16 |
| 46 | 126471 | 61 | 126510 | 873490 | 488963 | 9.999793 | 489170 | 510830 | 683665 | 9.999493 | 684172 | 315828 | 14 |
| 48 | 144953 | 58 | 144996 | 855004 | 497078 | 86 | 497293 | 502707 | 688863 | 81 | 689381 | 310619 | 12 |
| 50 | 162681 | 54 | 162727 | 837273 | 505045 | 78 | 505267 | 494733 | 693998 | 69 | 694529 | 305471 | 10 |
| 52 | 179713 | 50 | 179763 | 820237 | 512867 | 69 | 513098 | 486902 | 699073 | 56 | 699617 | 300383 | 8 |
| 54 | 196102 | 46 | 196156 | 803844 | 520551 | 64 | 520790 | 479210 | 704090 | 43 | 704646 | 295354 | 6 |
| 56 | 211895 | 42 | 211953 | 788047 | 528102 | 53 | 528349 | 471651 | 709049 | 31 | 709618 | 290382 | 4 |
| 58 | 227134 | 38 | 227195 | 772805 | 535523 | 44 | 535779 | 464221 | 713952 | 18 | 714534 | 285465 | 2 |
| 60 | 241855 | 34 | 241921 | 758079 | 542819 | 35 | 543084 | 456916 | 718800 | 04 | 719396 | 280604 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
|  | 89 Deg. |  |  |  | 88 Deg. |  |  |  | 87 Deg. |  |  |  |  |

| | 3 Deg. | | | | 4 Deg. | | | | 5 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 8,718800 | 9.999404 | 8,719396 | 11.280604 | 8.843585 | 9.998941 | 8,844644 | 11.155356 | 8,940296 | 9.998344 | 8,941952 | 11,058048 | 60 |
| 2 | 723595 | 9.999391 | 724204 | 275796 | 847183 | 23 | 848260 | 151740 | 943174 | 22 | 944852 | 055148 | 58 |
| 4 | 728337 | 78 | 728959 | 271041 | 850751 | 05 | 851846 | 148154 | 946034 | 00 | 947734 | 052266 | 56 |
| 6 | 733027 | 64 | 733663 | 266337 | 854291 | 9.998887 | 855403 | 144597 | 948874 | 9.998277 | 950597 | 049403 | 54 |
| 8 | 737667 | 50 | 738317 | 261683 | 857801 | 69 | 858932 | 141068 | 951696 | 55 | 953441 | 046559 | 52 |
| 10 | 742259 | 36 | 742922 | 257078 | 861283 | 51 | 862433 | 137567 | 954499 | 32 | 956267 | 043733 | 50 |
| 12 | 746802 | 22 | 747479 | 252521 | 864738 | 32 | 865906 | 134094 | 957284 | 09 | 959075 | 040925 | 48 |
| 14 | 751297 | 08 | 751989 | 248011 | 868165 | 13 | 869351 | 130649 | 960052 | 9,998186 | 961866 | 038134 | 46 |
| 16 | 755747 | 9,999294 | 756453 | 243547 | 871565 | 9.998795 | 872770 | 127230 | 962801 | 63 | 964639 | 035361 | 44 |
| 18 | 760151 | 79 | 760872 | 239128 | 874938 | 76 | 876162 | 123838 | 965534 | 39 | 967394 | 032606 | 42 |
| 20 | 764511 | 65 | 765246 | 234754 | 878285 | 57 | 879529 | 120471 | 968249 | 16 | 970133 | 029867 | 40 |
| 22 | 8,768828 | 9.999250 | 8,769578 | 11.230422 | 8.881607 | 9.998738 | 8,882869 | 11,117131 | 8,970947 | 9.998092 | 8,972855 | 11,027145 | 38 |
| 24 | 773101 | 35 | 773866 | 226134 | 884903 | 18 | 886185 | 113815 | 973628 | 68 | 975560 | 024440 | 36 |
| 26 | 777333 | 20 | 778114 | 221886 | 888174 | 9,998699 | 889476 | 110524 | 976293 | 44 | 978248 | 021752 | 34 |
| 28 | 781524 | 05 | 782320 | 217680 | 891421 | 79 | 892742 | 107258 | 978941 | 20 | 980921 | 019079 | 32 |
| 30 | 785675 | 9.999189 | 786486 | 213514 | 894643 | 59 | 895984 | 104016 | 981573 | 9,997996 | 983577 | 016423 | 30 |
| 32 | 789787 | 74 | 790613 | 209387 | 897842 | 39 | 899203 | 100797 | 984189 | 72 | 986217 | 013783 | 28 |
| 34 | 793859 | 58 | 794701 | 205299 | 901017 | 19 | 902398 | 097602 | 986789 | 47 | 988842 | 011158 | 26 |
| 36 | 797894 | 42 | 798752 | 201248 | 904169 | 9,998599 | 905570 | 094430 | 989374 | 22 | 991451 | 008549 | 24 |
| 38 | 801892 | 26 | 802765 | 197235 | 907297 | 78 | 908719 | 091281 | 991943 | 9,997897 | 994045 | 005955 | 22 |
| 40 | 805852 | 10 | 806742 | 193258 | 910404 | 58 | 911846 | 088154 | 994497 | 72 | 996624 | 003376 | 20 |
| 42 | 8,809777 | 9,999094 | 8.810683 | 11.189317 | 8,913488 | 9.998537 | 8,914951 | 11.085049 | 8,997036 | 9.997847 | 8,999188 | 11.000812 | 18 |
| 44 | 813667 | 77 | 814589 | 185411 | 916550 | 16 | 918034 | 081966 | 999560 | 22 | 9.001738 | 10,998262 | 16 |
| 46 | 817522 | 61 | 818461 | 181539 | 919591 | 9.998495 | 921096 | 078904 | 002069 | 9.997797 | 004272 | 995728 | 14 |
| 48 | 821343 | 44 | 822298 | 177702 | 922610 | 74 | 924136 | 075864 | 004563 | 71 | 006792 | 993208 | 12 |
| 50 | 825130 | 27 | 826103 | 173897 | 925609 | 53 | 927156 | 072844 | 007044 | 45 | 009298 | 990702 | 10 |
| 52 | 828884 | 10 | 829874 | 170126 | 928587 | 31 | 930155 | 069845 | 009510 | 19 | 011790 | 988210 | 8 |
| 54 | 832607 | 9.998993 | 833613 | 166387 | 931544 | 10 | 933134 | 066866 | 011962 | 9,997693 | 014268 | 985732 | 6 |
| 56 | 836297 | 76 | 837321 | 162679 | 934481 | 9,998388 | 936093 | 063907 | 014400 | 67 | 016732 | 983268 | 4 |
| 58 | 839956 | 58 | 840998 | 159002 | 937398 | 66 | 939032 | 060968 | 016824 | 41 | 019183 | 980817 | 2 |
| 60 | 843585 | 41 | 844644 | 155356 | 940296 | 44 | 941952 | 058048 | 019235 | 14 | 021620 | 978380 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine | Sine. | Cotang. | Tang. | ′ |
| | 86 Deg. | | | | 85 Deg. | | | | 84 Deg. | | | | |

|  | 6 Deg. |  |  |  | 7 Deg. |  |  |  | 8 Deg. |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9.019235 | 9.997614 | 9.021620 | 10.978380 | 9.085894 | 9.996751 | 9.089144 | 10.910856 | 9.143555 | 9.995753 | 9.147803 | 10.852197 | 60 |
| 2 | 9.021632 | 588 | 4044 | 5956 | 7947 | 720 | 9.091228 | 10.908772 | 5349 | 717 | 9632 | 0368 | 58 |
| 4 | 4016 | 561 | 6455 | 3545 | 9990 | 688 | 3302 | 6698 | 7136 | 681 | 9.151454 | 10.848546 | 56 |
| 6 | 6386 | 534 | 8852 | 1148 | 9.092024 | 657 | 5367 | 4633 | 8915 | 646 | 3269 | 6731 | 54 |
| 8 | 8744 | 507 | 9.031237 | 10.968763 | 4047 | 625 | 7422 | 2578 | 9.150686 | 610 | 5077 | 4923 | 52 |
| 10 | 9.031089 | 480 | 3609 | 6391 | 6062 | 594 | 9468 | 0532 | 2451 | 573 | 6877 | 3123 | 50 |
| 12 | 3421 | 452 | 5969 | 4031 | 8066 | 562 | 9.101504 | 10.898496 | 4208 | 537 | 8671 | 1329 | 48 |
| 14 | 5741 | 425 | 8316 | 1684 | 9.100062 | 530 | 3532 | 6468 | 5957 | 501 | 9.160457 | 10.839543 | 46 |
| 16 | 8048 | 397 | 9.040651 | 10.959349 | 2048 | 498 | 5550 | 4450 | 7700 | 464 | 2236 | 7764 | 44 |
| 18 | 9.040342 | 369 | 2973 | 7027 | 4025 | 465 | 7559 | 2441 | 9435 | 427 | 4008 | 5992 | 42 |
| 20 | 2625 | 341 | 5284 | 4716 | 5992 | 433 | 9559 | 0441 | 9.161164 | 390 | 5774 | 4226 | 40 |
| 22 | 9.044895 | 9.997313 | 9.047582 | 10.952418 | 9.107951 | 9.996400 | 9.111551 | 10.888449 | 9.162885 | 9.995353 | 9.167532 | 10.832468 | 38 |
| 24 | 7154 | 285 | 9869 | 0131 | 9901 | 368 | 3533 | 6467 | 4600 | 316 | 9284 | 0716 | 36 |
| 26 | 9400 | 257 | 9.052144 | 10.947856 | 9.111842 | 335 | 5507 | 4493 | 6307 | 278 | 9.171029 | 10.828971 | 34 |
| 28 | 9.051635 | 228 | 4407 | 5593 | 3774 | 302 | 7472 | 2528 | 8008 | 241 | 2767 | 7233 | 32 |
| 30 | 3859 | 199 | 6659 | 3341 | 5698 | 269 | 9429 | 0571 | 9702 | 203 | 4499 | 5501 | 30 |
| 32 | 6071 | 170 | 8900 | 1100 | 7613 | 235 | 9.121377 | 10.878623 | 9.171389 | 165 | 6224 | 3776 | 28 |
| 34 | 8271 | 141 | 9.061130 | 10.938870 | 9519 | 202 | 3317 | 6683 | 3070 | 127 | 7942 | 2058 | 26 |
| 36 | 9.060460 | 112 | 3348 | 6652 | 9.121417 | 168 | 5249 | 4751 | 4744 | 089 | 9655 | 0345 | 24 |
| 38 | 2639 | 083 | 5556 | 4444 | 3306 | 134 | 7172 | 2828 | 6411 | 051 | 9.181360 | 10.818640 | 22 |
| 40 | 4806 | 053 | 7752 | 2248 | 5187 | 100 | 9087 | 0913 | 8072 | 013 | 3059 | 6941 | 20 |
| 42 | 9.066962 | 9.997024 | 9.069938 | 10.930062 | 9.127060 | 9.996066 | 9.130994 | 10.869006 | 9.179726 | 9.994974 | 9.184752 | 10.815248 | 18 |
| 44 | 9107 | 9.996994 | 9.072113 | 10.927887 | 8925 | 032 | 2893 | 7107 | 9.181374 | 935 | 6439 | 3561 | 16 |
| 46 | 9.071242 | 964 | 4278 | 5722 | 9.130781 | 9.995998 | 4784 | 5216 | 3016 | 896 | 8120 | 1880 | 14 |
| 48 | 3366 | 934 | 6432 | 3568 | 2630 | 963 | 6667 | 3333 | 4651 | 857 | 9794 | 0206 | 12 |
| 50 | 5480 | 904 | 8576 | 1424 | 4470 | 928 | 8542 | 1458 | 6280 | 818 | 9.191462 | 10.808538 | 10 |
| 52 | 7583 | 874 | 9.080710 | 10.919290 | 6303 | 894 | 9.140409 | 10.859591 | 7903 | 779 | 3124 | 6876 | 8 |
| 54 | 9676 | 843 | 2833 | 7167 | 8128 | 859 | 2269 | 7731 | 9519 | 739 | 4780 | 5220 | 6 |
| 56 | 9.081759 | 812 | 4947 | 5053 | 9944 | 823 | 4121 | 5879 | 9.191130 | 700 | 6430 | 3570 | 4 |
| 58 | 3832 | 782 | 7050 | 2950 | 9.141754 | 788 | 5966 | 4034 | 2734 | 660 | 8074 | 1926 | 2 |
| 60 | 5894 | 751 | 9144 | 0856 | 3555 | 753 | 7803 | 2197 | 4332 | 620 | 9713 | 0287 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
|  | 83 Deg. |  |  |  | 82 Deg. |  |  |  | 81 Deg. |  |  |  |  |

| | 9 Deg. | | | | 10 Deg. | | | | 11 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9.194332 | 9.994620 | 9.199713 | 10.800287 | 9.239670 | 9.993351 | 9.246319 | 10.753681 | 9.280599 | 9.991947 | 9.288652 | 10.711348 | 60 |
| 2 | 5925 | 580 | 9.201345 | 10.798655 | 9.241101 | 307 | 7794 | 2206 | 1897 | 897 | 9999 | 0001 | 58 |
| 4 | 7511 | 540 | 2971 | 7029 | 2526 | 262 | 9264 | 0736 | 3190 | 848 | 9.291342 | 10.708658 | 56 |
| 6 | 9091 | 499 | 4592 | 5408 | 3947 | 217 | 9.250730 | 10.749270 | 4480 | 799 | 2682 | 7318 | 54 |
| 8 | 9.200666 | 459 | 6207 | 3793 | 5363 | 172 | 2191 | 7809 | 5766 | 749 | 4017 | 5983 | 52 |
| 10 | 2234 | 418 | 7817 | 2183 | 6775 | 127 | 3648 | 6352 | 7048 | 699 | 5349 | 4651 | 50 |
| 12 | 3797 | 377 | 9420 | 0580 | 8181 | 081 | 5100 | 4900 | 8326 | 649 | 6677 | 3323 | 48 |
| 14 | 5354 | 336 | 9.211018 | 10.788982 | 9583 | 036 | 6547 | 3453 | 9600 | 599 | 8001 | 1999 | 46 |
| 16 | 6906 | 295 | 2611 | 7389 | 9.250980 | 9.992990 | 7990 | 2010 | 9.290870 | 549 | 9322 | 0678 | 44 |
| 18 | 8452 | 254 | 4198 | 5802 | 2373 | 944 | 9429 | 0571 | 2137 | 498 | 9.300638 | 10.699362 | 42 |
| 20 | 9992 | 212 | 5780 | 4220 | 3761 | 898 | 9.260863 | 10.739137 | 3399 | 448 | 1951 | 8049 | 40 |
| 22 | 9.211526 | 9.994171 | 9.217355 | 10.782645 | 9.255144 | 9.992852 | 9.262292 | 10.737708 | 9.294658 | 9.991397 | 9.303261 | 10.696739 | 38 |
| 24 | 3055 | 129 | 8926 | 1074 | 6523 | 806 | 3717 | 6283 | 5913 | 346 | 4567 | 5433 | 36 |
| 26 | 4579 | 087 | 9.220492 | 10.779508 | 7898 | 759 | 5138 | 4862 | 7164 | 295 | 5869 | 4131 | 34 |
| 28 | 6097 | 045 | 2052 | 7948 | 9268 | 713 | 6555 | 3445 | 8412 | 244 | 7168 | 2832 | 32 |
| 30 | 7609 | 003 | 3607 | 6393 | 9.260633 | 666 | 7967 | 2033 | 9655 | 193 | 8463 | 1537 | 30 |
| 32 | 9116 | 9.993960 | 5156 | 4844 | 1994 | 619 | 9375 | 0625 | 9.300895 | 141 | 9754 | 0246 | 28 |
| 34 | 9.220618 | 918 | 6700 | 3300 | 3351 | 572 | 9.270779 | 10.729221 | 2132 | 090 | 9.311042 | 10.688958 | 26 |
| 36 | 2115 | 875 | 8239 | 1761 | 4703 | 525 | 2178 | 7822 | 3364 | 038 | 2327 | 7673 | 24 |
| 38 | 3606 | 832 | 9773 | 0227 | 6051 | 478 | 3573 | 6427 | 4593 | 9.990986 | 3608 | 6392 | 22 |
| 40 | 5092 | 789 | 9.231302 | 10.768698 | 7395 | 430 | 4964 | 5036 | 5819 | 934 | 4885 | 5115 | 20 |
| 42 | 9.226573 | 9.993746 | 9.232826 | 10.767174 | 9.268734 | 9.992382 | 9.276351 | 10.723649 | 9.307041 | 9.990882 | 9.316159 | 10.683841 | 18 |
| 44 | 8048 | 703 | 4345 | 5655 | 9.270069 | 335 | 7734 | 2266 | 8259 | 829 | 7430 | 2570 | 16 |
| 46 | 9518 | 660 | 5859 | 4141 | 1400 | 287 | 9113 | 0887 | 9474 | 777 | 8697 | 1303 | 14 |
| 48 | 9.230984 | 616 | 7368 | 2632 | 2726 | 239 | 9.280488 | 10.719512 | 9.310685 | 724 | 9961 | 0039 | 12 |
| 50 | 2444 | 572 | 8872 | 1128 | 4049 | 190 | 1858 | 8142 | 1893 | 671 | 9.321222 | 10.678778 | 10 |
| 52 | 3899 | 528 | 9.240371 | 10.759629 | 5367 | 142 | 3225 | 6775 | 3097 | 618 | 2479 | 7521 | 8 |
| 54 | 5349 | 484 | 1865 | 8135 | 6681 | 093 | 4588 | 5412 | 4297 | 565 | 3733 | 6267 | 6 |
| 56 | 6795 | 440 | 3354 | 6646 | 7991 | 044 | 5947 | 4053 | 5495 | 511 | 4983 | 5017 | 4 |
| 58 | 8235 | 396 | 4839 | 5161 | 9297 | 9.991996 | 7301 | 2699 | 6689 | 458 | 6231 | 3769 | 2 |
| 60 | 9670 | 351 | 6319 | 3681 | 9.280599 | 947 | 8652 | 1348 | 7879 | 404 | 7475 | 2525 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 80 Deg. | | | | 79 Deg. | | | | 78 Deg. | | | | |

|  | 12 Deg. |  |  |  | 13 Deg. |  |  |  | 14 Deg. |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9.317879 | 9.990404 | 9.327474 | 10.672526 | 9.352088 | 9.988724 | 9.363364 | 10.636636 | 9.393675 | 9.986904 | 9.396771 | 10.603229 | 60 |
| 2 | 9066 | 351 | 8715 | 1285 | 3181 | 666 | 4515 | 5485 | 4687 | 841 | 7846 | 2154 | 58 |
| 4 | 9.320249 | 297 | 9953 | 0047 | 4271 | 607 | 5664 | 4336 | 5697 | 778 | 8919 | 1081 | 56 |
| 6 | 1430 | 243 | 9.331187 | 10.668813 | 5358 | 548 | 6810 | 3190 | 6704 | 714 | 9990 | 0010 | 54 |
| 8 | 2607 | 188 | 2418 | 7582 | 6443 | 489 | 7953 | 2047 | 7709 | 651 | 9.401058 | 10.598942 | 52 |
| 10 | 3780 | 134 | 3646 | 6354 | 7524 | 430 | 9094 | 0906 | 8711 | 587 | 2124 | 7876 | 50 |
| 12 | 4950 | 079 | 4871 | 5129 | 8603 | 371 | 9.370232 | 10.629768 | 9711 | 523 | 3187 | 6813 | 48 |
| 14 | 6117 | 025 | 6093 | 3907 | 9678 | 312 | 1367 | 8633 | 9.390708 | 459 | 4249 | 5751 | 46 |
| 16 | 7281 | 9.989970 | 7311 | 2689 | 9.360752 | 252 | 2409 | 7501 | 1703 | 395 | 5308 | 4692 | 44 |
| 18 | 8442 | 915 | 8527 | 1473 | 1822 | 193 | 3629 | 6371 | 2695 | 331 | 6364 | 3636 | 42 |
| 20 | 9599 | 860 | 9739 | 0261 | 2889 | 133 | 4756 | 5244 | 3685 | 266 | 7419 | 2581 | 40 |
| 22 | 9.330753 | 9.989804 | 9.340948 | 10.659052 | 9.363954 | 9.988073 | 9.375881 | 10.624119 | 9.394673 | 9.986202 | 9.408471 | 10.591529 | 38 |
| 24 | 1903 | 749 | 2155 | 7845 | 5016 | 013 | 7003 | 2997 | 5658 | 137 | 9521 | 0479 | 36 |
| 26 | 3051 | 693 | 3358 | 6642 | 6075 | 9.987953 | 8122 | 1878 | 6641 | 072 | 9.410569 | 10.589431 | 34 |
| 28 | 4195 | 637 | 4558 | 5442 | 7131 | 892 | 9239 | 0761 | 7621 | 007 | 1615 | 8385 | 32 |
| 30 | 5337 | 582 | 5755 | 4245 | 8185 | 832 | 9.380354 | 10.619646 | 8600 | 9.985942 | 2658 | 7342 | 30 |
| 32 | 6475 | 525 | 6949 | 3051 | 9236 | 771 | 1466 | 8534 | 9575 | 876 | 3699 | 6301 | 28 |
| 34 | 7610 | 469 | 8141 | 1859 | 9.370285 | 710 | 2575 | 7425 | 9.400549 | 811 | 4738 | 5262 | 26 |
| 36 | 8742 | 413 | 9329 | 0671 | 1330 | 649 | 3682 | 6318 | 1520 | 745 | 5775 | 4225 | 24 |
| 38 | 9871 | 356 | 9.350514 | 10.649486 | 2373 | 588 | 4786 | 5214 | 2489 | 679 | 6810 | 3190 | 22 |
| 40 | 9.340996 | 300 | 1697 | 8303 | 3414 | 526 | 5888 | 4112 | 3455 | 613 | 7842 | 2158 | 20 |
| 42 | 9.342119 | 9.989243 | 9.352876 | 10.647124 | 9.374452 | 9.987465 | 9.386087 | 10.613013 | 9.404420 | 9.985547 | 9.418873 | 10.581127 | 18 |
| 44 | 3239 | 186 | 4053 | 5947 | 5487 | 403 | 8084 | 1916 | 5382 | 480 | 9901 | 0099 | 16 |
| 46 | 4355 | 128 | 5227 | 4773 | 6519 | 341 | 9178 | 0822 | 6341 | 414 | 9.420927 | 10.579073 | 14 |
| 48 | 5469 | 071 | 6398 | 3602 | 7549 | 279 | 9.390270 | 10.609730 | 7299 | 347 | 1952 | 8048 | 12 |
| 50 | 6579 | 014 | 7566 | 2434 | 8577 | 217 | 1360 | 8640 | 8254 | 280 | 2974 | 7026 | 10 |
| 52 | 7687 | 9.988956 | 8731 | 1269 | 9601 | 155 | 2447 | 7553 | 9207 | 213 | 3993 | 6007 | 8 |
| 54 | 8792 | 898 | 9893 | 0107 | 9.380624 | 092 | 3531 | 6469 | 9.410157 | 146 | 5011 | 4989 | 6 |
| 56 | 9893 | 840 | 9.361053 | 10.638947 | 1643 | 030 | 4614 | 5386 | 1106 | 079 | 6027 | 3973 | 4 |
| 58 | 9.350992 | 782 | 2210 | 7790 | 2661 | 9.986967 | 5694 | 4306 | 2052 | 011 | 7041 | 2959 | 2 |
| 60 | 2088 | 724 | 3364 | 6636 | 3675 | 904 | 6771 | 3229 | 2996 | 9.984944 | 8052 | 1948 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Taug. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
|  | 77 Deg. |  |  |  | 76 Deg. |  |  |  | 75 Deg. |  |  |  |  |

|  | 15 Deg. |  |  |  | 16 Deg. |  |  |  | 17 Deg. |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ' | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ' |
| 0 | 9.412996 | 9,984944 | 9.428052 | 10.571948 | 9.440338 | 9.982842 | 9.457496 | 10.542504 | 9,465935 | 9,980596 | 9.485339 | 10,514661 | 60 |
| 2 | 3938 | 876 | 9062 | 0938 | 1218 | 769 | 8449 | 1551 | 6761 | 519 | 6242 | 3758 | 58 |
| 4 | 4878 | 808 | 9.430070 | 10,569930 | 2096 | 696 | 9400 | 0600 | 7585 | 442 | 7143 | 2857 | 56 |
| 6 | 5815 | 740 | 1075 | 8925 | 2973 | 624 | 9.460349 | 10.539651 | 8407 | 364 | 8043 | 1957 | 54 |
| 8 | 6751 | 672 | 2079 | 7921 | 3847 | 551 | 1297 | 8703 | 9227 | 286 | 8941 | 1059 | 52 |
| 10 | 7684 | 603 | 3080 | 6920 | 4720 | 477 | 2242 | 7758 | 9.470046 | 208 | 9838 | 0162 | 50 |
| 12 | 8615 | 535 | 4080 | 5920 | 5590 | 404 | 3186 | 6814 | 0863 | 130 | 9.490733 | 10.509267 | 48 |
| 14 | 9544 | 466 | 5078 | 4922 | 6459 | 331 | 4128 | 5872 | 1679 | 052 | 1627 | 8373 | 46 |
| 16 | 9,420470 | 397 | 6073 | 3927 | 7326 | 257 | 5069 | 4931 | 2492 | 9.979973 | 2519 | 7481 | 44 |
| 18 | 1395 | 328 | 7067 | 2933 | 8191 | 183 | 6008 | 3992 | 3304 | 895 | 3410 | 6590 | 42 |
| 20 | 2318 | 259 | 8059 | 1941 | 9054 | 109 | 6945 | 3055 | 4115 | 816 | 4299 | 5701 | 40 |
| 22 | 9,423238 | 9,984190 | 9.439048 | 10,560952 | 9,449915 | 9,982035 | 9.467880 | 10.532120 | 9.474923 | 9,979737 | 9.495186 | 10,504814 | 38 |
| 24 | 4156 | 120 | 9.440036 | 10,559964 | 9.450775 | 9.981961 | 8814 | 1186 | 5730 | 658 | 6073 | 3927 | 36 |
| 26 | 5073 | 050 | 1022 | 8978 | 1632 | 886 | 9746 | 0254 | 6536 | 579 | 6957 | 3043 | 34 |
| 28 | 5987 | 9,983981 | 2006 | 7994 | 2488 | 812 | 9.470676 | 10.529324 | 7340 | 499 | 7841 | 2159 | 32 |
| 30 | 6899 | 911 | 2988 | 7012 | 3342 | 737 | 1605 | 8395 | 8142 | 420 | 8722 | 1278 | 30 |
| 32 | 7809 | 840 | 3968 | 6032 | 4194 | 662 | 2532 | 7468 | 8942 | 340 | 9603 | 0397 | 28 |
| 34 | 8717 | 770 | 4947 | 5053 | 5044 | 587 | 3457 | 6543 | 9741 | 260 | 9.500481 | 10,499519 | 26 |
| 36 | 9623 | 700 | 5923 | 4077 | 5893 | 512 | 4381 | 5619 | 9,480539 | 180 | 1359 | 8641 | 24 |
| 38 | 9.430527 | 629 | 6898 | 3102 | 6739 | 436 | 5303 | 4697 | 1334 | 100 | 2235 | 7765 | 22 |
| 40 | 1429 | 558 | 7870 | 2130 | 7584 | 361 | 6223 | 3777 | 2128 | 019 | 3109 | 6891 | 20 |
| 42 | 9.432329 | 9,983487 | 9.448841 | 10.551159 | 9.458427 | 9,981285 | 9.477142 | 10.522858 | 9,482921 | 9,978939 | 9.503982 | 10,496018 | 18 |
| 44 | 3226 | 416 | 9810 | 0190 | 9268 | 209 | 8059 | 1941 | 3712 | 858 | 4854 | 5146 | 16 |
| 46 | 4122 | 345 | 9.450777 | 10.549223 | 9.460108 | 133 | 8975 | 1025 | 4501 | 777 | 5724 | 4276 | 14 |
| 48 | 5016 | 273 | 1743 | 8257 | 0946 | 057 | 9889 | 0111 | 5289 | 696 | 6593 | 3407 | 12 |
| 50 | 5908 | 202 | 2706 | 7294 | 1782 | 9.980981 | 9.480801 | 10.519199 | 6075 | 615 | 7460 | 2540 | 10 |
| 52 | 6798 | 130 | 3668 | 6332 | 2616 | 904 | 1712 | 8288 | 6860 | 533 | 8326 | 1674 | 8 |
| 54 | 7686 | 058 | 4628 | 5372 | 3448 | 827 | 2621 | 7379 | 7643 | 452 | 9191 | 0809 | 6 |
| 56 | 8572 | 9.982986 | 5586 | 4414 | 4279 | 750 | 3529 | 6471 | 8424 | 370 | 9,510054 | 10.489946 | 4 |
| 58 | 9456 | 914 | 6542 | 3458 | 5108 | 673 | 4435 | 5565 | 9204 | 288 | 0916 | 9084 | 2 |
| 60 | 9.440338 | 842 | 7496 | 2504 | 5935 | 596 | 5339 | 4661 | 9982 | 206 | 1776 | 8224 | 0 |
| ' | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ' |
|  | 74 Deg. |  |  |  | 73 Deg. |  |  |  | 72 Deg. |  |  |  |  |

| | 18 Deg. | | | | 19 Deg. | | | | 20 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9.489982 | 9.978206 | 9.511776 | 10.488224 | 9.512642 | 9.975670 | 9.536972 | 10.463028 | 9.534052 | 9.972986 | 9.561066 | 10.438934 | 60 |
| 2 | 0759 | 124 | 2635 | 7365 | 3375 | 583 | 7792 | 2208 | 4745 | 894 | 1851 | 8149 | 58 |
| 4 | 1535 | 042 | 3493 | 6507 | 4107 | 496 | 8611 | 1389 | 5438 | 802 | 2636 | 7364 | 56 |
| 6 | 2308 | 9.977959 | 4349 | 5651 | 4837 | 408 | 9429 | 0571 | 6129 | 709 | 3419 | 6581 | 54 |
| 8 | 3081 | 877 | 5204 | 4796 | 5566 | 321 | 9.540245 | 10.459755 | 6818 | 617 | 4202 | 5798 | 52 |
| 10 | 3851 | 794 | 6057 | 3943 | 6294 | 233 | 1061 | 8939 | 7507 | 524 | 4983 | 5017 | 50 |
| 12 | 4621 | 711 | 6910 | 3090 | 7020 | 145 | 1875 | 8125 | 8194 | 431 | 5763 | 4237 | 48 |
| 14 | 5388 | 628 | 7761 | 2239 | 7745 | 057 | 2688 | 7312 | 8880 | 338 | 6542 | 3458 | 46 |
| 16 | 6154 | 544 | 8610 | 1390 | 8468 | 9.974969 | 3499 | 6501 | 9565 | 245 | 7320 | 2680 | 44 |
| 18 | 6919 | 461 | 9458 | 0542 | 9190 | 880 | 4310 | 5690 | 9.540249 | 151 | 8098 | 1902 | 42 |
| 20 | 7682 | 377 | 9.520305 | 10.479695 | 9911 | 792 | 5119 | 4881 | 0931 | 058 | 8873 | 1127 | 40 |
| 22 | 9.498444 | 9.977293 | 9.521151 | 10.478849 | 9.520631 | 9.974703 | 9.545928 | 10.454072 | 9.541613 | 9.971964 | 9.569648 | 10.430352 | 38 |
| 24 | 9204 | 209 | 1995 | 8005 | 1349 | 614 | 6735 | 3265 | 2293 | 870 | 9.570422 | 10.429578 | 36 |
| 26 | 9963 | 125 | 2838 | 7162 | 2066 | 525 | 7540 | 2460 | 2971 | 776 | 1195 | 8805 | 34 |
| 28 | 9.500721 | 041 | 3680 | 6320 | 2781 | 436 | 8345 | 1655 | 3649 | 682 | 1967 | 8033 | 32 |
| 30 | 1476 | 9.976957 | 4520 | 5480 | 3495 | 347 | 9149 | 0851 | 4325 | 588 | 2738 | 7262 | 30 |
| 32 | 2231 | 872 | 5359 | 4641 | 4209 | 257 | 9951 | 0049 | 5000 | 493 | 3507 | 6493 | 28 |
| 34 | 2984 | 787 | 6197 | 3803 | 4920 | 167 | 9.550752 | 10.449248 | 5674 | 398 | 4276 | 5724 | 26 |
| 36 | 3735 | 702 | 7033 | 2967 | 5630 | 077 | 1552 | 8448 | 6347 | 303 | 5044 | 4956 | 24 |
| 38 | 4485 | 617 | 7868 | 2132 | 6339 | 9.973987 | 2351 | 7649 | 7019 | 208 | 5810 | 4190 | 22 |
| 40 | 5234 | 532 | 8702 | 1298 | 7046 | 897 | 3149 | 6851 | 7689 | 113 | 6576 | 3424 | 20 |
| 42 | 9.505981 | 9.976446 | 9.529535 | 10.470465 | 9.527753 | 9.973807 | 9.553946 | 10.446054 | 9.548359 | 9.971018 | 9.577341 | 10.422659 | 18 |
| 44 | 6727 | 361 | 9.530366 | 10.469634 | 8458 | 716 | 4741 | 5259 | 9027 | 9.970922 | 8104 | 1896 | 16 |
| 46 | 7471 | 275 | 1196 | 8804 | 9161 | 625 | 5536 | 4464 | 9693 | 827 | 8867 | 1133 | 14 |
| 48 | 8214 | 189 | 2025 | 7975 | 9864 | 535 | 6329 | 3671 | 9.550359 | 731 | 9629 | 0371 | 12 |
| 50 | 8956 | 103 | 2853 | 7147 | 9.530565 | 444 | 7121 | 2879 | 1024 | 635 | 9.580389 | 10.419611 | 10 |
| 52 | 9696 | 017 | 3679 | 6321 | 1265 | 352 | 7913 | 2087 | 1687 | 538 | 1149 | 8851 | 8 |
| 54 | 9.510434 | 9.975930 | 4504 | 5496 | 1963 | 261 | 8702 | 1298 | 2349 | 442 | 1907 | 8093 | 6 |
| 56 | 1172 | 844 | 5328 | 4672 | 2661 | 169 | 9491 | 0509 | 3010 | 345 | 2665 | 7335 | 4 |
| 58 | 1907 | 757 | 6150 | 3850 | 3357 | 078 | 9.560279 | 10.439721 | 3670 | 249 | 3422 | 6578 | 2 |
| 60 | 2642 | 670 | 6972 | 3028 | 4052 | 9.972986 | 1066 | 8934 | 4329 | 152 | 4177 | 5823 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 71 Deg. | | | | 70 Deg. | | | | 69 Deg. | | | | |

| | 21 Deg. | | | | 22 Deg. | | | | 23 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9,554329 | 9.970152 | 9.584177 | 10.415823 | 9.573575 | 9.967166 | 9,606410 | 10.393590 | 9,591878 | 9,964026 | 9.627852 | 10.372148 | 60 |
| 2 | 4987 | 055 | 4932 | 5068 | 4200 | 064 | 7137 | 2863 | 2473 | 9,963919 | 8554 | 1446 | 58 |
| 4 | 5643 | 9.969957 | 5686 | 4314 | 4824 | 9.966961 | 7863 | 2137 | 3067 | 811 | 9255 | 0745 | 56 |
| 6 | 6299 | 860 | 6439 | 3561 | 5447 | 859 | 8588 | 1412 | 3659 | 704 | 9956 | 0044 | 54 |
| 8 | 6953 | 762 | 7190 | 2810 | 6069 | 756 | 9312 | 0688 | 4251 | 596 | 9,630656 | 10.369344 | 52 |
| 10 | 7606 | 665 | 7941 | 2059 | 6689 | 653 | 9,610036 | 10,389964 | 4842 | 488 | 1355 | 8645 | 50 |
| 12 | 8258 | 567 | 8691 | 1309 | 7309 | 550 | 0759 | 9241 | 5432 | 379 | 2053 | 7947 | 48 |
| 14 | 8909 | 469 | 9440 | 0560 | 7927 | 447 | 1480 | 8520 | 6021 | 271 | 2750 | 7250 | 46 |
| 16 | 9558 | 370 | 9.590188 | 10,409812 | 8545 | 344 | 2201 | 7799 | 6609 | 163 | 3447 | 6553 | 44 |
| 18 | 9,560207 | 272 | 0935 | 9065 | 9162 | 246 | 2921 | 7079 | 7196 | 054 | 4143 | 5857 | 42 |
| 20 | 0855 | 173 | 1681 | 8319 | 9777 | 136 | 3641 | 6359 | 7783 | 9,962945 | 4838 | 5162 | 40 |
| 22 | 9,561501 | 9,969075 | 9.592426 | 10,407574 | 9,580392 | 9,966033 | 9,614359 | 10,385641 | 9,598368 | 9.962836 | 9.635532 | 10.364468 | 38 |
| 24 | 2146 | 9,968076 | 3171 | 6829 | 1005 | 9,965929 | 5077 | 4923 | 8952 | 727 | 6226 | 3774 | 36 |
| 26 | 2790 | 877 | 3914 | 6086 | 1618 | 824 | 5793 | 4207 | 9536 | 617 | 6919 | 3081 | 34 |
| 28 | 3433 | 777 | 4656 | 5344 | 2229 | 720 | 6509 | 3491 | 9,600118 | 508 | 7611 | 2389 | 32 |
| 30 | 4075 | 678 | 5398 | 4602 | 2840 | 615 | 7224 | 2776 | 0700 | 398 | 8302 | 1698 | 30 |
| 32 | 4716 | 578 | 6138 | 3862 | 3449 | 511 | 7939 | 2061 | 1280 | 288 | 8992 | 1008 | 28 |
| 34 | 5356 | 479 | 6878 | 3122 | 4058 | 406 | 8652 | 1348 | 1860 | 178 | 9682 | 0318 | 26 |
| 36 | 5995 | 379 | 7616 | 2384 | 4665 | 301 | 9364 | 0636 | 2439 | 067 | 9.640371 | 10.359629 | 24 |
| 38 | 6632 | 278 | 8354 | 1646 | 5272 | 195 | 9.620076 | 10.379924 | 3017 | 9,961957 | 1060 | 8940 | 22 |
| 40 | 7269 | 178 | 9091 | 0909 | 5877 | 090 | 0787 | 9213 | 3594 | 846 | 1747 | 8253 | 20 |
| 42 | 9.567904 | 9.968078 | 9.599827 | 10.400173 | 9,586482 | 9.964984 | 9,621497 | 10.378503 | 9.604170 | 9.961735 | 9,642434 | 10.357566 | 18 |
| 44 | 8539 | 9.967977 | 9.600562 | 10.399438 | 7085 | 879 | 2207 | 7793 | 4745 | 624 | 3120 | 6880 | 16 |
| 46 | 9172 | 876 | 1296 | 8704 | 7688 | 773 | 2915 | 7085 | 5319 | 513 | 3806 | 6194 | 14 |
| 48 | 9804 | 775 | 2029 | 7971 | 8289 | 666 | 3623 | 6377 | 5892 | 402 | 4490 | 5510 | 12 |
| 50 | 9,570435 | 674 | 2761 | 7239 | 8890 | 560 | 4330 | 5670 | 6465 | 290 | 5174 | 4826 | 10 |
| 52 | 1066 | 573 | 3493 | 6507 | 9489 | 454 | 5036 | 4964 | 7036 | 179 | 5857 | 4143 | 8 |
| 54 | 1695 | 471 | 4223 | 5777 | 9,590088 | 347 | 5741 | 4259 | 7607 | 067 | 6540 | 3460 | 6 |
| 56 | 2323 | 370 | 4953 | 5047 | 0686 | 240 | 6445 | 3555 | 8177 | 9,960955 | 7222 | 2778 | 4 |
| 58 | 2950 | 268 | 5682 | 4318 | 1282 | 133 | 7149 | 2851 | 8745 | 843 | 7903 | 2097 | 2 |
| 60 | 3575 | 166 | 6410 | 3590 | 1878 | 026 | 7852 | 2148 | 9313 | 730 | 8583 | 1417 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 68 Deg. | | | | 67 Deg. | | | | 66 Deg. | | | | |

| | 24 Deg. | | | | 25 Deg. | | | | 26 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ' | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ' |
| 0 | 9.609313 | 9.960730 | 9.648583 | 10.351417 | 9.625948 | 9.957276 | 9.668673 | 10.331327 | 9.641842 | 9.953660 | 9.688182 | 10.311818 | 60 |
| 2 | 9880 | 618 | 9263 | 0737 | 6490 | 158 | 9332 | 0668 | 2360 | 537 | 8823 | 1177 | 58 |
| 4 | 9.610447 | 505 | 9942 | 0058 | 7030 | 040 | 9991 | 0009 | 2877 | 413 | 9463 | 0537 | 56 |
| 6 | 1012 | 392 | 9.650620 | 10.349380 | 7570 | 9.956921 | 9.670649 | 10.329351 | 3393 | 290 | 9.690103 | 10.309897 | 54 |
| 8 | 1576 | 279 | 1297 | 8703 | 8109 | 803 | 1306 | 8694 | 3909 | 169 | 0742 | 9258 | 52 |
| 10 | 2140 | 165 | 1974 | 8026 | 8647 | 684 | 1963 | 8037 | 4423 | 042 | 1381 | 8619 | 50 |
| 12 | 2702 | 052 | 2650 | 7350 | 9185 | 566 | 2619 | 7381 | 4936 | 9.952918 | 2019 | 7981 | 48 |
| 14 | 3264 | 9.959988 | 3326 | 6674 | 9721 | 447 | 3274 | 6726 | 5450 | 793 | 2656 | 7344 | 46 |
| 16 | 3825 | 825 | 4000 | 6000 | 9.630257 | 327 | 3929 | 6071 | 5962 | 669 | 3293 | 6707 | 44 |
| 18 | 4385 | 711 | 4674 | 5326 | 0792 | 208 | 4584 | 5416 | 6474 | 544 | 3930 | 6070 | 42 |
| 20 | 4944 | 596 | 5348 | 4652 | 1326 | 089 | 5237 | 4763 | 6984 | 419 | 4566 | 5434 | 40 |
| 22 | 9.615502 | 9.959482 | 9.656020 | 10.343980 | 9.631859 | 9.955969 | 9.675890 | 10.324110 | 9.647494 | 9.952294 | 9.695201 | 10.304799 | 38 |
| 24 | 6060 | 368 | 6692 | 3308 | 2392 | 849 | 6543 | 3457 | 8004 | 168 | 5836 | 4164 | 36 |
| 26 | 6616 | 253 | 7364 | 2636 | 2923 | 729 | 7194 | 2806 | 8512 | 043 | 6470 | 3530 | 34 |
| 28 | 7172 | 138 | 8034 | 1966 | 3454 | 609 | 7846 | 2154 | 9020 | 9.951917 | 7103 | 2897 | 32 |
| 30 | 7727 | 023 | 8704 | 1296 | 3984 | 488 | 8496 | 1504 | 9527 | 791 | 7736 | 2264 | 30 |
| 32 | 8281 | 9.958908 | 9373 | 0627 | 4514 | 368 | 9146 | 0854 | 9.650034 | 665 | 8369 | 1631 | 28 |
| 34 | 8834 | 792 | 9.660042 | 10.339958 | 5042 | 247 | 9795 | 0205 | 0539 | 539 | 9001 | 0999 | 26 |
| 36 | 9386 | 677 | 0710 | 9290 | 5570 | 126 | 9.680444 | 10.319556 | 1044 | 412 | 9632 | 0368 | 24 |
| 38 | 9938 | 561 | 1377 | 8623 | 6097 | 005 | 1092 | 8908 | 1549 | 286 | 9.700263 | 10.299737 | 22 |
| 40 | 9.620488 | 445 | 2043 | 7957 | 6623 | 9.954883 | 1740 | 8260 | 2052 | 159 | 0893 | 9107 | 20 |
| 42 | 9.621038 | 9.958329 | 9.662709 | 10.337291 | 9.637148 | 9.954762 | 9.682387 | 10.317613 | 9.652555 | 9.951032 | 9.701523 | 10.298477 | 18 |
| 44 | 1587 | 213 | 3375 | 6625 | 7673 | 640 | 3033 | 6967 | 3057 | 9.950905 | 2152 | 7848 | 16 |
| 46 | 2135 | 096 | 4039 | 5961 | 8197 | 518 | 3679 | 6321 | 3558 | 778 | 2780 | 7220 | 14 |
| 48 | 2682 | 9.957979 | 4703 | 5297 | 8720 | 396 | 4324 | 5676 | 4059 | 650 | 3409 | 6591 | 12 |
| 50 | 3229 | 863 | 5366 | 4634 | 9242 | 274 | 4968 | 5032 | 4558 | 522 | 4036 | 5964 | 10 |
| 52 | 3774 | 746 | 6029 | 3971 | 9764 | 152 | 5612 | 4388 | 5058 | 394 | 4663 | 5337 | 8 |
| 54 | 4319 | 628 | 6691 | 3309 | 9.640284 | 029 | 6255 | 3745 | 5556 | 266 | 5290 | 4710 | 6 |
| 56 | 4863 | 511 | 7352 | 2648 | 0804 | 9.953906 | 6898 | 3102 | 6054 | 138 | 5916 | 4084 | 4 |
| 58 | 5406 | 393 | 8013 | 1987 | 1324 | 783 | 7540 | 2460 | 6551 | 010 | 6541 | 3459 | 2 |
| 60 | 5948 | 276 | 8672 | 1328 | 1842 | 660 | 8182 | 1818 | 7047 | 9.949881 | 7166 | 2834 | 0 |
| ' | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ' |
| | 65 Deg. | | | | 64 Deg. | | | | 63 Deg. | | | | |

| ′ | 27 Deg. | | | | 28 Deg. | | | | 29 Deg. | | | | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | |
| 0 | 9.657047 | 9.949881 | 9.707166 | 10.292834 | 9.671609 | 9.945935 | 9.725674 | 10.274326 | 9.685571 | 9.941819 | 9.743752 | 10.256248 | 60 |
| 2 | 7542 | 752 | 7790 | 2210 | 2084 | 800 | 6284 | 3716 | 6027 | 679 | 4348 | 5652 | 58 |
| 4 | 8037 | 623 | 8414 | 1586 | 2558 | 666 | 6892 | 3108 | 6482 | 539 | 4943 | 5057 | 56 |
| 6 | 8531 | 494 | 9037 | 0963 | 3032 | 531 | 7501 | 2499 | 6936 | 398 | 5538 | 4462 | 54 |
| 8 | 9025 | 364 | 9660 | 0340 | 3505 | 396 | 8109 | 1891 | 7389 | 258 | 6132 | 3868 | 52 |
| 10 | 9517 | 235 | 9.710282 | 10.289718 | 3977 | 261 | 8716 | 1284 | 7843 | 117 | 6726 | 3274 | 50 |
| 12 | 9.660009 | 105 | 0904 | 9096 | 4448 | 125 | 9323 | 0677 | 8295 | 9.940975 | 7319 | 2681 | 48 |
| 14 | 0501 | 9.948975 | 1525 | 8475 | 4919 | 9.944990 | 9929 | 0071 | 8747 | 834 | 7913 | 2087 | 46 |
| 16 | 0991 | 845 | 2146 | 7854 | 5390 | 854 | 9.730535 | 10.269465 | 9198 | 693 | 8505 | 1495 | 44 |
| 18 | 1481 | 715 | 2766 | 7234 | 5859 | 718 | 1141 | 8859 | 9648 | 551 | 9097 | 0903 | 42 |
| 20 | 1970 | 584 | 3386 | 6614 | 6328 | 582 | 1746 | 8254 | 9.690098 | 409 | 9689 | 0311 | 40 |
| 22 | 9.662459 | 9.948454 | 9.714005 | 10.285995 | 9.676796 | 9.944446 | 9.732351 | 10.267649 | 9.690548 | 9.940267 | 9.750281 | 10.249719 | 38 |
| 24 | 2946 | 323 | 4624 | 5376 | 7264 | 309 | 2955 | 7045 | 0996 | 125 | 0872 | 9128 | 36 |
| 26 | 3433 | 192 | 5242 | 4758 | 7731 | 172 | 3558 | 6442 | 1444 | 9.939982 | 1462 | 8538 | 34 |
| 28 | 3920 | 060 | 5860 | 4140 | 8197 | 096 | 4162 | 5838 | 1892 | 840 | 2052 | 7948 | 32 |
| 30 | 4406 | 9.947929 | 6477 | 3523 | 8663 | 9.943899 | 4764 | 5236 | 2339 | 697 | 2642 | 7358 | 30 |
| 32 | 4891 | 797 | 7093 | 2907 | 9128 | 761 | 5367 | 4633 | 2785 | 554 | 3231 | 6769 | 28 |
| 34 | 5375 | 665 | 7709 | 2291 | 9592 | 624 | 5969 | 4031 | 3231 | 410 | 3820 | 6180 | 26 |
| 36 | 5859 | 533 | 8325 | 1675 | 9.680056 | 486 | 6570 | 3430 | 3676 | 267 | 4409 | 5591 | 24 |
| 38 | 6342 | 401 | 8940 | 1060 | 0519 | 348 | 7171 | 2829 | 4120 | 123 | 4997 | 5003 | 22 |
| 40 | 6824 | 269 | 9555 | 0445 | 0982 | 210 | 7771 | 2229 | 4564 | 9.938980 | 5585 | 4415 | 20 |
| 42 | 9.667305 | 9.947196 | 9.720169 | 10.279831 | 9.681443 | 9.943072 | 9.738371 | 10.261629 | 9.695007 | 9.938836 | 9.756172 | 10.243828 | 18 |
| 44 | 7786 | 004 | 0783 | 9217 | 1905 | 9.942934 | 8971 | 1029 | 5450 | 691 | 6759 | 3241 | 16 |
| 46 | 8267 | 9.946871 | 1396 | 8604 | 2365 | 795 | 9570 | 0430 | 5892 | 547 | 7345 | 2655 | 14 |
| 48 | 8746 | 738 | 2009 | 7991 | 2825 | 656 | 9.740169 | 10.259831 | 6334 | 402 | 7931 | 2069 | 12 |
| 50 | 9225 | 604 | 2621 | 7379 | 3284 | 517 | 0767 | 9233 | 6775 | 258 | 8517 | 1483 | 10 |
| 52 | 9703 | 471 | 3232 | 6768 | 3743 | 378 | 1365 | 8635 | 7215 | 113 | 9102 | 0898 | 8 |
| 54 | 9.670181 | 337 | 3844 | 6156 | 4201 | 239 | 1962 | 8038 | 7654 | 9.937967 | 9687 | 0313 | 6 |
| 56 | 0658 | 203 | 4454 | 5546 | 4658 | 099 | 2559 | 7441 | 8094 | 822 | 9.760272 | 10.239728 | 4 |
| 58 | 1134 | 069 | 5065 | 4935 | 5115 | 9.941959 | 3156 | 6844 | 8532 | 676 | 0856 | 9144 | 2 |
| 60 | 1609 | 9.945935 | 5674 | 4326 | 5571 | 819 | 3752 | 6248 | 8970 | 531 | 1439 | 8561 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 62 Deg. | | | | 61 Deg. | | | | 60 Deg. | | | | |

| | 30 Deg. | | | | 31 Deg. | | | | 32 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9.698970 | 9.937531 | 9.761439 | 10.238561 | 9.711839 | 9.933066 | 9.778774 | 10.221226 | 9.724210 | 9.928420 | 9.795789 | 10.204211 | 60 |
| 2 | 9407 | 385 | 2023 | 7977 | 2260 | 9.932914 | 9346 | 0654 | 4614 | 263 | 6351 | 3649 | 58 |
| 4 | 9844 | 238 | 2606 | 7394 | 2679 | 762 | 9918 | 0082 | 5017 | 104 | 6913 | 3087 | 56 |
| 6 | 9.700280 | 092 | 3188 | 6812 | 3098 | 609 | 9.780489 | 10.219511 | 5420 | 9.927946 | 7475 | 2525 | 54 |
| 8 | 0716 | 9.936946 | 3770 | 6230 | 3517 | 457 | 1060 | 8940 | 5823 | 787 | 8036 | 1964 | 52 |
| 10 | 1151 | 799 | 4352 | 5648 | 3935 | 304 | 1631 | 8369 | 6225 | 629 | 8596 | 1404 | 50 |
| 12 | 1585 | 652 | 4933 | 5067 | 4352 | 151 | 2201 | 7799 | 6626 | 470 | 9157 | 0843 | 48 |
| 14 | 2019 | 505 | 5514 | 4486 | 4769 | 9.931998 | 2771 | 7229 | 7027 | 310 | 9717 | 0283 | 46 |
| 16 | 2452 | 357 | 6095 | 3905 | 5186 | 845 | 3341 | 6658 | 7428 | 151 | 9.800277 | 10.199723 | 44 |
| 18 | 2885 | 210 | 6675 | 3325 | 5602 | 691 | 3910 | 6090 | 7828 | 9.926991 | 0836 | 9164 | 42 |
| 20 | 3317 | 062 | 7255 | 2745 | 6017 | 537 | 4479 | 5521 | 8227 | 831 | 1396 | 8604 | 40 |
| 22 | 9.703749 | 9.935914 | 9.767834 | 10.232166 | 9.716432 | 9.931383 | 9.785048 | 10.214952 | 9.728626 | 9.926671 | 9.801955 | 10.198045 | 38 |
| 24 | 4179 | 766 | 8414 | 1586 | 6846 | 229 | 5616 | 4384 | 9024 | 511 | 2513 | 7487 | 36 |
| 26 | 4610 | 618 | 8992 | 1008 | 7259 | 075 | 6184 | 3816 | 9422 | 351 | 3072 | 6928 | 34 |
| 28 | 5040 | 469 | 9570 | 0430 | 7673 | 9.930921 | 6752 | 3248 | 9820 | 190 | 3630 | 6370 | 32 |
| 30 | 5469 | 320 | 9.770148 | 10.229852 | 8085 | 766 | 7319 | 2681 | 9.730217 | 029 | 4187 | 5813 | 30 |
| 32 | 5898 | 171 | 0726 | 9274 | 8497 | 611 | 7886 | 2114 | 0613 | 9.925868 | 4745 | 5255 | 28 |
| 34 | 6326 | 022 | 1303 | 8697 | 8909 | 456 | 8453 | 1547 | 1009 | 707 | 5302 | 4698 | 26 |
| 36 | 6753 | 9.934873 | 1880 | 8120 | 9320 | 300 | 9019 | 0981 | 1404 | 545 | 5859 | 4141 | 24 |
| 38 | 7180 | 723 | 2457 | 7543 | 9730 | 145 | 9585 | 0415 | 1799 | 384 | 6415 | 3585 | 22 |
| 40 | 7606 | 574 | 3033 | 6967 | 9.720140 | 9.929989 | 9.790151 | 10.209849 | 2193 | 222 | 6971 | 3029 | 20 |
| 42 | 9.708032 | 9.934424 | 9.773608 | 10.226392 | 9.720549 | 9.929833 | 9.790716 | 10.209284 | 9.732587 | 9.925060 | 9.807527 | 10.192473 | 18 |
| 44 | 8458 | 274 | 4184 | 5816 | 0958 | 677 | 1281 | 8719 | 2980 | 9.924897 | 8083 | 1917 | 16 |
| 46 | 8882 | 123 | 4759 | 5241 | 1366 | 521 | 1846 | 8154 | 3373 | 735 | 8638 | 1362 | 14 |
| 48 | 9306 | 9.933973 | 5333 | 4667 | 1774 | 364 | 2410 | 7590 | 3765 | 572 | 9193 | 0807 | 12 |
| 50 | 9730 | 822 | 5908 | 4092 | 2181 | 207 | 2974 | 7026 | 4157 | 409 | 9748 | 0252 | 10 |
| 52 | 9.710153 | 671 | 6482 | 3518 | 2588 | 050 | 3538 | 6462 | 4549 | 246 | 9.810302 | 10.189698 | 8 |
| 54 | 0575 | 520 | 7055 | 2945 | 2994 | 9.928893 | 4101 | 5899 | 4939 | 083 | 0857 | 9143 | 6 |
| 56 | 0997 | 369 | 7628 | 2372 | 3400 | 736 | 4661 | 5336 | 5330 | 9.923919 | 1410 | 8590 | 4 |
| 58 | 1419 | 217 | 8201 | 1799 | 3805 | 578 | 5227 | 4773 | 5719 | 755 | 1964 | 8036 | 2 |
| 60 | 1839 | 066 | 8774 | 1226 | 4210 | 420 | 5789 | 4211 | 6109 | 591 | 2517 | 7483 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine | Cotang. | Tang | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 59 Deg. | | | | 58 Deg. | | | | 57 Deg. | | | | |

| | 33 Deg. | | | | 34 Deg. | | | | 35 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9,736109 | 9,923591 | 9.812517 | 10.187483 | 9.747562 | 9,918574 | 9.828987 | 10,171013 | 9.758591 | 9,913365 | 9,845227 | 10,154773 | 60 |
| 2 | 6498 | 427 | 3070 | 6930 | 7936 | 404 | 9532 | 0468 | 8952 | 187 | 5764 | 4236 | 58 |
| 4 | 6886 | 263 | 3623 | 6377 | 8310 | 233 | 9,830077 | 10,169923 | 9312 | 010 | 6302 | 3698 | 56 |
| 6 | 7274 | 098 | 4175 | 5825 | 8683 | 062 | 0621 | 9379 | 9672 | 9,912833 | 6839 | 3161 | 54 |
| 8 | 7661 | 9,922933 | 4728 | 5272 | 9056 | 9,917891 | 1165 | 8835 | 9.760031 | 655 | 7376 | 2624 | 52 |
| 10 | 8048 | 768 | 5279 | 4721 | 9429 | 719 | 1709 | 8291 | 0390 | 477 | 7913 | 2087 | 50 |
| 12 | 8434 | 603 | 5831 | 4169 | 9801 | 548 | 2253 | 7747 | 0748 | 299 | 8449 | 1551 | 48 |
| 14 | 8820 | 438 | 6382 | 3618 | 9.750172 | 376 | 2796 | 7204 | 1106 | 121 | 8986 | 1014 | 46 |
| 16 | 9206 | 272 | 6933 | 3067 | 0543 | 204 | 3339 | 6661 | 1464 | 9,911942 | 9522 | 0478 | 44 |
| 18 | 9590 | 106 | 7484 | 2516 | 0914 | 032 | 3882 | 6118 | 1821 | 763 | 9,850058 | 10,149942 | 42 |
| 20 | 9975 | 9.921940 | 8035 | 1965 | 1284 | 9.916859 | 4425 | 5575 | 2177 | 584 | 0593 | 9407 | 40 |
| 22 | 9,740359 | 9,921774 | 9,818585 | 10.181415 | 9.751654 | 9,916687 | 9.834967 | 10,165033 | 9.762534 | 9.911405 | 9,851129 | 10.148871 | 38 |
| 24 | 0742 | 607 | 9135 | 0865 | 2023 | 514 | 5509 | 4491 | 2889 | 226 | 1664 | 8336 | 36 |
| 26 | 1125 | 441 | 9684 | 0316 | 2392 | 341 | 6051 | 3949 | 3245 | 046 | 2199 | 7801 | 34 |
| 28 | 1508 | 274 | 9,820234 | 10,179766 | 2760 | 167 | 6593 | 3407 | 3600 | 9.910866 | 2733 | 7267 | 32 |
| 30 | 1889 | 107 | 0783 | 9217 | 3128 | 9.915994 | 7134 | 2866 | 3954 | 686 | 3268 | 6732 | 30 |
| 32 | 2271 | 9.920939 | 1332 | 8668 | 3495 | 820 | 7675 | 2325 | 4308 | 506 | 3802 | 6198 | 28 |
| 34 | 2652 | 772 | 1880 | 8120 | 3862 | 646 | 8216 | 1784 | 4662 | 325 | 4336 | 5664 | 26 |
| 36 | 3033 | 604 | 2429 | 7571 | 4229 | 472 | 8757 | 1243 | 5015 | 144 | 4870 | 5130 | 24 |
| 38 | 3413 | 436 | 2977 | 7023 | 4595 | 297 | 9297 | 0703 | 5367 | 9,909963 | 5404 | 4596 | 22 |
| 40 | 3792 | 268 | 3524 | 6476 | 4960 | 123 | 9838 | 0162 | 5720 | 782 | 5938 | 4062 | 20 |
| 42 | 9,744171 | 9,920099 | 9,824072 | 10.175928 | 9.755326 | 9,914948 | 9.840378 | 10,159622 | 9.766072 | 9,909601 | 9,856471 | 10,143529 | 18 |
| 44 | 4550 | 9,919931 | 4619 | 5381 | 5690 | 773 | 0917 | 9083 | 6423 | 419 | 7004 | 2996 | 16 |
| 46 | 4928 | 762 | 5166 | 4834 | 6054 | 598 | 1457 | 8543 | 6774 | 237 | 7537 | 2463 | 14 |
| 48 | 5306 | 593 | 5713 | 4287 | 6418 | 422 | 1996 | 8004 | 7124 | 055 | 8069 | 1931 | 12 |
| 50 | 5683 | 424 | 6259 | 3741 | 6782 | 246 | 2535 | 7465 | 7475 | 9,908873 | 8602 | 1398 | 10 |
| 52 | 6060 | 254 | 6805 | 3195 | 7144 | 070 | 3074 | 6926 | 7824 | 690 | 9134 | 0866 | 8 |
| 54 | 6436 | 085 | 7351 | 2649 | 7507 | 9.913894 | 3612 | 6388 | 8173 | 507 | 9666 | 0334 | 6 |
| 56 | 6812 | 9,918915 | 7897 | 2103 | 7869 | 718 | 4151 | 5849 | 8522 | 324 | 9,860198 | 10,139802 | 4 |
| 58 | 7187 | 745 | 8442 | 1558 | 8230 | 541 | 4689 | 5311 | 8871 | 141 | 0730 | 9270 | 2 |
| 60 | 7562 | 574 | 8987 | 1013 | 8591 | 365 | 5227 | 4773 | 9219 | 9,907958 | 1261 | 8739 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 56 Deg. | | | | 55 Deg. | | | | 54 Deg. | | | | |

| | 36 Deg. | | | | 37 Deg. | | | | 38 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9.769219 | 9.907958 | 9.861261 | 10.138739 | 9.779163 | 9.902349 | 9.877114 | 10.122886 | 9.789342 | 9.896532 | 9.892810 | 10.107190 | 60 |
| 2 | 9566 | 774 | 1792 | 8208 | 9798 | 158 | 7640 | 2360 | 9665 | 335 | 3331 | 6669 | 58 |
| 4 | 9913 | 590 | 2323 | 7677 | 9.780133 | 9.901967 | 8165 | 1835 | 9988 | 137 | 3851 | 6149 | 56 |
| 6 | 9.770260 | 406 | 2854 | 7146 | 0467 | 776 | 8691 | 1309 | 9.790310 | 9.895939 | 4371 | 5629 | 54 |
| 8 | 0606 | 222 | 3385 | 6615 | 0801 | 585 | 9216 | 0784 | 0632 | 741 | 4892 | 5108 | 52 |
| 10 | 0952 | 037 | 3915 | 6085 | 1134 | 394 | 9741 | 0259 | 0954 | 542 | 5412 | 4588 | 50 |
| 12 | 1298 | 9.906852 | 4445 | 5555 | 1468 | 202 | 9.880265 | 10.119735 | 1275 | 343 | 5932 | 4068 | 48 |
| 14 | 1643 | 667 | 4975 | 5025 | 1800 | 010 | 0790 | 9210 | 1596 | 145 | 6452 | 3548 | 46 |
| 16 | 1987 | 482 | 5505 | 4495 | 2132 | 9.900818 | 1314 | 8686 | 1917 | 9.894945 | 6971 | 3029 | 44 |
| 18 | 2331 | 296 | 6035 | 3965 | 2464 | 626 | 1839 | 8161 | 2237 | 746 | 7491 | 2509 | 42 |
| 20 | 2675 | 111 | 6564 | 3436 | 2796 | 433 | 2363 | 7637 | 2557 | 546 | 8010 | 1990 | 40 |
| 22 | 9.773018 | 9.905925 | 9.867094 | 10.132906 | 9.783127 | 9.900240 | 9.882887 | 10.117113 | 9.792876 | 9.894346 | 9.898530 | 10.101470 | 38 |
| 24 | 3361 | 739 | 7623 | 2377 | 3458 | 047 | 3410 | 6590 | 3195 | 146 | 9049 | 0951 | 36 |
| 26 | 3704 | 552 | 8152 | 1848 | 3788 | 9.899854 | 3934 | 6066 | 3514 | 9.893946 | 9568 | 0432 | 34 |
| 28 | 4046 | 366 | 8680 | 1320 | 4118 | 660 | 4457 | 5543 | 3832 | 745 | 9.900086 | 10.099914 | 32 |
| 30 | 4388 | 179 | 9209 | 0791 | 4447 | 467 | 4980 | 5020 | 4150 | 514 | 0605 | 9395 | 30 |
| 32 | 4729 | 9.904992 | 9737 | 0263 | 4776 | 273 | 5503 | 4497 | 4467 | 343 | 1124 | 8876 | 28 |
| 34 | 5070 | 804 | 9.870265 | 10.129735 | 5105 | 078 | 6026 | 3974 | 4784 | 142 | 1642 | 8358 | 26 |
| 36 | 5410 | 617 | 0793 | 9207 | 5433 | 9.898884 | 6549 | 3451 | 5101 | 9.892940 | 2160 | 7840 | 24 |
| 38 | 5750 | 429 | 1321 | 8679 | 5761 | 689 | 7072 | 2928 | 5417 | 739 | 2679 | 7321 | 22 |
| 40 | 6090 | 241 | 1849 | 8151 | 6089 | 494 | 7594 | 2406 | 5733 | 536 | 3197 | 6803 | 20 |
| 42 | 9.776429 | 9.904053 | 9.872376 | 10.127624 | 9.786416 | 9.898299 | 9.888116 | 10.111884 | 9.796049 | 9.892334 | 9.903714 | 10.096286 | 18 |
| 44 | 6768 | 9.903864 | 2903 | 7097 | 6742 | 104 | 8639 | 1361 | 6364 | 132 | 4232 | 5768 | 16 |
| 46 | 7106 | 676 | 3430 | 6570 | 7069 | 9.897908 | 9160 | 0840 | 6679 | 9.891929 | 4750 | 5250 | 14 |
| 48 | 7444 | 487 | 3957 | 6043 | 7395 | 712 | 9682 | 0318 | 6993 | 726 | 5267 | 4733 | 12 |
| 50 | 7781 | 298 | 4484 | 5516 | 7720 | 516 | 9.890204 | 10.109796 | 7307 | 523 | 5784 | 4216 | 10 |
| 52 | 8119 | 108 | 5010 | 4990 | 8045 | 320 | 0725 | 9275 | 7621 | 319 | 6302 | 3698 | 8 |
| 54 | 8455 | 9.902919 | 5536 | 4464 | 8370 | 123 | 1247 | 8753 | 7934 | 115 | 6819 | 3181 | 6 |
| 56 | 8792 | 729 | 6063 | 3937 | 8694 | 9.896926 | 1768 | 8232 | 8247 | 9.890911 | 7336 | 2664 | 4 |
| 58 | 9128 | 539 | 6589 | 3411 | 9018 | 729 | 2289 | 7711 | 8560 | 707 | 7852 | 2148 | 2 |
| 60 | 9463 | 349 | 7114 | 2886 | 9342 | 532 | 2810 | 7190 | 8872 | 503 | 8369 | 1631 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 53 Deg. | | | | 52 Deg. | | | | 51 Deg. | | | | |

| | 39 Deg. | | | | 40 Deg. | | | | 41 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9.798872 | 9.890503 | 9.908369 | 10.091631 | 9.808067 | 9.884254 | 9.923813 | 10.076187 | 9.816943 | 9.877780 | 9.939163 | 10.060837 | 60 |
| 2 | 9181 | 298 | 8886 | 1114 | 8368 | 042 | 4327 | 5673 | 7233 | 560 | 9673 | 0327 | 58 |
| 4 | 9495 | 093 | 9402 | 0598 | 8669 | 9.883829 | 4840 | 5160 | 7524 | 340 | 9.940183 | 10.059817 | 56 |
| 6 | 9806 | 9.889888 | 9918 | 0082 | 8969 | 617 | 5352 | 4648 | 7813 | 120 | 0694 | 9306 | 54 |
| 8 | 9.800117 | 682 | 9.910435 | 10.089565 | 9269 | 404 | 5865 | 4135 | 8103 | 9.876899 | 1204 | 8796 | 52 |
| 10 | 0427 | 477 | 0951 | 9049 | 9569 | 191 | 6378 | 3622 | 8392 | 678 | 1714 | 8286 | 50 |
| 12 | 0737 | 271 | 1467 | 8533 | 9868 | 9.882977 | 6890 | 3110 | 8681 | 457 | 2223 | 7777 | 48 |
| 14 | 1047 | 064 | 1982 | 8018 | 9.810167 | 764 | 7403 | 2597 | 8969 | 236 | 2733 | 7267 | 46 |
| 16 | 1356 | 9.888858 | 2498 | 7502 | 0465 | 550 | 7915 | 2085 | 9257 | 014 | 3243 | 6757 | 44 |
| 18 | 1665 | 651 | 3014 | 6986 | 0763 | 336 | 8427 | 1573 | 9545 | 9.875793 | 3752 | 6248 | 42 |
| 20 | 1973 | 444 | 3529 | 6471 | 1061 | 121 | 8940 | 1060 | 9832 | 571 | 4262 | 5738 | 40 |
| 22 | 9.802282 | 9.888237 | 9.914044 | 10.085956 | 9.811358 | 9.881907 | 9.929452 | 10.070548 | 9.820120 | 9.875348 | 9.944771 | 10.055229 | 38 |
| 24 | 2589 | 030 | 4560 | 5440 | 1655 | 692 | 9964 | 0036 | 0406 | 126 | 5281 | 4719 | 36 |
| 26 | 2897 | 9.887822 | 5075 | 4925 | 1952 | 477 | 9.930475 | 10.069525 | 0693 | 9.874903 | 5790 | 4210 | 34 |
| 28 | 3204 | 614 | 5590 | 4410 | 2248 | 261 | 0987 | 9013 | 0979 | 680 | 6299 | 3701 | 32 |
| 30 | 3511 | 406 | 6104 | 3896 | 2544 | 046 | 1499 | 8501 | 1265 | 456 | 6808 | 3192 | 30 |
| 32 | 3817 | 198 | 6619 | 3381 | 2840 | 9.880830 | 2010 | 7990 | 1550 | 232 | 7318 | 2682 | 28 |
| 34 | 4123 | 9.886989 | 7134 | 2866 | 3135 | 613 | 2522 | 7478 | 1835 | 009 | 7826 | 2174 | 26 |
| 36 | 4428 | 780 | 7648 | 2352 | 3430 | 397 | 3033 | 6967 | 2120 | 9.873784 | 8336 | 1664 | 24 |
| 38 | 4734 | 571 | 8163 | 1837 | 3725 | 180 | 3545 | 6455 | 2404 | 560 | 8844 | 1156 | 22 |
| 40 | 5039 | 362 | 8677 | 1323 | 4019 | 9.879963 | 4056 | 5944 | 2688 | 335 | 9353 | 0647 | 20 |
| 42 | 9.805343 | 9.886152 | 9.919191 | 10.080809 | 9.814313 | 9.879746 | 9.934567 | 10.065433 | 9.822972 | 9.873110 | 9.949862 | 10.050138 | 18 |
| 44 | 5647 | 9.885942 | 9705 | 0295 | 4607 | 529 | 5078 | 4922 | 3255 | 9.872885 | 9.950370 | 10.049630 | 16 |
| 46 | 5951 | 732 | 9.920219 | 10.079781 | 4900 | 311 | 5589 | 4411 | 3539 | 659 | 0879 | 9121 | 14 |
| 48 | 6254 | 522 | 0733 | 9267 | 5193 | 093 | 6100 | 3900 | 3821 | 434 | 1388 | 8612 | 12 |
| 50 | 6557 | 311 | 1247 | 8753 | 5485 | 9.878875 | 6610 | 3390 | 4104 | 208 | 1896 | 8104 | 10 |
| 52 | 6860 | 100 | 1760 | 8240 | 5778 | 656 | 7121 | 2879 | 4386 | 9.871981 | 2405 | 7595 | 8 |
| 54 | 7163 | 9.884889 | 2274 | 7726 | 6069 | 438 | 7632 | 2368 | 4668 | 755 | 2913 | 7087 | 6 |
| 56 | 7465 | 677 | 2787 | 7213 | 6361 | 219 | 8142 | 1858 | 4949 | 528 | 3421 | 6579 | 4 |
| 58 | 7766 | 466 | 3300 | 6700 | 6652 | 9.877999 | 8653 | 1347 | 5230 | 301 | 3929 | 6071 | 2 |
| 60 | 8067 | 254 | 3813 | 6187 | 6943 | 780 | 9163 | 0837 | 5511 | 073 | 4437 | 5563 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 50 Deg. | | | | 49 Deg. | | | | 48 Deg. | | | | |

| | 42 Deg. | | | | 43 Deg. | | | | 44 Deg. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | Sine. | Cosine. | Tang. | Cotang. | ′ |
| 0 | 9.825511 | 9.871073 | 9.954437 | 10.045563 | 9.833783 | 9.864127 | 9.969656 | 10.030344 | 9.841771 | 9.856934 | 9.984837 | 10.015163 | 60 |
| 2 | 5791 | 9.870846 | 4945 | 5055 | 4054 | 9.863892 | 9.970162 | 10.029838 | 2033 | 690 | 5343 | 4657 | 58 |
| 4 | 6071 | 618 | 5454 | 4546 | 4325 | 656 | 0669 | 9331 | 2294 | 446 | 5848 | 4152 | 56 |
| 6 | 6351 | 390 | 5961 | 4039 | 4595 | 419 | 1175 | 8825 | 2555 | 201 | 6354 | 3646 | 54 |
| 8 | 6631 | 161 | 6469 | 3531 | 4865 | 183 | 1682 | 8318 | 2815 | 9.855056 | 6860 | 3140 | 52 |
| 10 | 6910 | 9.869933 | 6977 | 3023 | 5134 | 9.862946 | 2188 | 7812 | 3076 | 711 | 7365 | 2635 | 50 |
| 12 | 7189 | 704 | 7485 | 2515 | 5403 | 709 | 2694 | 7306 | 3336 | 465 | 7871 | 2129 | 48 |
| 14 | 7467 | 474 | 7993 | 2007 | 5672 | 471 | 3201 | 6799 | 3595 | 219 | 8376 | 1624 | 46 |
| 16 | 7745 | 245 | 8500 | 1500 | 5941 | 234 | 3707 | 6293 | 3855 | 9.854973 | 8882 | 1118 | 44 |
| 18 | 8023 | 015 | 9008 | 0992 | 6209 | 9.861996 | 4213 | 5787 | 4114 | 727 | 9387 | 0613 | 42 |
| 20 | 8301 | 9.868785 | 9516 | 0484 | 6477 | 758 | 4719 | 5281 | 4372 | 480 | 9893 | 0107 | 40 |
| 22 | 9.828578 | 9.868555 | 9.960023 | 10.039977 | 9.836745 | 9.861519 | 9.975226 | 10.024774 | 9.844631 | 9.854233 | 9.990398 | 10.009602 | 38 |
| 24 | 8855 | 324 | 0531 | 9469 | 7012 | 280 | 5732 | 4268 | 4889 | 9.853986 | 0903 | 9097 | 36 |
| 26 | 9131 | 093 | 1038 | 8962 | 7279 | 041 | 6238 | 3762 | 5147 | 738 | 1409 | 8591 | 34 |
| 28 | 9407 | 9.867862 | 1545 | 8455 | 7546 | 9.860802 | 6744 | 3256 | 5405 | 490 | 1914 | 8086 | 32 |
| 30 | 9683 | 631 | 2052 | 7948 | 7812 | 562 | 7250 | 2750 | 5662 | 242 | 2420 | 7580 | 30 |
| 32 | 9959 | 399 | 2560 | 7440 | 8078 | 322 | 7756 | 2244 | 5919 | 9.852994 | 2925 | 7075 | 28 |
| 34 | 9.830234 | 167 | 3067 | 6933 | 8344 | 082 | 8262 | 1738 | 6175 | 745 | 3430 | 6570 | 26 |
| 36 | 0509 | 9.866935 | 3574 | 6426 | 8610 | 9.859842 | 8768 | 1232 | 6432 | 496 | 3936 | 6064 | 24 |
| 38 | 0784 | 703 | 4081 | 5919 | 8875 | 601 | 9274 | 0726 | 6688 | 247 | 4441 | 5559 | 22 |
| 40 | 1058 | 470 | 4588 | 5412 | 9140 | 360 | 9780 | 0220 | 6944 | 9.851997 | 4947 | 5053 | 20 |
| 42 | 9.831332 | 9.866237 | 9.965095 | 10.034905 | 9.839404 | 9.859119 | 9.980286 | 10.019714 | 9.847199 | 9.851747 | 9.995452 | 10.004548 | 18 |
| 44 | 1606 | 004 | 5602 | 4398 | 9668 | 9.858877 | 0791 | 9209 | 7454 | 497 | 5957 | 4043 | 16 |
| 46 | 1879 | 9.865770 | 6109 | 3891 | 9932 | 635 | 1297 | 8703 | 7709 | 246 | 6463 | 3537 | 14 |
| 48 | 2152 | 536 | 6616 | 3384 | 9.840196 | 393 | 1803 | 8197 | 7964 | 9.850996 | 6968 | 3032 | 12 |
| 50 | 2425 | 302 | 7123 | 2877 | 0459 | 151 | 2309 | 7691 | 8218 | 745 | 7473 | 2527 | 10 |
| 52 | 2697 | 068 | 7629 | 2371 | 0722 | 9.857908 | 2814 | 7186 | 8472 | 493 | 7979 | 2021 | 8 |
| 54 | 2969 | 9.864833 | 8136 | 1864 | 0985 | 665 | 3320 | 6680 | 8726 | 242 | 8184 | 1516 | 6 |
| 56 | 3241 | 598 | 8643 | 1357 | 1247 | 422 | 3826 | 6174 | 8979 | 9.849990 | 8989 | 1011 | 4 |
| 58 | 3512 | 363 | 9149 | 0851 | 1509 | 178 | 4331 | 5669 | 9232 | 738 | 9495 | 0505 | 2 |
| 60 | 3783 | 127 | 9656 | 0344 | 1771 | 9.856934 | 4837 | 5163 | 9485 | 485 | 10.00000 | 0000 | 0 |
| ′ | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | Cosine. | Sine. | Cotang. | Tang. | ′ |
| | 47 Deg. | | | | 46 Deg. | | | | 45 Deg. | | | | |

# TABLE III.

*Table of useful numbers with their logarithms.—(Babbage.)*

| *Constants.* | | *Log.* | *Ar. comp Log.* |
|---|---|---|---|
| Diameter = 1, circumference = $\pi$ | 3.1415927 | 0.4971499 | 9.5028501 |
| ———— area of circle = $\frac{\pi}{4}$ | 0.7853982 | 9.8950899 | 0.1049101 |
| ———— content of sphere = $\frac{\pi}{6}$ | 0.5235988 | 9.7189986 | 0.2810014 |
| $\sqrt{\pi}$ | 1.7724539 | 0.2485750 | 9.7514250 |
| $\pi^2$ | 9.8696044 | 0.9942997 | 9.0057003 |
| Hyp. log. $\pi$ | 1.1447299 | 0.0587030 | 9.9412970 |
| Length of arc 1″ = sin. 1″ | .000004848 | 4.6855749 | 5.3144251 |
| ———— 2″ = sin. 2″ | .000009696 | 4.9866049 | 5.0133951 |
| ———— 3″ = sin. 3″ | .000014544 | 5.1626961 | 4.8373039 |
| ———— 1′ = sin. 1′ | .000290888 | 6.4637261 | 3.5362739 |
| ———— 1° ———— | .017453293 | 8.2418774 | 1.7581226 |
| Sin. 1° | .017452406 | 8.2418553 | 1.7581447 |
| 360° expressed in seconds | 1296000 | 6.1126050 | 3.8873950 |
| Radius reduced to seconds | 206264.8 | 5.3144251 | 4.6855749 |
| Radius reduced to minutes | 3437.74677 | 3.5362739 | 6.4637261 |
| Radius reduced to degrees | 57.295780 | 1.7581226 | 8.2418774 |
| Number whose hyp. log. is unity | 2.718281829 | 0.4342945 | 9.5657055 |
| Modulus of common logs. | .434294482 | 9.6377843 | 0.3622156 |
| French toise = in metres | 1.949040 | 0.2898200 | 9.7101800 |
| ———— = in English yards | 2.1315308 | 0.3286916 | 9.6713084 |
| ———— = in English feet | 6.3945925 | 0.8058129 | 9.1941871 |
| French metre = in English yards | 1.0936331 | 0.0388716 | 9.9611284 |
| ———— = in English feet | 3.2808992 | 0.5159929 | 9.4840071 |
| ———— = in English inches | 39.37079 | 1.5951741 | 8.4048259 |
| French foot = in English feet | 1.0657654 | 0.0276616 | 9.9723384 |
| French are = in English acres | .02471143 | 8.3928979 | 1.6071021 |
| French gramme = imperial lbs. Troy | .00268098 | 7.4282928 | 2.5717072 |
| French gramme = in imperial lbs. avoirdupois | .00220606 | 7.3436173 | 2.6563827 |
| French litre in imperial gallons | .22009687 | 9.3426139 | 0.6573861 |

| *Constants.* | | *Log.* | *Ar. comp Log.* |
|---|---|---|---|
| Centes. degree = in sex. degrees | .9 | 9.9542425 | 0.0457575 |
| —— minute = —— minutes | .54 | 9.7323938 | 0.2676062 |
| —— second = —— seconds | .324 | 9.5105450 | 0.4894550 |
| Mean circumference of the earth in miles | 24856 | 4.3954312 | 5.6045688 |
| Diameter | 7912 | 3.8982863 | 6.1017137 |
| Radius of Equator | 3962.349 | 3 5979528 | 6.4020472 |
| Semipolar axis | 3949.669 | 3.5965608 | 6.4034392 |
| Difference | 12.680 | 1.1031193 | 8.8968807 |
| Circumference of the Equator | 24896 | 4.3961296 | 5 6038704 |
| Geographical mile in feet | 6075.6 | 3.7835892 | 6.2164108 |
| 24 hours expressed in seconds | 86400 | 4.9365137 | 5.0634863 |
| Diurnal acceleration of stars in mean solar seconds | 235.9093 | 2.3727451 | 7.6272549 |
| Sidereal day (23*h*. 56*m*. 4.09*s*.) in mean solar days | .99726967 | 9.9988126 | 0.0011874 |
| Solar mean day (24*h*. 3*m*. 56.5554*s*) in sidereal days | 1.00273791 | 0.0011874 | 9.9988126 |
| Sidereal revolution of earth in mean solar days | 365.25636 | 2.5625978 | 7.4374022 |
| Tropical revolution of earth in mean solar days | 365.24224 | 2.5625810 | 7.4374190 |
| Cubic inch of distilled water in grains (Bar. 30 in. Fah. Therm. 62°) | 252.458 | 2.4021891 | 7 5978109 |
| An ounce of water in cubic inches | 1.73298 | 0.2387924 | 9.7612076 |
| Cubic inches in the Imperial gallon | 277.276 | 2.4429124 | 7.5570876 |
| Length of seconds pendulum at London | 39.1393 | 1.5926130 | 8.4073870 |
| Force of gravity at London in feet | 32.19084 | 1.5077222 | 8.4922778 |

787 3

# EXPLANATION AND USE OF THE TABLES.

## TABLE I.

1. *To find the log. of any given number.*

*If the given number be under* 100, its log. is found in the first page of the Table, immediately opposite to it. Thus log. 66 is 1.819544.

*If the No. consist of three figures*, find the given number in the column under N, and opposite to it in the next column, marked 0 at the top, is the decimal part of the logarithm required, before which put an index, which is always less by unity than the number of integral figures in the natural number. Thus log. 448 is 2.651278. If the number should consist wholly of decimals, the index of the log. is then *negative*, and it is indicated by the place occupied by the first figure in the decimal. Thus the index of the log. of .04 is — 2; of .006 is — 3. But to avoid the confusion that might arise by the addition and subtraction of negative indices, it is customary to take the arithmetical complement of the negative indices, and to consider these complements as positive; thus 8 is put as the index of .04; 7 as the index of .006.

*If the No. consist of four figures*, the three first are to be found as before in the side column under N; and under the 4th at the top will be found the logarithm required, to which prefix the index as before. Thus log. 7218 is 3.858417. If the No. be odd, and ∴ not contained in the Table, take the difference of the logs. of the Nos. next greater and less than the given one; and add ½ this difference to the less log. Thus if log. 7217 were required, we have by Table

Log. 7218 ............................ 3.858417
Log. 7216 ............................ 3.858297
120

the ½ of which, or 60, added to 3.858297 gives 3.858357, the log required.

*If the No. consist of* 5 *figures or more*, find the difference between the logs. answering to the first four figures of the given No., and the next immediately following; multiply this difference by the remaining figures in the given number, strike off as many figures from the right hand as there are in the multiplier; and the remainder added to the log., answering to the first 4 figures, will be the log. required nearly. Thus if log. 100176 were required, we have by last case,

Log. 1001 ............................ 000434
1002 ............................ 000868
434

∴ 434 × 76 is 32984. From this cut off two figures, and it becomes 329.84 or 330 nearly. Whence to 000434 add 330 and supply the index, and we have the required log. = 5.000764.

2. *To find the natural No. corresponding to any given logarithm.*

Look in the different columns for the decimal part of the given log.; but if you cannot find it exactly, take the next less tabular log., and in a line with the log. found in the col. on the left marked N, you have three figures of the number sought, and at the top of the column in which the log. is, you have one figure more, which annex to the other three. As, however, the Table contains only the logs. of the even Nos., it should be observed that if the given log. falls between any two of the tabular logs, and differs considerably from both; in that case we must find the log. of the intermediate odd No. as directed above, and compare it with the given one; by which means the 4th figure of the No. sought (whether it be even or odd) may be correctly ascertained. The number of integers is always one more than the number expressed by the index. Thus the

No. answering to 2.993789 is 985.8. If the number be required to a greater No. of places than four, find the difference between the given and the next less log. To this annex on the right hand as many ciphers as there are figures required above four. Divide the whole by the difference between the next less and next greater log., and the quotient annexed to the four figures formerly found will be the natural number required. Thus required the No. to 6 places answering to the log. 4.687956. The nearest less log. than this is 687886 corresponding to which is the No. 4874. The difference between 687956 and 687886 is 70, to this annex 2 ciphers and it becomes 7000, which being divided by 89, the difference between the next less and next greater log. gives 79, ∴ the number required is 48747.9.

## TABLE II.

1. *To find the logarithmic sine, cosine, &c. answering to any given degree or minute.*

Find the given degrees at the top of the page, if less than 45°, and the minutes in the left hand column; opposite to which, and under the word sine, cosine, &c. is the number required. But if the given degrees be greater than 45° and less than 90°, find them at the bottom, and the required sine, cosine, &c. will be found above the word sine, cosine, &c. opposite to the given number of minutes in the *right* hand column. If the given arc exceed 90°, find the sine, cosine, &c. of its supplement. Thus the log. sine of 23°. 28′ is 9.600118; and the cotangent of 55°. 57′ is 9.829805. If the No. of minutes be odd, and ∴ not contained in the Table, proceed as directed for the odd numbers, Table I.

*To find the logarithmic sine, tangent, &c. of an arc expressed in degrees, minutes, and seconds.*

Find the sine, tangent, &c. corresponding to the given degree and minute, and also that answering to the next greater minute; multiply the difference between them by the given number of seconds, and divide the product by 60; then the quotient added to the sine, tangent, &c. of the given degree and minute, or subtracted from the cosine, cotangent, &c. will give the quantity required nearly.

*Ex.* Required the log. sine of 23°. 27′ 40″.

Log. sin. 23° 27′ ...................... 9.599827
23 28 ...................... 9.600118
Difference ......... 291

which multiplied by 40, and divided by 60, gives 194, and this added to 9.599827 gives the required logarithm 9.600021.

2. *To find the degrees and minutes answering to any given logarithmic sine, tangent, &c.*

Find the nearest log. to that given in the proper column: if the title be at the top of the column, you have the number of degrees at the top of the page, and the minutes in the column on the left hand; but should the title be at the bottom of the column, you have the degrees at the bottom of the page, and the minutes in the column on the right hand. If the given log. seems to belong to the odd minutes, proceed as directed Art. 2. Table I. Thus log. sin. 9.457584 answers to 16°. 40′. Log. tan. 10.535401 answers to 73°. 45′. But if the seconds in the arc are also required, we seek in the proper column for the logarithm which is next less than the given one, when the logs. in the column are increasing; but next greater, when they are decreasing, and take the degrees and minutes corresponding to that logarithm for the degrees and minutes in the required arc. Then to the difference between the logarithm so found and the given log. we annex two ciphers, and divide the result by $\frac{5}{3}$ of the difference between the next less and next greater log.; and the quotient is the seconds to be added to the degrees and minutes before taken out.

*Ex*. Required the degrees, minutes, and seconds corresponding to the log. sin. 9.641357.

The sin. 25°. 58′. is 9.641324 which is the log. *next less* than the given one. The difference of these two logs. is 33, which by adding two ciphers becomes 3300, and this divided by $\frac{5}{3}$ of 260, or by 433, gives 8 nearly for the number of seconds; ∴ required arc is 25°. 58′. 8″.

When the arc is small, a particular process is necessary as follows:—

*To find the log. sine of a small arc less than 3°.*

Add 4.685575 to the common log. of the arc reduced to seconds; from the sum subtract one-third of the log. secant less radius of the arc, and the remainder will be the required log. sine.

*To find the log. tangent of a small arc.*

Add together the common log. of the arc, reduced to seconds, ⅔ of the log. secant less radius of the arc, and 4.685575; and the sum will be the required tangent. We have hence the following rules for performing the reverse operations:—

*To find a small arc whose log. sine is given.*

To ⅓ of the log. secant of the arc in the Table, whose log. sine most nearly corresponds with the given log. sine, add the given log. sine, and 5.314425, and the sum will be the common log. of the seconds in the required arc.

*To find a small arc when its log. tangent is given.*

To the log. tangent add 5.314425, and from the sum subtract ⅔ of the log. secant of the arc in the Table, whose tangent most nearly agrees with the given tangent; and the remainder will be the log. of the seconds in the required arc.

*Ex*. 1. Required the log. sine of 1°. 28′. 13″. or the log. cosine of 88°. 31′. 47″.

1°. 28′. 13″ = 5293″ ................ log. 3.723702
Constant No. .................... 4.685575
8.409277
⅓ log. secant 1°. 28′ ............... sub. .000047
1°. 28′. 13″. log. sine ...................... 8.409230

*Ex*. 2. Required the arc to the log. sine 7.963214.

⅓ log. sec. 0°. 32′ ........................ .000006
7.963214
Constant No. ........................ 5.314425
1895″ ................................ log. 3.277645

Whence the required arc is 31′. 35″

Hence the arc to log. cosine 7.963214 is 89°. 28′. 25″.

FINIS.

# ERRATA.

Page 16. line 3. for Young's *read* Young.
P. 21. l. 26. This series is the same as the last, the higher powers of being neglected.
P. 22. l. 7. for *y* *read* *y* (= equation of the centre.)
P. 55. l. 5. for $A + SA^2 + B + SB^2 + C + SC^2 +$ &c. *read* $A \times SA + B \times SB^2 + C \times SC^2 +$ &c.
P. 86. l. 3. for spheriod *read* spheroid.
P. 88. l. 16. for with *read* of.
P. 107. March 7, *read* 11. 14; April 14, *read* 0. 14; June 13, *read* 0. 21
P. 147. In some copies the Figure has been inverted by mistake.
P. 169. l. 20. for .43424918 *read* .43429448.
P. 175. l. 21. for mix. *read* min.
P. 252. l. 26. for $2g \times Ws - Wv^2$; *read* $2g \times Ws = Wv^2$.
P. 273. l. 11. for Berege *read* Barege.
P. 302. Art. Thermometer, for Centrigrade *read* Centigrade.

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